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Obtaining base edge correspondence in stereo images via quantitative measures along C-diagonals

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Abstract

We propose a novel approach to solving the object edge correspondence problem for stereo images. For an object placed on a calibration plate (C-plate), the proposed approach first obtains the correspondences of the object edges lying on the C-plate (the base edges) via quantitative measures of locations of the intersections of the extended line of the edges and the diagonals of the C-plate (C-diagonals) using cross ratios. The measures are viewpoint invariant for an object base edge, and are expressed in the number of image pixels. Special cases which need only coarse calculations, as well as those which require extra measures for additional point features, are also considered. The proposed approach requires 2-D image data only, and is robust in the presence of errors in the feature detection. Experimental results are presented for polyhedral and curved objects to demonstrate the effectiveness of the proposed approach. © 1997 Elsevier Science B.V.

Keywords: Stereo vision; Feature correspondence; Calibration plate; Viewpoint invariant; Cross ratio; Quantitative measure

1. Introduction

In stereo vision, reconstructing the 3-D information of an object requires finding a feature correspondence from two perspective views of the object. Techniques for solving the correspondence problem can be divided into two categories (Barnard and Fischler, 1982; Dhond and Aggarwal, 1989): area-based stereo techniques and feature-based stereo techniques. Since area-based stereo techniques are sensitive to the change of the perspective distortions and intensity, and are slower, most researchers use the feature-based stereo techniques (Leu and Pherwani, 1989; Horaud and Skordas, 1989; Grosso et

al., 1989; Tu and Dubuisson, 1990; Boraie and Sid-Ahmed, 1992; Lee and Leou, 1994), which choose image features such as corner points, junctions, line segments, etc., as primitives for correspondence matching. Most feature-based stereo techniques employ the epipolar line approach (Leu and Pherwani, 1989; Horaud and Skordas, 1989; Tu and Dubuisson, 1990; Boraie and Sid-Ahmed, 1992; Lee and Leou, 1994) such that a designated feature in one image is matched with features on the epipolar line in the other image. Although the epipolar line approach searches corresponding features in a 2-D image, the approach needs to use the 3-D information of two camera lens centers to construct an epipolar plane before building the epipolar line of each feature and is thus inefficient.

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Lei (1990) presents a method to match planar polygons of more than four vertices in two images using the cross ratios. It is shown that the value of the cross ratio of each of the vertices is viewpoint invariant. Since the method does not use 3-D information, it is simple. The method can be used to match polyhedral objects with the requirement that at least one of the polyhedral faces has more than four vertices. For every possible pair of corresponding polyhedral faces from two images, the method must compute and cyclically match the cross ratios for every vertex. Therefore, it requires much computation time and matching frequency.

In this paper, we propose a novel approach to solving the vertex/edge correspondence in the stereo images. The images are obtained with an auxiliary planar calibration plate (C-plate) placed under the object. In the proposed approach, the C-plate vertex correspondence is first determined using cross ratios, and the result is then used to reduce the search space for determining object vertex/edge correspondence. The major task of this paper is to develop an effective procedure to determine the correspondence of object edges lying on the C-plate (called the object base edges). The correspondence of other object edges can then be found straightforwardly using the approach proposed in (Chiu et al., to appear) which only requires the correspondence of a single matched base edge pair. The approach first obtains the additional correspondence of object edges, in the clockwise (or counterclockwise) order, of the object face containing the matched base edge pair in the stereo images. The process is then continued for the object faces containing the newly matched object edges until all the object faces visible from both of the stereo images are considered.

Since a visible object base edge must be an object boundary edge in an image, only object boundary edges will be considered for possible base edge correspondences. To that end, simple geometric relationships between an object boundary edge and some C-plate features are examined to coarsely detect invalid correspondences. These relationships include (i) the partitioning of the endpoints of the object edge by the C-diagonals, and (ii) the partitioning of the intersections of the C-diagonals by the extended line of the object edge (see Section 3 for details).

For a base edge of the object, the above simple

relationships remain unchanged in different images. However, such a statement is not always true for a non-base object feature, whose location in an image corresponds to the image of its perspective projection on the C-plate (called C-projection) with the viewpoint as the projection center. For each object edge pair having the above geometric relationships unchanged from different views, the proposed approach develops a way to measure the difference in the associated C-projections quantitatively along the C-diagonals to identify a correspondence involving a non-base object edge.

The remainder of this paper is organized as follows. Section 2 discusses the determination of C-plate vertex correspondences using cross ratios. Section 3 describes the procedure of coarse screening for possible object base edge correspondences according to relationships (i) and (ii) mentioned above. Section 4 presents the procedure for finding object base edge correspondences via some quantitative measures based on cross ratios. Section 5 illustrates the implementation of our method and presents some experimental results. Some concluding remarks are given in Section 6.

2. Determination of C-plate vertex correspondence

To determine the vertex correspondence of the C-plates in two images, it is assumed that the number of vertices of the C-plate is greater than or equal to five. Thus, the method proposed by Lei (1990) can be applied. Consider the polygon shown in Fig. 1, according to Lei (1990), the cross ratio of V_1 is viewpoint invariant and can be computed as

$$R_1 = \frac{\sin(\theta_1 + \theta_2) \sin(\theta_2 + \theta_3)}{\sin(\theta_2) \sin(\theta_1 + \theta_2 + \theta_3)},$$

where θ_1 , θ_2 and θ_3 are angles formed by the rays originating from V_1 and pointing toward its four neighboring vertices (V_2 , V_3 , V_4 and V_5) arranged in the counterclockwise (or clockwise) direction, with $\sum_{i=1}^3 \theta_i < 180^\circ$. Let R_j^i be the cross ratio of the j th vertex, V_j^i , of the C-plate in the i th image, $i = 1, 2$; $j = 1, 2, \dots, V \geq 5$. For the vertex sequence $\{V_j^1 \mid 1 \leq j \leq V\}$, the vertex sequence $\{V_{\langle j+k \rangle}^2 \mid 1 \leq$

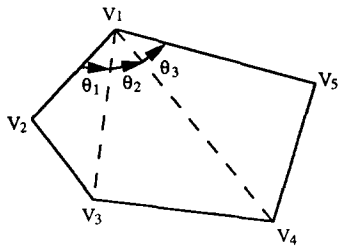


Fig. 1. Three angles of a pentagon formed by V_1V_2 , V_1V_3 , V_1V_4 and V_1V_5 for the cross ratio calculation.

$j \leq V, 1 \leq k \leq V - 1$, $\langle j + k \rangle = (j + k)_{\text{mod } V}$, whose corresponding cross ratio sequence satisfies the following criterion is identified as the matched vertex sequence:

$$\min_k \sum_{j=1}^V |R_j^1 - R_{\langle j+k \rangle}^2|^2.$$

It is shown in the following that if we first determine the vertex correspondence of the C-plates in the two images by using cross ratios, the correspondence, as an auxiliary information, can speed up the procedures for finding the base vertex/edge correspondence by reducing the search space of the matching process.

3. Coarse screening for possible object base edge correspondences

Assume that an object is placed on the C-plate and the boundary of the C-plate is not occluded by the object. It is obvious that any visible object base edge has to be a boundary edge of the object in an image. Thus, only these boundary edges will need to be considered in the determination of base vertex/edge correspondence. For example, Fig. 2

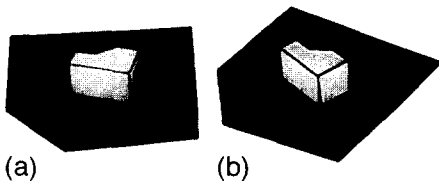


Fig. 2. Two views of a polyhedron placed on the C-plate. (a) Image 1. (b) Image 2.

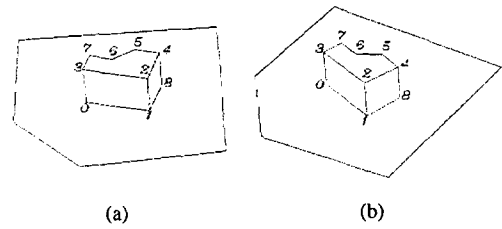


Fig. 3. Edges extracted from (a) Image 1 and (b) Image 2 of Fig. 2.

shows two images of a polyhedron placed on the C-plate. The extracted object edges are shown in Fig. 3 with endpoints marked with identification numbers. The boundary edges, represented in counter-clockwise vertex pairs, include (0, 1), (1, 8), (8, 4), (4, 5), (5, 6), (6, 7), (7, 3) and (3, 0). Among these boundary edges, edges (0, 1), and (1, 8) are visible base edges.

In the proposed approach, the C-plate is used in the development of effective algorithms for finding the object base vertex/edge correspondences in stereo images. In addition to the C-diagonals, with the correspondences of the C-plate vertices obtained in the previous section, the following C-plate features can also be defined in each of the images:

1. *C-vertices*: the intersections of the C-diagonals, including the C-plate vertices.
2. *C-partitions*: the partitions of the C-plate by the C-diagonals.

For example, for the pentagonal C-plate as shown in Fig. 2, there are ten C-vertices and eleven C-partitions.

For non-occluded object base vertices/edges, the following geometric relationships with the above C-plate features will remain unchanged in images obtained from different views: (i) the C-partition to which an object base vertex belongs, and (ii) the partitioning of the C-vertices by the extended line of an object base edge. For a non-base object feature, on the other hand, its location in an image corresponds to the image of its C-projection with the viewpoint as the projection center. Since their C-projections vary with different viewpoints, non-base object features may not have the above geometric relationships unchanged in different images. Accordingly, for the coarse screening of possible object base vertex/edge correspondences, the stereo im-

ages of an object boundary edge are analyzed to see if the following conditions are satisfied:

C1. Each of its two endpoints belongs to the same C-partition in the two images.

C2. Its extended line partitions the C-vertices in the same way in the two images.

For simplicity, an object edge which is collinear with a C-vertex, or one of its endpoints is collinear with a C-diagonal, is removed from the candidates for possible base edge correspondence.

For a non-base object boundary edge, if either (or both) of the above tests fails, the edge will not be considered further for possible object base edge correspondence. However, if the difference in the C-projections of a non-base object edge is not large enough in two views, the above two conditions will still be satisfied. More accurate, and quantitative, measures for such a difference can be developed to resolve the problem, as discussed next.

4. Finding object base edge correspondences via quantitative measures

In this section, quantities which are viewpoint invariant for an object base edge are established to measure the difference in the C-projections of a non-base object edge due to images obtained from different views. Consider the C-projection, $\overline{A_1B_1}$, of a non-base \overline{AB} due to viewpoint O_1 . The extended line of $\overline{A_1B_1}$ corresponds to the intersection of the plane containing the C-plate and the plane containing \overline{AB} and O_1 (called projection plane P_1). For a second viewpoint $O_2 \neq O_1$, the C-projection of \overline{AB} , $\overline{A_2B_2}$, will not coincide with $\overline{A_1B_1}$. The difference between $\overline{A_1B_1}$ and $\overline{A_2B_2}$ can be analyzed for the following two cases: (i) O_2 is not coplanar with P_1 , and (ii) O_2 is coplanar with P_1 .

For (i), the extended line of $\overline{A_2B_2}$ corresponds to the intersection of the plane containing the C-plate and the plane containing \overline{AB} and O_2 (called projection plane P_2). Since $P_1 \neq P_2$, the extended lines of $\overline{A_1B_1}$ and $\overline{A_2B_2}$ are different. Therefore, at least one of the C-plate diagonals (C-diagonals) will intersect the two lines with two different intersection points

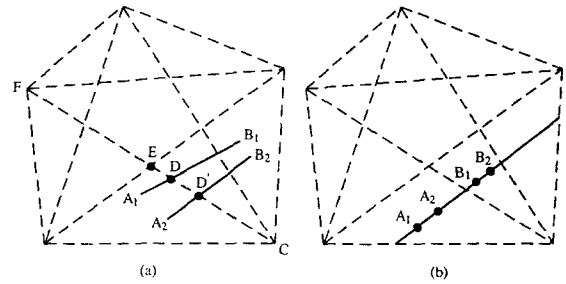


Fig. 4. Non-coincident C-projections of a non-base object edge. (a) Non-collinear C-projections. (b) Collinear C-projections.

(see C-diagonal \overline{CF} in Fig. 4(a)). For (ii), the extended line of $\overline{A_2B_2}$ is identical to that of $\overline{A_1B_1}$. However, since $O_1 \neq O_2$, we have (see Fig. 4(b))

$$\max(|\overline{A_1A_2}|, |\overline{B_1B_2}|) > 0. \quad (1)$$

In the following subsections, measures for the difference in the C-projections of a non-base object edge due to different viewpoints, which do not exist in theory for an object base edge, are developed. Object edge correspondences with negligible differences in these measures are considered as valid base edge correspondences.

4.1. Identification of a non-base object edge whose C-projections in the stereo views are non-collinear

Consider the situation shown in Fig. 4(a) wherein the two C-projections of object edge \overline{AB} , $\overline{A_1B_1}$ and $\overline{A_2B_2}$, are non-collinear. At least two C-diagonals will intersect the extended lines of $\overline{A_1B_1}$ and $\overline{A_2B_2}$. Furthermore, unlike the situation with an object base edge, at least one of the C-diagonals will have different intersections with the lines containing $\overline{A_1B_1}$ and $\overline{A_2B_2}$, respectively. Therefore, to determine if $\overline{A_1B_1}$ and $\overline{A_2B_2}$ are associated with a corresponding base edge pair in the stereo images, one only needs to examine if the extended lines of $\overline{A_1B_1}$ and $\overline{A_2B_2}$ have identical intersection points with each C-diagonal.

In order to determine whether the extended lines of the C-projections of an object edge in different images intersect a C-diagonal at the same location, the cross ratio (Duda and Hart, 1973) can be used. Let \overline{CF} be a C-diagonal, D be the intersection of \overline{CF} and the extended line of the C-projection of an

object edge due to viewpoint O_1 , and E be a fixed point on \overline{CF} (e.g., one of the two intersection points of \overline{CF} and two other C-diagonals), as shown in Fig. 4(a). If the object edge is a base edge, D is also a fixed point on \overline{CF} , and the cross ratio

$$R_1 = \frac{|\overline{CE}| \cdot |\overline{DF}|}{|\overline{DE}| \cdot |\overline{CF}|} \equiv \frac{|\overline{CE}| \cdot y}{x \cdot |\overline{CF}|} \quad (2)$$

is the same from different views. That is,

$$R_1 = \frac{|\overline{C_1E_1}| \cdot |\overline{D_1F_1}|}{|\overline{D_1E_1}| \cdot |\overline{C_1F_1}|} = \frac{|\overline{C_2E_2}| \cdot |\overline{D_2F_2}|}{|\overline{D_2E_2}| \cdot |\overline{C_2F_2}|}, \quad (3)$$

where subscript i indicates quantities measured in image obtained from O_i , $i = 1, 2$.

On the other hand, if the object edge is not a base edge, a second viewpoint, say O_2 , may result in an intersection, $D' \neq D$, of \overline{CF} and the extended line of the corresponding C-projection of the object edge, similar to the situation shown in Fig. 4(a). The corresponding cross ratio calculated for the image of the object edge obtained from O_2 is equal to

$$R_2 = \frac{|\overline{C_2E_2}| \cdot |\overline{D_2F_2}|}{|\overline{D_2E_2}| \cdot |\overline{C_2F_2}|} = \frac{|\overline{CE}| \cdot |\overline{D'F}|}{|\overline{D'E}| \cdot |\overline{CF}|} = \frac{|\overline{CE}| \cdot (y + \Delta x)}{(x + \Delta x) \cdot |\overline{CF}|}, \quad (4)$$

where Δx is the displacement from D to D' , i.e., $\Delta x > 0$ if $D' \in \overline{CD}$ and $\Delta x < 0$ if $D' \in \overline{DE}$. ($D' \in \overline{CE}$ if the object edge has passed the coarse screening.) Let $R^* = R_1/R_2$; we have

$$R^* = \frac{y(x + \Delta x)}{x(y + \Delta x)} \quad (5)$$

and

$$\Delta x = \frac{(R^* - 1)xy}{y - R^*x}. \quad (6)$$

One can show that the denominator of (6) is always greater than zero.

In practice, R_1 (R_2) can be calculated directly from one of the stereo images using the images of C , D (D'), E and F . However, x and y are the lengths of the C-projections of \overline{DE} and \overline{DF} , respectively, and can not be measured directly from the stereo images;

therefore, the displacement Δx can not be calculated using (6). Alternatively, one can calculate

$$\Delta X^i = \frac{(R^* - 1)X^iY^i}{Y^i - R^*X^i}, \quad i = 1, 2, \quad (7)$$

where $X^i = |\overline{D_iE_i}|$ and $Y^i = |\overline{D_iF_i}|$ can be calculated in the image obtained from O_i , and $|\Delta X^i|$ is equal to the length of $\overline{DD'}$ measured in that image. For the examples presented in this paper, the displacement Δx is estimated using one of the images which results in a larger $|\Delta X^i|$ in (7), i.e., $|\Delta X| \equiv \max(|\Delta X^1|, |\Delta X^2|)$. Such an estimation can prevent the displacement from being overlooked when one of the viewpoints, say O_j , is almost collinear with $\overline{DD'}$, that is, $|\Delta X^j| \cong 0$.

In order to improve the numerical stability of the calculations associated with (3) and (4), point E can be chosen from one of the two intersection points of \overline{CF} and the two C-diagonals such that extremely small values of $|\overline{D_1E_1}|$ and $|\overline{D_2E_2}|$ in (3) and (4), respectively, can be avoided. Accordingly, the situation shown in Fig. 4(a) can be identified for an object edge correspondence in the stereo images involving a non-base object edge with the following procedure.

Procedure Collinearity-Check

- Step 1.* Identify the C-diagonals, d_i , $1 \leq i \leq N$, which intersect with the extended line of the object edge in the images. (For a pentagonal C-plate, $N = 2$ or 4.)
- Step 2.* Along each d_i , select one of the two intersection points with the other two C-diagonals that is located farther away from the intersection point obtained in Step 1.
- Step 3.* Calculate $|\Delta X_i|$, along each d_i using one of the images which results in a larger $|\Delta X|$ in (7).
- Step 4.* Sort the $|\Delta X_i|$ such that $|\Delta X_i| \leq |\Delta X_j|$ if $i < j$.
- Step 5.* If $|\Delta X_1| + |\Delta X_2| \geq T_1$, conclude that the two object edges in the stereo images do not form a base edge correspondence.

The selection associated with Step 2 can be carried out by taking both of the stereo images into account. Nonetheless, a simpler implementation is adopted for the examples presented in this paper which makes the selection using just one of the

images. In Step 5, $|\Delta X_1| + |\Delta X_2| < T_1$ is allowed for base edge correspondence to accommodate possible noise in the calculations. Larger $|\Delta X_i|$ are not used for $N = 4$, since they are resulting from smaller intersection angles between the C-diagonals and the extended line of the object edge in an image, and are thus more sensitive to the error in feature extraction.

For $N = 4$, it is possible for $|\Delta X_1| + |\Delta X_2| < T_1$ with ΔX_1 and ΔX_2 corresponding to the intersections of the extended line of the object edge with two C-diagonals, respectively, near the intersection of the two C-diagonals, i.e., one of the corresponding C-vertices. The difference in the two non-collinear C-projections of the object edge due to two different views will not be measured properly using ΔX_1 and ΔX_2 . Such a problem can be resolved by using ΔX_3 in place of ΔX_2 in Step 5.

4.2. Identification of a non-base object edge whose C-projections in the stereo views are collinear

In some rare situations, the stereo images could be obtained from two views such that the extended lines of the C-projections of a non-base object edge coincide, e.g., $\overline{A_1 B_1}$ and $\overline{A_2 B_2}$ in Fig. 4(b). The procedure presented in the previous subsection will not indicate that the edge is not a base edge. An additional procedure is developed in the following to resolve such a problem.

According to (1), the collinear C-projections of the object edge will have differences in the location for at least one of their endpoints. Since the extended line of these C-projections will intersect the boundary of the C-plate as well as at least two C-diagonals at fixed locations, cross ratios can be used to find the above differences similar to that discussed in the previous subsection. Accordingly, the following procedure can be carried out to identify a non-base edge in stereo images whose two C-projections are collinear.

Procedure Coincidence-Check

Step 1. Identify the two C-diagonals, d_i , $i = 1, 2$, each having almost identical intersection with the extended line of the C-projections of the object edge, i.e., the two d_i used in Step 5 in the previous subsection.

Step 2. Along the extended line of the object edge in each of the stereo images, identify the two intersection points with the C-plate boundary.

Step 3. For each endpoint of the object edge, identify one of the two intersection points with the above d_i which is farther away from that endpoint.

Step 4. Calculate the displacements $|\Delta X_A|$ and $|\Delta X_B|$, separately, for each endpoint of the object edge in the stereo images (cf. $|\overline{A_1 A_2}|$ and $|\overline{B_1 B_2}|$ in Eq. (1)), similar to that done in (7).

Step 5. If $\max(|\Delta X_A|, |\Delta X_B|) > T_2$, conclude that the object edge pair does not correspond to a base edge correspondence.

4.3. Identification of other erroneous correspondences

Although the discussion of the previous two subsections are concerned with the identification of the images of the same non-base object edge, other erroneous object edge correspondences not being identified by the aforementioned coarse screening procedure can be detected as well. These erroneous correspondences include:

1. The correspondence between a non-base object edge with a base edge for the case where the C-projection of the former does not coincide with the latter.
2. The correspondence between two non-base object edges whose C-projections do not coincide.

On the other hand, if the C-projection of a non-base object edge almost coincides with an object base edge (or the C-projection of another non-base object

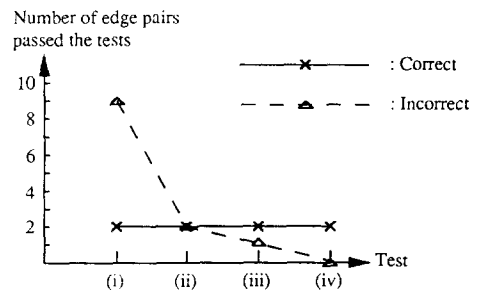


Fig. 5. The number of correct and incorrect base edge correspondences which pass the consecutive tests for (i) condition C1 and (ii) condition C2 in Section 3, (iii) procedure Colinearity-Check and (iv) procedure Coincidence-Check, based on $T_1 = 4$ pixels and $T_2 = 8$ pixels. The results are obtained for examples shown in Fig. 3.

Table 1

The displacement measures of each object edge pair shown in Fig. 3 which has passed Tests (i) and (ii)

Displacement measure	Edge pair			
	$(0, 1)^1 / (0, 1)^2$	$(1, 8)^1 / (1, 8)^2$	$(5, 6)^1 / (5, 6)^2$	$(6, 7)^1 / (4, 5)^2$
$ \Delta X_1 + \Delta X_2 $	0.14	1.80	4.19	0.21
$\max(\Delta X_A , \Delta X_B)$	1.32	6.36	15.77	22.55

Note: Boldface indicates displacements larger than the thresholds allowed for a matched base edge pair.

edge), the proposed approach will generate incorrect base edge correspondences. The situation can only occur if the former is coplanar with the latter, and they are edges of a convex polygon with their endpoints as vertices. It is easy to show that if the polygon is a tetragon (triangle), the above ambiguity can only occur at a single viewpoint (viewpoints along a line). Additional images obtained from viewpoints away from these special locations can be used to resolve the problem.

5. Experimental results

In this section, experimental results for the determination of base edge correspondences for an object in two images are presented. For an object placed on the C-plate, two images are taken from two different viewing angles, denoted as Image 1 and Image 2 of the object, respectively. Fig. 2 shows the 512×512 images of the polyhedron with edges of the polyhedron marked with dark lines.

Once stereo images of an object are obtained, the lines are obtained through line fitting of manually selected edge points for the object as well as the

C-plate. Fig. 5 shows the numbers of correct (and incorrect) base edge correspondences obtained for the above examples which pass the consecutive tests for (i) condition C1 and (ii) condition C2 in Section 3, (iii) procedure Collinearity-Check, and (iv) procedure Coincidence-Check, respectively, based on $T_1 = 4$ pixels and $T_2 = 8$ pixels. Both T_1 and T_2 are empirical values. Table 1 shows the displacement measures used in Tests (iii) and (iv), respectively, calculated for object edges shown in Fig. 3 which have passed Tests (i) and (ii). The correct base edge correspondences include $(0, 1)^1 / (0, 1)^2$ and $(1, 8)^1 / (1, 8)^2$, where the superscript i of each vertex pair indicates the image from which the edge is obtained.

While a larger value is used for T_2 , a smaller value is used for T_1 , which can be justified as follows. In procedure Collinearity-Check, the locations of (i) the intersections of the lines containing edges of C-plates, i.e., the C-plate vertices, (ii) the intersections of C-diagonals with other C-diagonals, and (iii) the intersections of C-diagonals with the extended line of the object edge under consideration are required. *Smaller* quantization errors, due to line extraction, in the calculation of (ii) are expected

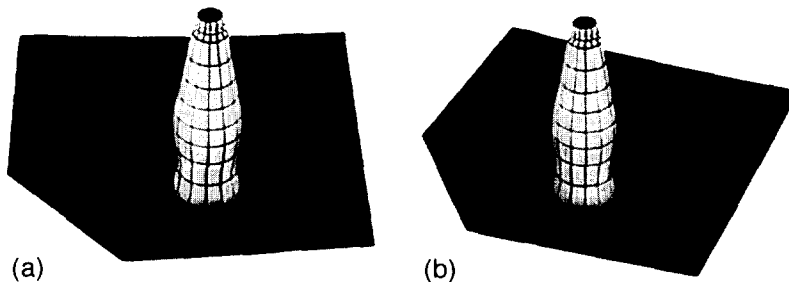


Fig. 6. The two views of the bottle-shaped object placed on the C-plate with identification numbers marked for some vertices. (a) Image 1. (b) Image 2.

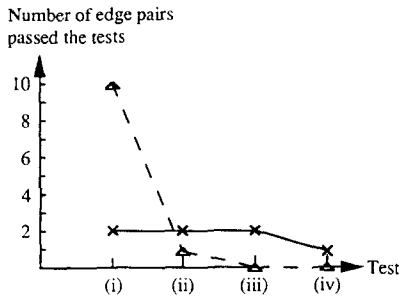


Fig. 7. The base edge correspondence results, similar to that shown in Fig. 5, obtained for selected object edges shown in Fig. 6.

compared with that for (i), since (ii) are in effect obtained by interpolating the locations of the C-plate vertices. In procedure Coincidence-Check, on the other hand, the locations of the intersections of the extended line of the object edge with (i*) C-plate boundary, (ii*) some C-diagonals, and (iii*) other extended lines of object edges, which give the two end points of the object edge under consideration, are required. It is obvious that the calculation of the locations associated with (ii*) is identical to that for (iii). However, the calculations of the locations associated with (ii*) and (iii*) will in general have *larger* quantization errors than that for C-plate vertices. This is because the object edges are always shorter than the edges of the C-plate for the examples considered in this paper, and thus have lower accuracy in the corresponding line extraction.

The proposed approach also works for curved objects. Fig. 6 shows the image of the curved object with dark grid lines pasted on its surface, and with endpoints of selected object edges marked with identification numbers. In practice, the grid lines can be generated by casting a grid pattern onto the object surface using a structured light (see also Chen et al., 1993). Fig. 7 shows the numbers of base edge correspondences obtained for these edges similar to that shown in Fig. 5. There is only a single correctly matched base edge pair, i.e., $(1, 2)^1 / (0, 1)^2$. Because the edges of curved objects are much shorter, the corresponding line fittings are more noise sensitive than that for simple polyhedra, resulting in some false rejections of actual base pairs. (A similar situation may occur if low-contrast images are employed.) Such false rejections of actual base edge pairs are

acceptable in this case since only one correct base edge pair is needed in finding other object edge correspondences.

6. Conclusion

In this paper, we have presented a novel approach which uses the extra information provided by a calibration polygon, the C-plate, to solve the object base edge correspondence problem. Such a correspondence problem is important in that a matched base edge pair can be used to find other object edge correspondences in a straightforward manner. We first determine the C-plate vertex correspondence in two images by comparing the cross ratios of C-plate vertices. The result is then used to simplify the subsequent procedure for finding the object base edge correspondence. For an object base edge, the partitioning of the endpoints of the object edge by the C-diagonals and the partitioning of the intersections of the C-diagonals by the extended line of the object edge are viewpoint invariant. The non-base object edges which do not possess these viewpoint invariant properties in stereo images can thus be ruled out from the candidates of the base edge correspondences. For each of the remaining object edge pairs, the proposed approach develops a way to measure the difference in their C-projections quantitatively to identify and delete a correspondence involving a non-base object edge.

The proposed approach has several advantages. First, only the 2-D image data is needed in determining the object base edge correspondence. Second, the number of vertices of each object face can be less than five, and the quantization errors in the location of object features can be absorbed partially. Third, the quantitative measures are numerically stable and are expressed in the number of image pixels, which makes the selection of the thresholds for the determination of base edge correspondences convenient. Finally, the ambiguity in the feature correspondence, if any, only occurs at special viewpoints for a non-base object edge whose C-projection coincides with (i) an object base edge or (ii) the C-projection of another non-base edge. The proposed approach in fact works for two or more objects lying on the C-plate as long

as the complete boundary of C-plate can be derived and at least one of the base edges of each object are visible in the stereo images.

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