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Research

Robust Design for Multiple Dynamic Quality Characteristics Using Data Envelopment Analysis

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The Taguchi method is extensively adopted in various industries to continuously improve product design in response to customer requirements. The dynamic system of the Taguchi method is frequently implemented to design products with flexible applications. However, Taguchi's dynamic system can be employed only for individual quality characteristic, and the relationship between the quality characteristic and the signal factor is assumed to be linear. Because of these restrictions, Taguchi's dynamic system is ineffective for multiple quality characteristics or when the quality characteristic has a nonlinear relationship with the signal factor. This study describes a novel procedure for optimizing a dynamic system based on data envelopment analysis. The proposed procedure overcomes the limitations of Taguchi's dynamic system. Two cases are analyzed to demonstrate the effectiveness of the proposed procedure. The results show that the proposed procedure can enhance multiple dynamic quality characteristics.

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KEY WORDS: Taguchi method; dynamic system; data envelopment analysis; multiple quality characteristics; robust design

1. INTRODUCTION

The Taguchi method is extensively adopted to improve industrial/manufacturing procedures and product quality. The Taguchi method uses parameter design to determine a robust, optimal factor-level combination that can intervene in noise factors. Noise factors produce product/process variances. These factors are difficult or costly to control. The Taguchi method includes static and dynamic systems. A dynamic system contains signal factors for expressing the intended output. The relationship between a quality characteristic and a signal factor is assumed to be linear. In other words, the quality characteristic is a linear function of the signal factor. However, product/process design tends to be rather complex to comply with constantly changing customer requirements and production technology. Multiple responses

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must be evaluated simultaneously to determine overall product/process quality. Accordingly, the linear relationship is in fact rarely achieved in the current complex product/process design. Such a scenario limits the application of the conventional Taguchi method. Product/process design with a dynamic system offers the flexibility needed to satisfy customer requirements than that of a static system and can enhance a manufacturer's competitiveness, particularly in an extremely competitive market. Therefore, quality improvement involving a dynamic system is an important issue.

Recent studies of dynamic system optimization, including Miller and Wu[1], Miller and Wu[2], Wasserman[3], Lunani *et al.*[4], McCaskey and Tsui[5], Chen[6], and Bae and Tsui[7], have noted some of the limitations of Taguchi's dynamic system. Wasserman[3] observed that the factor-level combination of a dynamic system using Taguchi's signal-to-noise ratio (SNR) might not be optimal. Lunani *et al.*[4] noted that using SNR as a quality performance measure might produce inaccuracies due to a biased dispersion effect, thus making it impossible to minimize quality loss. Accordingly, Lunani *et al.*[4] combined half-normal graph schemes with analysis of variance to replace SNR to alleviate the drawbacks of the Taguchi method. Lunani *et al.*[4] thereby determined the vital factors affecting the dispersion effect in a dynamic system. Chen[6] developed a stochastic optimization modeling procedure that incorporated a sequential quadratic programming technique to determine the optimal factor-level combination in a dynamic system. The above studies focused on optimizing single-response problems in a dynamic system.

Given that multiple responses must be simultaneously evaluated to enhance product/process quality, several studies have recommended procedures for optimizing multiple responses in a dynamic system. For instance, Tong *et al.*[8] adopted principal component analysis and technique for order preference by similarity to ideal solution to optimize a dynamic system with multiple quality characteristics. Hsieh *et al.*[9] applied regression analysis and desirability function to optimize a dynamic system with multiple quality characteristics. Su *et al.*[10] also employed a desirability function to optimize dynamic multiple quality characteristics. Additionally, determining the optimal factor-level combination with the Taguchi method is restricted to the initially designed factor levels. However, an optimal factor-level combination might fall in the interval of factor levels when the factor is continuous. Vining and Mylers[11] discussed optimization of a product/process involving continuous factors by employing a response surface methodology. Tong *et al.*[12] employed a dual response surface methodology to optimize a dynamic multi-response problem with continuous factors.

While focusing mainly on a dynamic system with multiple quality characteristics, this study develops a novel optimization procedure using data envelopment analysis (DEA). The proposed procedure does not require any specific relationship between quality characteristics and signal factors, allowing it to efficiently determine an optimal factor-level combination in a dynamic system. In other words, the developed procedure is more practical than procedures that require certain assumptions. The relative efficiencies of the location effect and dispersion effect for a dynamic system are initially estimated using DEA. Therefore, overall relative efficiencies are obtained as an overall quality performance (OQP) of a dynamic system with multiple responses. A prediction model of OQP on design factors is then established, and the optimal factor-level combination in the feasible region of design factors is thus determined. Finally, two cases are utilized to demonstrate the effectiveness of the proposed procedure.

2. DYNAMIC SYSTEM OF THE TAGUCHI METHOD

Without advanced or costly equipment, the Taguchi method can obtain an optimized parameters setting, which is robust to interference with noise factors. Moreover, experiments allocating Taguchi's orthogonal array can reduce the quantity of experiments to lower the experimental cost. The parameter design of the Taguchi method includes both a static system and a dynamic system. These two systems differ in that the dynamic system includes signal factors to express intended outputs. In a dynamic system, a quality characteristic is valued according to the levels of the signal factor. Taguchi's dynamic system assumes a linear relationship between quality characteristics and signal factors. The linear function can be expressed as

$$Y = \beta M + \varepsilon \tag{1}$$

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where Y denotes the quality characteristic of a dynamic system, M represents the signal factor, β is the slope or sensitivity of the system, and ε denotes the random error.

Based on the linear function assumption, the Taguchi method uses linear regression to obtain β, the slope between the quality characteristic and the signal factor. Additionally, the deviation from the regression line of the quality characteristic represents the variability of the dynamic system (namely σ_e^2). Both β and σ_e^2 can be obtained using the following equations, respectively[13]:

$$\beta = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} M_i)}{\sum_{i=1}^{m} \sum_{j=1}^{n} M_i^2} \tag{2}$$

$$\sigma_e^2 = \frac{1}{mn-1} \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} - \beta M_i)^2 \tag{3}$$

where β represents the slope between the quality characteristic and the signal factor, and σ_e^2 indicates the variability of a dynamic system. M_i denotes the ith signal factor, $i = 1, \ldots, m$. y_{ij} is the quality characteristic with the combination of the ith signal factor, $i = 1, \ldots, m$, and the jth noise factor, $j = 1, \ldots, n$.

Taguchi applies the SNR as a quality performance measure. The SNR of a dynamic system is defined as in the following equation. A higher SNR implies a higher quality.

$$\eta = 10 \log \frac{\beta^2}{\sigma_e^2} \tag{4}$$

3. DATA ENVELOPMENT ANALYSIS

DEA can clarify the complex relationship between inputs and outputs. Therefore, DEA is effective for assessing the relative efficiency in a group of decision-making units (DMUs)[14]. Charnes *et al.*[15] introduced a DEA model based on the concept of weighted virtual inputs and outputs, referred to as the CCR model. Subsequently, Banker *et al.*[16] released the limitation of constant returns to scale in the CCR model and presented an alternative DEA model, referred to as the BCC model. Both models are widely used in practical applications.

The DEA model evaluates the relative efficiency of each DMU by using the ratio of weighted virtual output to weighted virtual input. Weights in the DEA model are obtained from the data rather than given in advance. Therefore, each DMU is assigned the optimum set of weights with values that may vary from one DMU to another. By selecting the weight that maximizes the relative efficiency value of the evaluated DMU, the relative efficiency value of the kth DMU can obtained using the following equation:

$$h_k = \frac{\sum_r u_r Y_{kr}}{\sum_i v_i X_{ki}} \tag{5}$$

where h_k denotes the relative efficiency value of the kth DMU, X_{ki} represents the ith input of the kth DMU, and v_i is the corresponding weight. Additionally, Y_{kr} denotes the rth output of the kth DMU, and u_r represents the corresponding weight.

The CCR model maximizes the relative efficiency value of the kth DMU by analyzing a reference set of DMUs. The CCR model is represented as

$$\begin{aligned} \text{Max} \quad & h_k = \frac{\sum_r u_r Y_{kr}}{\sum_i v_i X_{ki}} \\ \text{s.t.} \quad & \frac{\sum_r u_r Y_{qr}}{\sum_i v_i X_{qi}} \le 1, \quad q = 1, \ldots, L \\ & u_r \ge \varepsilon, \quad v_i \ge \varepsilon \end{aligned} \tag{6}$$

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where h_k denotes the relative efficiency value of the kth DMU; Y_{kr} and X_{ki} represent the rth output and the ith input, respectively, for the kth evaluated DMU; Y_{qr} and X_{qi} are the rth output and the ith input, respectively, for the qth DMU, $q=1,\ldots,L$; u_r denotes the rth output weight; v_i represents the ith input weight, and ε is a non-Archimedean constant, representing an extremely small positive value[14]. The first constraint in Equation (6) ensures that the relative efficiency value h_k does not exceed 1, whereas the second constraint restricts the CCR model to yield a positive weight.

As a nonlinear programming model may cause a complex procedure to derive a solution for Equation (6), Equation (6) can be transformed into a linear programming model, as shown in the following equation, to simplify the calculation:

$$\begin{aligned} \text{Max} \quad & h_k=\sum_r u_r Y_{kr} \\ \text{s.t.} \quad & \sum_i v_i X_{ki}=1 \\ & \sum_r u_r Y_{qr}-\sum_i v_i X_{qi}\le 0, \quad q=1,\ldots,L \\ & u_r\ge\varepsilon, \quad v_i\ge\varepsilon \end{aligned} \tag{7}$$

According to Equation (7), the maximum relative efficiency value of the kth DMU (denoted by h_k^*) can be determined. A high h_k^* value corresponds to a superior performance among a set of DMUs, and the value of h_k^* is between 0 and 1. When $h_k^*=1$, the kth DMU locates on the efficient frontier, which is a dominant solution for all evaluated DMUs.

4. PROPOSED METHOD

While focusing mainly on a dynamic system with multiple quality characteristics, this study presents a novel optimization procedure for enhancing product/process quality using DEA. Initially, the designed experiments for a dynamic system based on an orthogonal array are determined using the Taguchi method. Experiments are thereby conducted to obtain the response values. The proposed optimization procedure comprises the following six steps:

Step 1: *Design experiments for each quality characteristic.* This study uses an orthogonal array to design a dynamic system experiment. Control factors are located on the inner orthogonal array, and the signal factor level and noise factor level combinations are located on the outer orthogonal array. Accordingly, all experimental runs are conducted, and the corresponding response values are collected.

Step 2: *For each quality characteristic, estimate the relative efficiency of the location effect for each DMU.* Each treatment in the orthogonal array is regarded as a DMU when applying DEA. Additionally, experimental observations in each DMU are set as outputs, revealing location measures in a dynamic system, whereas ε is set as the input. As ε (i.e. input) is identical in its evaluation of the relative efficiency for each competing DMU, the returns to scale effect on relative efficiency is a constant. Therefore, this study uses the CCR model to determine relative efficiency for evaluating quality performance in a dynamic system. The relative efficiency of the location effect for the kth DMU can be calculated as follows:

$$\begin{aligned} \text{Max} \quad & h'_{kr}=\sum_{i=1}^{m}\sum_{j=1}^{n} u_{ijr} y_{ijkr} \\ \text{s.t.} \quad & v_r X_{kr}=1 \\ & \sum_{i=1}^{m}\sum_{j=1}^{n} u_{ijr} y_{ijqr}-v_r X_{qr}\le 0, \quad q=1,\ldots,L \\ & \forall X_{qr}\in\{\varepsilon\}, \quad q=1,\ldots,L \\ & u_{ijr}\ge\varepsilon, \quad v_r\ge\varepsilon \end{aligned} \tag{8}$$

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where y_{ijqr} is an experimental observation of the rth quality characteristic, $r=1,\ldots,s$, for the qth DMU with the combination of the ith signal factor level, $i=1,\ldots,m$, and the jth noise factor level, $j=1,\ldots,n$. Additionally, y_{ijkr} denotes a specific value of y_{ijqr} under the condition of $q=k$. h'_{kr} represents the relative efficiency value of the location effect of the rth quality characteristic for the kth DMU (i.e. the kth treatment); X_{kr} is the corresponding input value; u_{ijr} denotes an output weight of the rth quality characteristic with the combination of the ith signal factor level and the jth noise factor level; v_r represents an input weight of the rth quality characteristic; and X_{qr} is an input of the rth quality characteristic for the qth DMU.

Step 3: *For each quality characteristic*, *estimate the relative efficiency of the dispersion effect for each DMU.* Taguchi's dynamic system assumes a linear relationship between a quality characteristic and a signal factor. Accordingly, the predicted error is utilized as a measure of the dispersion effect. However, in actual practice, the relationship between a quality characteristic and a signal factor is never linear. The Taguchi method thus erroneously estimates dispersion effects. Therefore, this study applies the range of observations in each treatment (i.e. each DMU) as a dispersion effect of a dynamic system. The inverse dispersion effect is defined as

$$d_{qr}=\left\{\sum_{i=1}^{m}\left[\max_{j}\{y_{ijqr}\}-\min_{j}\{y_{ijqr}\}\right]\right\}^{-1}, \quad q=1,\ldots,L \tag{9}$$

where d_{qr} represents the inverse dispersion effect of the qth DMU under the rth quality characteristic. A large d_{qr} value produces a low dispersion effect in a dynamic system. Accordingly, the relative efficiency for the dispersion effect of the rth quality characteristic under the kth DMU can be determined based on the constructed CCR model as follows:

$$\begin{aligned} \text{Max} \quad & h''_{kr}=u_r d_{kr} \\ \text{s.t.} \quad & v_r X_{kr}=1 \\ & u_r d_{qr}-v_r X_{qr}\le 0, \quad q=1,\ldots,L \\ & \forall X_{qr}\in\{\varepsilon\}, \quad q=1,\ldots,L \\ & u_r\ge\varepsilon, \quad v_r\ge\varepsilon \end{aligned} \tag{10}$$

where h''_{kr} is the relative efficiency value of the dispersion effect of the rth quality characteristic for the kth DMU (i.e. the kth treatment). d_{kr} denotes the inverse dispersion effect of the rth quality characteristic under the specific kth DMU; X_{kr} represents the corresponding input (i.e. ε); and d_{qr} is the inverse dispersion effect of the rth quality characteristic under the qth DMU. X_{qr} is an input of the rth quality characteristic under the qth DMU; and u_r and v_r are the output and input weights of the dispersion effect for the kth DMU, respectively.

Step 4: *Calculate the OQP for each DMU of the system.* Robust product/process design attempts to minimize the variance of a response and to bring the mean response close to the target. Therefore, the location effect and the dispersion effect representing the mean performance and variance of a response, respectively, are utilized to establish the OQP of multiple dynamic quality systems. The formula of OQP is as follows:

$$O_q=\left(\prod_{r=1}^{s}((h'_{qr}h''_{qr})^{1/2})^{\alpha_r}\right)^{1/\sum_{r=1}^{s}\alpha_r}, \quad q=1,\ldots,L \tag{11}$$

where O_q denotes the OQP value of the qth DMU in the dynamic system, h'_{qr} represents a location effect of the qth DMU obtained from Step 2, and h''_{qr} indicates a dispersion effect of the qth DMU obtained from Step 3. Additionally, α_r is a weight of the rth quality characteristic, indicating the relative importance among multiple quality characteristics. A high OQP value yields a superior performance in a multiple dynamic quality system.

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Step 5: *Establish a prediction model of OQP on design factors.* According to the OQP value (i.e. O_q) of each DMU obtained in Step 4, an appropriate prediction model of OQP on design factors can be determined as follows:

$$\text{OQP} = f(C_z) = b_0 + \sum_{z=1}^{g} b_{z,w}(C_z)^w \tag{12}$$

where b_0 represents the intercept; C_z represents the level condition of a control factor, $z=1,\ldots,g$, for a certain factor-level combination; $(C_z)^w$ denotes C_z to the wth power, $w=1,2,\ldots$; and $b_{z,w}$ is the regression coefficient of $(C_z)^w$.

Step 6: *Determine the optimal product/process condition.* Because a large OQP value corresponds to a high OQP, the optimal product/process condition can be derived from the feasible region of design factor-level settings in the following equation:

$$\begin{aligned}
\text{Max} \quad & \text{OQP} = f(C_z) \\
\text{s.t.} \quad & 0 \le \text{OQP} \le 100\% \\
& \text{when } C_z \text{ implies a categorical level:} \\
& C_z = \sum_{z=1}^{g} \lambda_z D_{\lambda_z} \\
& \sum_{z=1}^{g} D_{\lambda_z} = 1 \\
& D_{\lambda_z} = 0 \text{ or } 1 \\
& \text{when } C_z \text{ implies a continuous level:} \\
& 1 \le C_z \le \text{Max}\{\lambda_z\} \\
& C_z \ge 0, \quad z = 1,\ldots,g
\end{aligned} \tag{13}$$

where D_{λ_z} is a dummy variable, and λ_z is all initial designed factor-level conditions for a control factor, $z=1,\ldots,g$.

5. ILLUSTRATIONS

The following two cases demonstrate the effectiveness of the proposed procedure.

Case 1: *Optimizing the process condition of a temperature control circuit.* Wu and Yeh[17] simulated experimental observations for a temperature control circuit introduced by Tomishima[18]. The dynamic system involves two equally important quality characteristics, $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$. Four control factors (A, B, C, and D), each with three levels, are located on the inner orthogonal array. A signal factor and a noise factor, each with three levels, are located on the outer orthogonal array. The L_{18} orthogonal array was used to design the experiments. Further details of this case can be found in Tomishima[18] and Wu and Yeh[17]. These experimental data for the temperature control circuit were reanalyzed using the proposed optimization procedure. The optimization results were compared with those of the conventional Taguchi method and Wu and Yeh[17]. Initially, the relative efficiencies of location effects (h'_{kr}) and dispersion effects (h''_{kr}) with respect to each quality characteristic $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$ were determined using Equations (8) and (10). Table I lists the calculations. Accordingly, the OQP of DMUs was derived using Equation (11). For example, the response values of the first experimental run for the first quality characteristic (i.e. $R_{T\text{-ON}}$), y_{ij11}, are $1.4258, 1.5426, 1.6763, 2.8516, 3.0851, 3.3527, 4.2774, 4.6277$, and 5.029, as shown in Table IV of

Table I. The efficiencies of location effects and dispersion effects with respect to $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$

No.	Control factors A	B	C	D	$R_{T\text{-ON}}$ (y_1) h'_{kr} (%)	h''_{kr} (%)	$R_{T\text{-OFF}}$ (y_2) h'_{kr} (%)	h''_{kr} (%)
1	1	1	1	1	41.51	24.74	44.45	49.81
2	2	2	2	2	36.61	33.45	44.45	49.81
3	3	3	3	3	33.81	40.70	44.45	49.81
4	1	1	2	3	31.76	49.30	47.15	44.44
5	2	2	3	1	36.29	41.68	47.15	44.44
6	3	3	1	2	51.48	19.32	37.00	72.77
7	1	2	1	2	58.73	21.89	61.70	38.98
8	2	3	2	3	52.51	28.14	61.70	38.98
9	3	1	3	1	16.28	100.00	21.85	92.32
10	1	3	3	2	76.05	20.10	100.00	22.14
11	2	1	1	3	23.86	66.97	29.63	74.72
12	3	2	2	1	29.95	42.06	29.63	74.72
13	1	2	3	3	46.34	38.67	70.73	29.62
14	2	3	1	1	100.00	7.12	55.49	48.50
15	3	1	2	2	16.67	96.90	20.96	100.00
16	1	3	2	1	100.00	10.38	92.55	25.98
17	2	1	3	2	21.71	81.70	32.77	61.51
18	3	2	1	3	26.77	52.91	27.42	87.68

Table II. The OQP value of each DMU

No.	Control factors A	B	C	D	O_q (%)
1	1	1	1	1	38.831591
2	2	2	2	2	40.578505
3	3	3	3	3	41.778897
4	1	1	2	3	42.559429
5	2	2	3	1	42.193214
6	3	3	1	2	40.452926
7	1	2	1	2	41.933218
8	2	3	2	3	43.418346
9	3	1	3	1	42.569682
10	1	3	3	2	42.891203
11	2	1	1	3	43.369040
12	3	2	2	1	40.865673
13	1	2	3	3	44.017941
14	2	3	1	1	37.205669
15	3	1	2	2	42.895592
16	1	3	2	1	39.746897
17	2	1	3	2	43.483652
18	3	2	1	3	42.957454

Wu and Yeh[17]. The relative efficiency value of the location effect of $R_{T\text{-ON}}$ for the first DMU, $h'_{11}=41.51\%$, can be obtained using Equation (8). Similarly, $h''_{11}=24.74\%$, $h'_{12}=44.45\%$, and $h''_{12}=49.81\%$ can be obtained using Equations (8) and (10), respectively. Finally, $O_q=38.831591\%$ for the first experimental run in Table II is obtained using Equation (11). The weights of responses $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$ were set to 1 (i.e. in Equation (11), $\alpha_1=1$ and $\alpha_2=1$), as the two quality characteristics are equally important. Table II lists the OQP values of each DMU, indicating an overall quality measure in a dynamic multi-response system.

According to the OQP values in Table II, the prediction model of OQP on design control factors was established by employing the stepwise regression approach as

$$\begin{aligned}\mathrm{OQP} = {} & 0.386672414171712 + 0.0042290508948704 \times A \\ & - 0.000421877582588344 \times A^2 + 0.0165591668577828 \times B \\ & - 0.00616582342127432 \times B^2 + 0.00503301213499012 \times C \times D \end{aligned} \tag{14}$$

Given that a large OQP value yields a superior quality performance for a dynamic system, the optimal factor-level combination is determined as $A_3B_1C_3D_3$ using Equations (13) and (14). Additionally, β and σ_e^2 of the dynamic system were derived using Equations (2) and (3). Table III lists β and σ_e^2 values. These values were used to establish the quadratic regression models of β and σ_e^2 on designed factors, respectively, as

For response $R_{T\text{-ON}}$ (y_1):

$$\begin{aligned}\hat{\beta}_1 = {} & 4.007057 - 0.385862 \times A + 0.04482 \times A^2 + 2.838296 \times B \\ & + 0.436832 \times B^2 - 1.774904 \times C + 0.233345 \times C^2 - 1.800993 \times D \\ & + 0.328771 \times D^2 - 0.625026 \times AB + 0.117892 \times AC - 0.36615 \times BC \\ & + 0.138594 \times AD - 0.548080 \times BD + 0.442152 \times CD \end{aligned} \tag{15}$$

$$\begin{aligned}\log(\hat{\sigma}_{1e}^2) = {} & -0.620458 - 0.234226 \times A - 0.030201 \times A^2 + 0.576154 \times B \\ & + 0.045826 \times B^2 - 0.561984 \times C + 0.035376 \times C^2 - 0.570838 \times D \\ & + 0.056997 \times D^2 + 0.02669 \times AB + 0.004556 \times AC - 0.055985 \times BC \\ & + 0.011346 \times AD - 0.09495 \times BD + 0.143968 \times CD \end{aligned} \tag{16}$$

Table III. The values of β and σ_e^2 for each quality characteristic

	Control factors				$R_{T\text{-ON}}$ (y_1)		$R_{T\text{-OFF}}$ (y_2)	
No.	A	B	C	D	β_1	σ_{1e}^2	β_2	σ_{2e}^2
1	1	1	1	1	3.096	0.055	1.667	0.003
2	2	2	2	2	2.699	0.030	1.667	0.003
3	3	3	3	3	2.475	0.020	1.667	0.003
4	1	1	2	3	2.308	0.014	1.765	0.004
5	2	2	3	1	2.642	0.019	1.765	0.004
6	3	3	1	2	3.839	0.090	1.380	0.001
7	1	2	1	2	4.318	0.070	2.308	0.005
8	2	3	2	3	3.828	0.042	2.308	0.005
9	3	1	3	1	1.181	0.003	0.817	0.001
10	1	3	3	2	5.534	0.083	3.751	0.014
11	2	1	1	3	1.733	0.007	1.111	0.001
12	3	2	2	1	2.205	0.019	1.111	0.001
13	1	2	3	3	3.345	0.022	2.648	0.008
14	2	3	1	1	7.434	0.665	2.069	0.003
15	3	1	2	2	1.211	0.004	0.785	0.001
16	1	3	2	1	7.461	0.312	3.462	0.010
17	2	1	3	2	1.568	0.005	1.225	0.002
18	3	2	1	3	1.956	0.012	1.026	0.001

For response $R_{T\text{-OFF}}$ (y_2):

$$\begin{aligned}\hat{\beta}_2 = {} & 1.706315-0.781251\times A+0.158126\times A^2+0.451129\times B \\ & +0.101454\times B^2+0.175218\times C-0.008988\times C^2+0.101215\times D \\ & -0.018349\times D^2-0.2249\times AB-0.067128\times AC+0.094456\times BC \\ & +0.007511\times AD-0.001094\times BD-0.027667\times CD \end{aligned} \tag{17}$$

$$\begin{aligned}\log(\hat{\sigma}^2_{2e}) = {} & -2.551005-0.359034\times A+0.000429\times A^2+0.236008\times B \\ & -0.02061\times B^2+0.127796\times C-0.020476\times C^2-0.004788\times D \\ & +0.000532\times D^2-0.001766\times AB+0.001689\times AC+0.038189\times BC \\ & +0.001946\times AD+0.00172\times BD-0.00165\times CD \end{aligned} \tag{18}$$

Instead of σ^2_e, $\log(\sigma^2_e)$ is employed to establish a regression model to satisfy the assumptions of a regression model. Accordingly, the predicted values of β and σ^2_e and the corresponding SNR under full factorial level combinations can be obtained. As the responses $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$ are equally important, the average SNR is determined as an overall quality measure for the temperature control circuit system. Table AI in Appendix lists these calculations. The factor-level combination $A_3B_1C_3D_3$ was determined as an optimal one based on Table AI in Appendix. Notably, this result is the optimum solution among all factor-level combinations of a full factorial design and is identical to the optimal factor-level combination obtained from the proposed procedure. Table IV compares the proposed procedure using DEA with the conventional Taguchi method, and that of Wu and Yeh[17]. The optimization of the proposed procedure is consistent with the results of Wu and Yeh[17], thus verifying the effectiveness of the proposed procedure.

Case 2: *Biological reduction in ethyl 4-chloro acetoacetate to produce an optically pure compound.* A dynamic system with multiple responses from Tong *et al.*[12] involves optimizing a procedure for biologically reducing ethyl 4-chloro acetoacetate to produce an optically pure compound. The responses S-CHBE and R-CHBE, which are two reactions produced by this procedure, are used as two quality characteristics. A large value is desired for response S-CHBE, whereas a small value is desired for response R-CHBE. Eight control factors (A, B, C, D, E, F, G, and H), one signal factor with three levels, and one noise factor with two levels were designed. Each control factor has three levels, except for factor A, which has two levels.

The response values of S-CHBE and R-CHBE were analyzed using the proposed procedure. Accordingly, the location effects, dispersion effects, and OQP values associated with each treatment in dynamic experiments were obtained using Equations (8), (10), and (11). The weights of the responses were set to equal 1 (namely $\alpha_1 = 1$ and $\alpha_2 = 1$) as both S-CHBE and R-CHBE are equally important. Table V shows the location effects and dispersion effects, and Table VI lists the OQP values. Based on the OQP values in Table VI, the prediction model of OQP on eight design factors obtained by the stepwise regression approach is

Table IV. Comparisons of different optimal approaches

	$R_{T\text{-ON}}$ (y_1)			$R_{T\text{-OFF}}$ (y_2)			
Method	$\hat{\beta}_1$	$\hat{\sigma}^2_{1e}$	$\text{SNR}_1(\eta_1)$	$\hat{\beta}_2$	$\hat{\sigma}^2_{2e}$	$\text{SNR}_2(\eta_2)$	Average SNR
Taguchi method	2.653600	0.012226	27.603794	1.846381	0.004068	29.233044	28.418419
Wu and Yeh[17]	2.529299	0.003358	32.799792	0.741390	0.000818	28.271109	30.535451
Proposed procedure	2.529299	0.003358	32.799792	0.741390	0.000818	28.271109	30.535451

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Table V. The efficiencies of location effects and dispersion effects with respect to S-CHBE and R-CHBE

	Control factors								S-CHBE (y_1)		R-CHBE (y_2)	
No.	A	B	C	D	E	F	G	H	h'_{kr} (%)	h''_{kr} (%)	h'_{kr} (%)	h''_{kr} (%)
1	1	1	1	1	1	1	1	1	100.00	51.13	86.11	13.17
2	1	1	2	2	2	2	2	2	92.88	31.54	100.00	26.82
3	1	1	3	3	3	3	3	3	93.59	28.04	94.23	100.00
4	1	2	1	1	2	2	3	3	100.00	25.62	100.00	7.88
5	1	2	2	2	3	3	1	1	100.00	42.35	80.81	11.68
6	1	2	3	3	1	1	2	2	78.10	43.31	100.00	10.37
7	1	3	1	2	1	3	2	3	84.92	27.67	100.00	6.03
8	1	3	2	3	2	1	3	1	85.36	57.99	92.63	29.98
9	1	3	3	1	3	2	1	2	100.00	32.02	68.13	31.44
10	2	1	1	3	2	2	2	1	88.02	31.24	92.18	18.33
11	2	1	2	1	1	3	3	2	91.96	32.31	73.35	16.15
12	2	1	3	2	3	1	1	3	85.03	39.44	100.00	21.46
13	2	2	1	2	3	1	3	2	95.12	29.86	86.11	43.74
14	2	2	2	3	1	2	1	3	72.29	48.59	100.00	12.09
15	2	2	3	1	2	3	2	1	100.00	34.51	72.77	29.94
16	2	3	1	3	2	3	1	2	72.25	52.44	86.15	18.40
17	2	3	2	1	3	1	2	3	92.84	27.00	61.21	49.93
18	2	3	3	2	1	2	3	1	91.90	100.00	90.70	10.56

given as

$$\begin{aligned}\mathrm{OQP} = {} & 0.728689051908353 - 0.108979662055521\times A - 0.303333300226461\times B\\ & + 0.0530479728745114\times B^2 + 0.0477032647622895\times C\\ & - 0.00267205243659638\times C^2 - 0.0448906596024196\times D\\ & + 0.0157781619586083\times D^2 + 0.0555060917557161\times A\times B\\ & + 0.0312508718770257\times E\times G - 0.0159688120501931\times G\times H \end{aligned} \tag{19}$$

The optimal factor-level combination $A_1B_1C_3D_1E_3F_1G_3H_1$ was determined using Equations (13) and (19).

Additionally, the values of β and σ_e^2 were calculated using Equations (2) and (3). Table VII lists β and σ_e^2 values. These values were used to establish the regression models of β and $\log(\sigma_e^2)$ on designed factors as follows:

For response S-CHBE (y_1):

$$\begin{aligned}\hat{\beta}_1 = {} & 0.565615 - 0.135774\times A + 0.077371\times B - 0.034239\times B^2\\ & + 0.045958\times C + 0.022482\times C^2 + 0.097202\times D + 0.002716\times D^2\\ & - 0.124232\times E + 0.031141\times E^2 + 0.049376\times F - 0.014238\times F^2\\ & - 0.0563\times G + 0.020129\times G^2 - 0.067481\times H + 0.014431\times H^2\\ & + 0.027852\times AB - 0.068517\times CD \end{aligned} \tag{20}$$

$$\begin{aligned}\log(\hat{\sigma}_{1e}^2) = {} & -0.174523 + 0.623949\times A + 0.335465\times B - 0.011088\times B^2\\ & - 0.97924\times C + 0.097114\times C^2 - 0.888545\times D + 0.029706\times D^2\\ & - 0.308284\times E + 0.103635\times E^2 + 0.333492\times F - 0.049443\times F^2\\ & + 0.185438\times G - 0.063798\times G^2 - 0.00995\times H - 0.032182\times H^2\\ & - 0.242662\times AB + 0.283379\times CD \end{aligned} \tag{21}$$

Table VI. The OQP value of each DMU

	Control factors								
No.	A	B	C	D	E	F	G	H	O_q (%)
1	1	1	1	1	1	1	1	1	49.071438
2	1	1	2	2	2	2	2	2	52.943258
3	1	1	3	3	3	3	3	3	70.517866
4	1	2	1	1	2	2	3	3	37.694357
5	1	2	2	2	3	3	1	1	44.713672
6	1	2	3	3	1	1	2	2	43.276736
7	1	3	1	2	1	3	2	3	34.501193
8	1	3	2	3	2	1	3	1	60.890242
9	1	3	3	1	3	2	1	2	51.175320
10	2	1	1	3	2	2	2	1	46.427242
11	2	1	2	1	1	3	3	2	43.313885
12	2	1	3	2	3	1	1	3	51.794626
13	2	2	1	2	3	1	3	2	57.190449
14	2	2	2	3	1	2	1	3	45.395505
15	2	2	3	1	2	3	2	1	52.364539
16	2	3	1	3	2	3	1	2	49.504355
17	2	3	2	1	3	1	2	3	52.610292
18	2	3	3	2	1	2	3	1	54.468658

Table VII. The values of β and σ_e^2 for each quality characteristic

	Control factors								S-CHBE (y_1)		R-CHBE (y_2)	
No.	A	B	C	D	E	F	G	H	β_1	σ_{1e}^2	β_2	σ_{2e}^2
1	1	1	1	1	1	1	1	1	0.453	0.171	0.104	0.018
2	1	1	2	2	2	2	2	2	0.422	0.047	0.122	0.011
3	1	1	3	3	3	3	3	3	0.408	0.070	0.112	0.011
4	1	2	1	1	2	2	3	3	0.461	0.116	0.108	0.010
5	1	2	2	2	3	3	1	1	0.455	0.138	0.097	0.003
6	1	2	3	3	1	1	2	2	0.376	0.053	0.140	0.016
7	1	3	1	2	1	3	2	3	0.396	0.063	0.127	0.065
8	1	3	2	3	2	1	3	1	0.395	0.024	0.106	0.007
9	1	3	3	1	3	2	1	2	0.508	0.101	0.074	0.001
10	2	1	1	3	2	2	2	1	0.405	0.067	0.106	0.004
11	2	1	2	1	1	3	3	2	0.400	0.172	0.068	0.026
12	2	1	3	2	3	1	1	3	0.361	0.097	0.108	0.024
13	2	2	1	2	3	1	3	2	0.438	0.076	0.103	0.002
14	2	2	2	3	1	2	1	3	0.350	0.034	0.137	0.012
15	2	2	3	1	2	3	2	1	0.469	0.121	0.081	0.003
16	2	3	1	3	2	3	1	2	0.347	0.026	0.113	0.013
17	2	3	2	1	3	1	2	3	0.368	0.056	0.060	0.002
18	2	3	3	2	1	2	3	1	0.427	0.051	0.104	0.010

For response R-CHBE (y_2):

$$\begin{aligned}\hat{\beta}_2 = {} & 0.070285 - 0.04084 \times A + 0.033234 \times B - 0.01369 \times B^2 \\ & - 0.009693 \times C + 0.00839 \times C^2 + 0.099493 \times D - 0.014809 \times D^2 \\ & + 0.0089 \times E - 0.006109 \times E^2 - 0.012399 \times F + 0.001442 \times F^2 \\ & - 0.022989 \times G + 0.005393 \times G^2 - 0.004835 \times H + 0.003962 \times H^2 \\ & + 0.010301 \times AB - 0.011869 \times CD \end{aligned} \tag{22}$$

Table VIII. Comparative analysis of optimal factor-level combination

	S-CHBE (y_1)			R-CHBE (y_2)			
Method	$\hat{\beta}_1$	$\hat{\sigma}^2_{1e}$	$\text{SNR}_1(\eta_1)$	$\hat{\beta}_2$	$\hat{\sigma}^2_{2e}$	$\text{SNR}_2(\eta_2)$	Average SNR
Initial	0.383167	0.074819	2.927631	0.092342	0.013715	−2.063979	0.431826
Taguchi method	0.356916	0.050206	4.043736	0.060674	0.001191	4.899849	4.471793
Tong *et al.*[12]	0.533061	0.120447	3.727584	0.063975	0.000558	8.654087	6.190836
Proposed procedure	0.637323	0.049053	9.180519	0.094251	0.000196	16.566749	12.873634

$$
\begin{aligned}
\log(\hat{\sigma}^2_{2e}) = & -1.29594 + 0.862421\times A - 0.931252\times B + 0.352817\times B^2 \\
& - 1.306126\times C + 0.082039\times C^2 + 0.340233\times D - 0.323628\times D^2 \\
& + 1.216981\times E - 0.387937\times E^2 - 0.702966\times F + 0.21409\times F^2 \\
& - 0.45689\times G + 0.071235\times G^2 + 0.387471\times H - 0.057581\times H^2 \\
& - 0.324895\times AB + 0.507063\times CD
\end{aligned} \tag{23}
$$

To further verify the effectiveness of the proposed procedure, the optimization result was compared with that of Taguchi's single-response method and the procedure developed by Tong *et al.*[12]. Table VIII lists these comparisons. According to Table VIII, the average SNR of the proposed procedure is 12.873634 dB, i.e. higher than that of the Taguchi method with an SNR value of 4.471793 dB, and the procedure of Tong *et al.*[12] with an SNR value of 6.190836 dB. The effectiveness of the proposed procedure was thus further verified. Therefore, the proposed optimization procedure using DEA obtains a more robust dynamic system than available procedures do.

6. CONCLUSIONS

This study describes a novel optimization procedure for a dynamic system with multiple responses using DEA. The relative efficiencies of location effects and dispersion effects resulting from DEA are used as quality performance measures for the product/process mean and variance, respectively. Therefore, capable of simultaneously accounting for the location effects and the dispersion effects for multiple quality characteristics, OQP can also be treated as an overall quality measure for a dynamic system with multiple responses. Consequently, a prediction model of OQP on design factors can be established to determine the optimal process condition.

In contrast with the conventional Taguchi method, the proposed procedure is not only effective for multiple quality characteristics, but also requires no assumptions between quality characteristics and signal factors. Moreover, the optimal process condition obtained from the proposed procedure is not restricted to the initially designed factor-level combinations. Therefore, the proposed procedure can determine the optimal process condition more efficiently than the Taguchi method and other optimization procedures can.

REFERENCES

1. Miller A, Wu CFJ. Parameter design for signal-response systems: A different look at Taguchi's dynamic parameter design. *Statistical Science* 1996; **11**:122–136.
2. Miller A, Wu CFJ. Commentary on Taguchi's parameter design with dynamic characteristics. *Quality and Reliability Engineering International* 1996; **12**:75–78.

DOI: 10.1002/qre

3. Wasserman GS. Parameter design with dynamic characteristics: A regression perspective. *Quality and Reliability Engineering International* 1996; **12**:113–117.
4. Lunani M, Nair VN, Wasserman GS. Graphical methods for robust design with dynamic characteristics. *Journal of Quality Technology* 1997; **29**:327–338.
5. McCaskey SD, Tsui KL. Analysis of dynamic robust design experiments. *International Journal of Production Research* 1997; **35**:1561–1574.
6. Chen SP. Robust design with dynamic characteristics using stochastic sequential quadratic programming. *Engineering Optimization* 2003; **35**:79–89.
7. Bae SJ, Tsui KL. Analysis of dynamic robust design experiment with explicit & hidden noise variables. *Quality Technology & Quantitative Management* 2006; **3**:55–75.
8. Tong LI, Wang CH, Chen CC, Chen CT. Dynamic multiple responses by ideal solution analysis. *European Journal of Operational Research* 2004; **156**:433–444.
9. Hsieh KL, Tong LI, Chiu HP, Yeh HY. Optimization of a multi-response problem in Taguchi's dynamic system. *Computers & Industrial Engineering* 2005; **49**:556–571.
10. Su CT, Chen MC, Chan HL. Applying neural network and scatter search to optimize parameter design with dynamic characteristics. *Journal of the Operational Research Society* 2005; **56**:1132–1140.
11. Vining GS, Mylers RH. Combining Taguchi and response surface philosophies: A dual response approach. *Journal of Quality Technology* 1990; **22**:38–45.
12. Tong LI, Wang CH, Houng JY, Chen JY. Optimizing dynamic multi-response problems by using dual response surface method. *Quality Engineering* 2002; **14**:115–125.
13. Phadke MS. *Quality Engineering Using Robust Design*. Prentice-Hall: Englewood Cliffs, NJ, 1989.
14. Cooper WW, Seiford LM, Tone K. *Data Envelopment Analysis*: *A Comprehensive Text with Models*, *Applications*, *References*, *and DEA-solver Software*. Kluwer Academic Publishers: Boston, MA, 2000.
15. Charnes A, Cooper WW, Rhodes E. Measuring efficiency of decision-making units. *European Journal of Operational Research* 1978; **2**:429–444.
16. Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 1984; **30**:1078–1092.
17. Wu FC, Yeh CH. Robust design of multiple dynamic quality characteristics. *International Journal of Advanced Manufacturing Technology* 2005; **25**:579–588.
18. Tomishima A. Tolerance design by performance analysis design: An example of temperature control device. *Reliability Design Case Studies for New Product Development*, Taguchi G (ed.). Japanese Standard Association: Japan, 1984; 213–220.

APPENDIX A

As the responses $R_{T\text{-ON}}$ and $R_{T\text{-OFF}}$ are equally important, the average SNR is determined as an overall quality measure for the temperature control circuit system. Table AI lists these calculations. The factor-level combination $A_3B_1C_3D_3$ was determined as an optimal one based on Table AI.

Table AI. Predicted SNR for all factor-level combinations of a full factorial design for Case 1

A	B	C	D	$\hat{\beta}_1$	$\hat{\sigma}^2_{1e}$	$\text{SNR}_1(\eta_1)$	$\hat{\beta}_2$	$\hat{\sigma}^2_{2e}$	$\text{SNR}_2(\eta_2)$	Average SNR
1	1	1	1	3.087	0.054	22.467	1.666	0.003	29.944	26.206
1	1	1	2	2.305	0.025	23.325	1.691	0.003	30.085	26.705
1	1	1	3	2.180	0.015	25.097	1.679	0.003	30.025	27.561
1	1	2	1	2.206	0.023	23.182	1.814	0.004	29.637	26.409
1	1	2	2	1.866	0.015	23.683	1.811	0.004	29.652	26.668
1	1	2	3	2.184	0.012	25.864	1.772	0.004	29.478	27.671
1	1	3	1	1.791	0.012	24.300	1.944	0.004	29.601	26.951
1	1	3	2	1.894	0.011	25.298	1.913	0.004	29.509	27.404
1	1	3	3	2.654	0.012	27.604	1.846	0.004	29.233	28.418
1	2	1	1	5.696	0.210	21.895	2.290	0.005	30.584	26.239

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Table AI. *Continued*.

A	B	C	D	$\hat{\beta}_1$	$\hat{\sigma}^2_{1e}$	$SNR_1(\eta_1)$	$\hat{\beta}_2$	$\hat{\sigma}^2_{2e}$	$SNR_2(\eta_2)$	Average SNR
1	2	1	2	4.366	0.077	23.930	2.314	0.005	30.668	27.299
1	2	1	3	3.694	0.037	25.681	2.301	0.005	30.604	28.142
1	2	2	1	4.449	0.080	23.942	2.532	0.006	30.030	26.986
1	2	2	2	3.561	0.041	24.913	2.529	0.006	30.028	27.470
1	2	2	3	3.331	0.027	26.097	2.488	0.006	29.888	27.992
1	2	3	1	3.669	0.036	25.752	2.757	0.008	29.749	27.750
1	2	3	2	3.223	0.026	26.092	2.725	0.008	29.676	27.884
1	2	3	3	3.435	0.024	26.970	2.657	0.008	29.473	28.221
1	3	1	1	9.179	1.006	19.229	3.117	0.007	31.550	25.390
1	3	1	2	7.301	0.297	22.535	3.140	0.007	31.591	27.063
1	3	1	3	6.081	0.114	25.100	3.126	0.007	31.519	28.309
1	3	2	1	7.566	0.337	22.303	3.454	0.010	30.632	26.468
1	3	2	2	6.130	0.139	24.329	3.449	0.010	30.613	27.471
1	3	2	3	5.352	0.074	25.864	3.407	0.010	30.491	28.178
1	3	3	1	6.420	0.133	24.921	3.773	0.014	29.999	27.460
1	3	3	2	5.426	0.076	25.875	3.740	0.014	29.934	27.904
1	3	3	3	5.089	0.057	26.594	3.671	0.014	29.771	28.182
2	1	1	1	2.467	0.028	23.342	1.075	0.001	29.694	26.518
2	1	1	2	1.823	0.013	23.999	1.107	0.001	29.945	26.972
2	1	1	3	1.838	0.008	26.207	1.103	0.001	29.893	28.050
2	1	2	1	1.704	0.012	23.715	1.155	0.002	29.261	26.488
2	1	2	2	1.502	0.008	24.465	1.160	0.002	29.306	26.885
2	1	2	3	1.959	0.007	27.470	1.128	0.002	29.061	28.266
2	1	3	1	1.407	0.006	24.935	1.218	0.002	29.068	27.001
2	1	3	2	1.648	0.006	26.710	1.195	0.002	28.928	27.819
2	1	3	3	2.547	0.007	29.751	1.136	0.002	28.498	29.125
2	2	1	1	4.451	0.116	22.309	1.474	0.002	30.332	26.320
2	2	1	2	3.260	0.044	23.834	1.505	0.002	30.489	27.162
2	2	1	3	2.726	0.022	25.371	1.500	0.002	30.423	27.897
2	2	2	1	3.322	0.045	23.915	1.649	0.003	29.863	26.889
2	2	2	2	2.573	0.024	24.485	1.653	0.003	29.874	27.180
2	2	2	3	2.481	0.016	25.821	1.620	0.003	29.679	27.750
2	2	3	1	2.660	0.020	25.422	1.806	0.004	29.619	27.521
2	2	3	2	2.352	0.015	25.708	1.782	0.004	29.511	27.609
2	2	3	3	2.703	0.014	27.125	1.722	0.004	29.207	28.166
2	3	1	1	7.309	0.594	19.539	2.076	0.003	31.613	25.576
2	3	1	2	5.570	0.180	22.359	2.106	0.003	31.697	27.028
2	3	1	3	4.488	0.071	24.523	2.099	0.003	31.617	28.070
2	3	2	1	5.814	0.201	22.259	2.345	0.005	30.848	26.553
2	3	2	2	4.517	0.085	23.806	2.348	0.005	30.832	27.319
2	3	2	3	3.877	0.047	25.080	2.314	0.005	30.668	27.874
2	3	3	1	4.785	0.080	24.567	2.597	0.006	30.316	27.441
2	3	3	2	3.930	0.047	25.158	2.572	0.006	30.223	27.690
2	3	3	3	3.732	0.036	25.870	2.510	0.006	29.992	27.931
3	1	1	1	1.936	0.013	24.666	0.800	0.001	30.676	27.671
3	1	1	2	1.432	0.006	25.211	0.839	0.001	31.072	28.141
3	1	1	3	1.584	0.004	28.119	0.843	0.001	31.068	29.593
3	1	2	1	1.291	0.006	24.689	0.813	0.001	29.743	27.216
3	1	2	2	1.229	0.004	25.985	0.825	0.001	29.863	27.924
3	1	2	3	1.823	0.003	30.003	0.801	0.001	29.580	29.792
3	1	3	1	1.113	0.003	26.231	0.809	0.001	29.027	27.629
3	1	3	2	1.492	0.003	29.068	0.793	0.001	28.866	28.967
3	1	3	3	2.529	0.003	32.800	0.741	0.001	28.271	30.535
3	2	1	1	3.296	0.056	22.858	0.974	0.001	30.300	26.579
3	2	1	2	2.243	0.022	23.633	1.013	0.001	30.595	27.114
3	2	1	3	1.848	0.011	24.926	1.015	0.001	30.558	27.742

Table AI. *Continued*.

A	*B*	*C*	*D*	$\hat{\beta}_1$	$\hat{\sigma}_{1e}^2$	$SNR_1(\eta_1)$	$\hat{\beta}_2$	$\hat{\sigma}_{2e}^2$	$SNR_2(\eta_2)$	Average SNR
3	2	2	1	2.285	0.022	23.777	1.082	0.001	29.752	26.765
3	2	2	2	1.674	0.012	23.753	1.093	0.001	29.814	26.783
3	2	2	3	1.721	0.008	25.530	1.067	0.001	29.570	27.550
3	2	3	1	1.740	0.010	24.806	1.172	0.002	29.396	27.101
3	2	3	2	1.571	0.008	25.158	1.155	0.002	29.261	27.210
3	2	3	3	2.060	0.007	27.610	1.102	0.002	28.830	28.220
3	3	1	1	5.529	0.305	20.007	1.351	0.001	31.467	25.737
3	3	1	2	3.928	0.095	22.105	1.388	0.001	31.645	26.875
3	3	1	3	2.985	0.038	23.646	1.390	0.001	31.579	27.613
3	3	2	1	4.152	0.104	22.180	1.553	0.002	30.837	26.509
3	3	2	2	2.993	0.045	22.965	1.563	0.002	30.848	26.906
3	3	2	3	2.492	0.026	23.860	1.537	0.002	30.643	27.252
3	3	3	1	3.241	0.042	23.983	1.738	0.003	30.378	27.181
3	3	3	2	2.524	0.025	24.000	1.720	0.003	30.262	27.131
3	3	3	3	2.465	0.020	24.842	1.666	0.003	29.944	27.393

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