HYDROLOGICAL PROCESSES
Hydrol. Process. **22**, 2959–2973 (2008)

An analytical method of stage–fall–discharge rating

Reuy-Bean Wu* and Jinn-Chuang Yang
National Chiao Tung University, Hsinchu, Taiwan, 30050

Abstract:

This paper presents an analytical method for establishing a stage–fall–discharge rating using hydraulic performance graphs (HPG). The rating curves derived from the HPG are used as the basis to establish the functional relation of stage, fall and discharge through regression analysis following the USGS procedure. In doing so, the conventional trial-and-error process can be avoided and the associated uncertainties involved may be reduced. For illustration, the proposed analytical method is applied to establish stage–fall–discharge relations for the Keelung River in northern Taiwan to examine its accuracy and applicability in an actual river. Based on the data extracted from the HPG for the Keelung River, one can establish a stage–fall–discharge relation that is more accurate than the one obtained by the conventionally used relation. Furthermore, the discharges obtained from the proposed rating method are verified through backwater analysis for measured high water level events. The results indicate that the analytical stage–fall–discharge rating method is capable of circumventing the shortcomings of those based on single-station data and, consequently, enhancing the reliability of flood estimation and forecasting. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS rating curve; hydraulic performance graph; stage–fall–discharge method

Received 3 January 2007; Accepted 5 July 2007

INTRODUCTION

In hydrologic and hydraulic applications, the rating curve at a gauge is normally determined by the bestfit line through field measurements of water stages and the corresponding discharges. In fact, the unique one-to-one relation between stage and discharge exists only in the ideal case of steady uniform flow. For natural streams, channel cross-sections are irregular and can change in time. Sometimes, man-made stream-crossing structures such as bridges, channel stabilizing works, diversion weirs and check dams are present in stream channels. Their presence could create backwater effects affecting water stage at the gauge. For a given water level at a gauge, the water surface slope and the corresponding slope of energy line vary spatially due to the backwater effect caused by changes in channel cross-section geometries in the downstream reach of the gauge. Hence, the discharge cannot be determined solely by a simple stage–discharge relation and a proper correction for the discharge is necessary. The stage–fall–discharge rating method developed by the US Geological Survey (Corbett *et al*., 1943; Rantz *et al*., 1982) is frequently used for such discharge correction. The method transforms the looped stage–discharge relation to a single-valued base rating curve with an auxiliary graph accounting for the effect of water surface slope to provide correction for discharge under varying energy slope.

To have a useful rating curve, adequate measurement of intermediate and high flows is desirable. However, field measurement during high flow periods is difficult, and if available, the associated measurement error would generally be large. Therefore, the uncertainty in measurement-based stage–discharge rating curve could be large in high flow regions due to extrapolation beyond the measurement data range. Discharges during flood events are generally estimated from the established stage–discharge rating curve which are in turn used in frequency analysis to determine the design discharge for flood control projects. It is essential to establish an accurate stage–discharge rating relation so that reliable estimates of discharge may be obtained. The conventional method for determining stage–discharge rating curve does not consider backwater situations under various discharges. Furthermore, the stage–fall–discharge rating relation can only be developed on the basis of experience using limited field-measured data through a trial-and-error process. The hydraulic performance graph (HPG), proposed by Yen and González (1994, 1995) summarizes the relations between upstream and downstream stages (or water depths) of a reach under various discharges while taking into account of backwater effect. Such a feature may be utilized to supplement the deficiency of the conventional method in establishing a stage–fall–discharge relation. In this study, the theoretical rating curve derived from the HPG is used as a basis to derive stage–fall–discharge relations through regression analysis based on the data extracted from the HPG following the USGS procedure. Through application to the Keelung River in Taiwan the feasibility of this proposed technique is investigated and discussed.

---

* Correspondence to: Reuy-Bean Wu, Department of Civil Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, Taiwan, 30050. E-mail: rbwu@wra.gov.tw

REVIEW OF STAGE–DISCHARGE RATING METHOD

Conventionally, a stage–discharge rating curve and its associated correction are derived from concurrent measurements of flow discharge and stage. If the measured discharge (Q_m) and rated discharge (Q_r) differ by more than ±5%, then a more complex rating is needed. Besides slope rating, there exist other corrections to account for changes in stage such as index-velocity rating (Rantz *et al.*, 1982). The index-velocity rating is more commonly used nowadays. The stage-fall in the slope rating method serves as a surrogate for velocity in the correction approach. In the derivation of a stage–fall–discharge relation, non-inertia wave approximation (Tsai and Yen, 2001) is often used in which the water surface is assumed to be parallel to the energy line. The USGS stage–fall–discharge method (Rantz *et al.*, 1982) involves two gauges, a base gauge and an auxiliary gauge downstream. The discharge measurements at these two gauges (along with the fall between the two gauges) are used to establish the stage–fall–discharge relation by trial-and-error. Important studies on this subject include Corbett *et al.* (1943), Mitchell (1954) and Eisenlohr (1964).

Conventionally, graphs relating stage, fall and discharge are constructed empirically using measured discharge and stage at the base gauge and the corresponding fall in water surface between the base and auxiliary gauges as the following:

1. Under a uniform flow or fixed backwater condition, the base rating curve between measured stage and discharge is established, from which the rating discharge is called Q_r.
2. Under the condition of uniform flow or fixed backwater, the relation between stage and fall is established and the corresponding rating fall is called F_r. The stage–fall–discharge rating is divided into two categories according to the relation between stage and rating fall. One category is the ‘rating fall constant’ when uniform flow prevails in the prismatic channel reach between the base and auxiliary gauges, and the other is the ‘rating fall a function of stage’ when non-uniform flow occurs due to varying channel geometry in the non-prismatic channel reach between the two gauges.
3. As flow alters due to backwater effect, the ratio of measured discharge to rating discharge, Q_m/Q_r, is related to the ratio of observed fall to rating fall, F_m/F_r, through the functional form

$$\frac{Q_m}{Q_r} = f\left(\frac{F_m}{F_r}\right) \tag{1}$$

In common practice, the flow cross-sectional area and hydraulic radius for the rating and measured discharges are assumed to be identical. This implies that the local and convective accelerations are negligible and the rating discharge is steady, uniform flow with constant conveyance coefficient. The function form in Equation (1), in general, is expressed as

$$\frac{Q_m}{Q_r} = \left(\frac{F_m}{F_r}\right)^d \tag{2}$$

in which the theoretical value of d based on uniform flow is 0·5.

Since the backwater is also affected by the channel alignment and the shape of channel cross-section, in addition to the control at the downstream section, the stage–fall relation determined solely from the observed stages at the base and auxiliary gauges may not reflect the actual water surface profile therein. Hence, a stage–fall–discharge relation cannot be derived theoretically in this case and, consequently, it is established empirically through a trial-and-error process. After plots of stage versus rating fall (F_r), stage versus rating discharge (Q_r), and Q_m/Q_r versus F_m/F_r are established, the discharge corresponding to a given observation of stage and fall (F_m) may be determined as the following:

(1) Determine the rating fall, F_r, from the stage-rating fall plot for a given stage.
(2) Calculate the ratio F_m/F_r.
(3) Determine the ratio Q_m/Q_r from the Q_m/Q_r versus F_m/F_r plot.
(4) Determine the rating discharge, Q_r, corresponding to the given stage from the stage versus rating discharge plot.
(5) Multiply the ratio Q_m/Q_r by Q_r to yield Q_m.

Empirically, the exponent (d) of F_m/F_r in Equation (2) is found to be in the range 0·4–0·6. Since there always exist conditions in natural streams that are complex and difficult to control, such as the physical characteristics of the channel and the source causing backwater, it is difficult to employ the conventional method to establish the stage–fall–discharge relation. Therefore, establishment of the intended stage–discharge rating curve needs sufficient discharge measurements that cover a wide range of flow conditions at a site. The USGS method is more difficult to carry out for rating curve establishment when limited data are available and stage–discharge data during high flow conditions are lacking. Furthermore, extension of the rating curve beyond the measured data range is subject to large uncertainties. The open channel HPG (Yen and González, 1994, 1995) accounts for all relevant relations between stage and discharge as well as between stage and fall under uniform flow and fixed backwater conditions. It may be used to supplement the deficiency of the conventional method in the establishment of a rating curve and stage–fall–discharge rating relation. Relevant studies on this subject include Gonzalez-Castro and Yen (2000), and Schmidt (2002). In the following sections, the concept of HPG is briefly described, and its application to the establishment of rating curve is presented.

DOI: 10.1002/hyp

HPG AND THEORETICAL RATING CURVE

Yen and González (1994, 1995, 2000) utilized the HPG to determine the flow carrying capacity of prismatic channels and to establish a so-called theoretical stage–discharge rating curve. The open channel HPG (Figure 1) is a set of curves relating the upstream and downstream stages (or flow depths) of a channel reach under various discharges through the use of numerical backwater computations. In essence, the HPG includes all backwater curves that could possibly occur in a channel reach and may be used as a theoretical basis to derive a stage–discharge rating curve.

To establish a HPG for a prismatic open channel reach, the normal depth 'N-line' represents the relation between upstream stage H_u and downstream stage H_d for uniform flows under various discharges where stream flow gauges are located. The N-line is a straight line and is parallel to the Z-line, which represents the relation between upstream and downstream stages in the reach when zero discharge occurs, i.e. $H_d = H_u$. From the HPG, the values of uniform-flow discharge and corresponding stage at the upstream gauge, read from the intersection of the N-line and the hydraulic performance curve (*HPC*) at various discharges, are the rating curve under uniform flow conditions, that is, the theoretical rating curve with constant fall.

In natural streams, channel sections are irregular and stream-crossing structures such as bridges, channel-stabilizing works, diversion weirs and check dams, may exist in the channels. The relation between upstream and downstream stages given by the theoretical N-line obtained for natural streams does not satisfy the normal flow conditions ($S_f = S_0$). The flows are influenced by irregular channel cross-sections and the presence of stream-crossing structures. In addition to friction losses, the head losses due to geometry changes of varying channel sections must be included. The velocity head terms in the energy equation for backwater computations cannot be neglected. Hence, the normal depth N-line in the HPG will be a curve and its shape depends on the features of channel geometry that affect the energy loss computation. The N-line for natural streams should be determined through the backwater computation under fixed condition where the energy slope at the downstream section equals the average channel bed slope in the reach (Wu *et al.*, 2003).

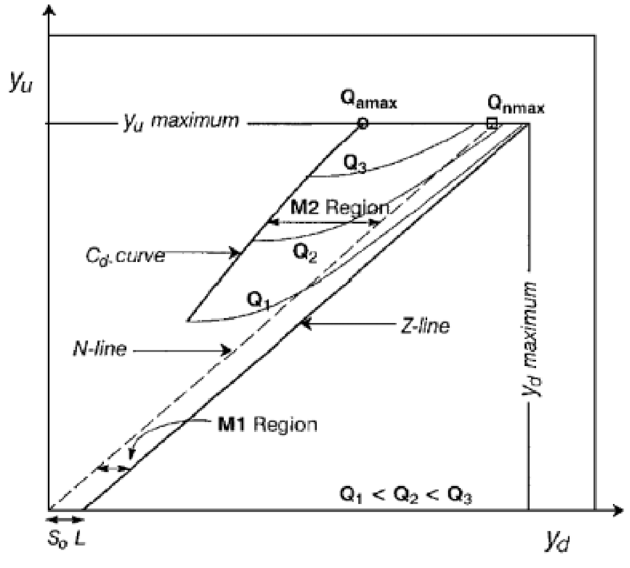

Figure 1. Schematic diagram of HPG for mild-slope open channel

The N-line thus determined represents the relation between the upstream and downstream stages under various discharges when the flows at the downstream section are at the corresponding normal depth. The stages and discharges at the intersection of the N-line and the HPG of various discharges represent the theoretical rating curve of the type where fall is a function of stage for the stream gauge being considered. The theoretical rating curve will be used as the base rating curve in the USGS rating method to establish a stage–fall–discharge functional relationship.

THE PROCEDURE OF ESTABLISHING HPG

The procedure to establish HPG is as follows:

1. Specify roughness coefficient and average channel slope S_0 for the channel reach.
2. Establish Z-line representing horizontal water surface in the reach under zero-discharge conditions, that is, upstream stage equals downstream stage.
3. Select a discharge (Q) to
    (1) compute critical depth y_c at downstream section;
    (2) perform backwater computations to find the upstream stage H_u; and
    (3) obtain pair($H_u, H_d | y_d = y_c$) to define a point on the C-curve of the HPG for the given discharge Q.
4. For the specified Q in step 3, select a downstream stage H_d such that $y_d > y_c$ and perform backwater computations to find the corresponding upstream stage H_u.
5. Select different values of H_d and repeat step 4 to find a set of ($H_{u,}H_d$) pairs to construct the HPC for the specified discharge Q.
6. Assuming the energy slope at the downstream section equals the average channel slope of the reach, compute normal depth y_n (hence H_d) for the discharge Q and perform backwater computations to obtain the pair ($H_{u,}H_d$).
7. Select a different value of Q and repeat steps 3–6 to establish the *HPC*'s for the discharge.
8. Construct the N-line by connecting all the points ($H_{u,}H_d$) found in step 6.

For backwater computations, various methods (Chow, 1959) and computer models are available. In this study, a one-dimensional hydraulic model, HEC-RAS version

3·01, developed by the US Army Corps of Engineers (2001) is employed for backwater computations.

FRAMEWORK OF ANALYTICAL RATING PROCESS

The head losses due to varying cross-section in a non-prismatic channel or a natural stream cannot be neglected in energy loss calculation because flow velocity is not constant and friction loss is not the total energy loss as in uniform flow, where $S_f = F/L$. The USGS rating method uses measured data that are often insufficient to derive reliable stage–fall–discharge relation in practice. Although the HPG for establishing the stage–fall–discharge relation contains all relevant information at base gauge and auxiliary gauge, the accuracy of the resulting rating curve may be influenced by human errors when interpolations between curves on HPG are made in finding the discharge. To simplify the determination of discharge, a functional relation of stage–fall–discharge, similar to the one used in the USGS rating method, is proposed:

$$\frac{Q_m}{Q_r} = c\left(\frac{F_m}{F_r}\right)^{d'} \tag{3}$$

in which c and d' are coefficients. For uniform flows in prismatic channels, the value of c would approach 1·0 and the value of d' would approach 0·5. Note that the USGS rating method uses $c = 1{\cdot}0$.

For a given channel reach, through repeated backwater computations using the HEC-RAS model, a database containing (H_u, H_d, Q) can be established to derive the functional relation stage–fall–discharge for the channel reach considered. The (H_u, H_d, Q) database could cover the range of medium to high discharges to supplement the measured stage–discharge data for constructing the rating curve. The procedure is summarized as follows:

1. From the N-line of HPG, select several discharges and the corresponding stages to establish the base rating curve for the upstream gauge.
2. For a given stage at the base gauge, extract fall F_r and the corresponding discharge Q_r from the base rating curve. In addition, obtain the fall F_m and the corresponding discharge Q_m from HPG.
3. Repeat step 2 for a number of stages at the base gauge.
4. Based on the data set of stage, F_r, Q_r, F_m, and Q_m, plot F_m/F_r versus Q_m/Q_r, conduct regression analysis to derive the functional relation in the form of Equation (3).
5. The accuracy of the rating curve depends very much on the flow conditions downstream from the base gauge. The backwater or drawdown downstream from the base gauge will have effects on the rating curve. To investigate the effects of different flow conditions on the stage–fall–discharge relationship and correction for discharge, the relation between discharge, stage and fall is analysed separately for backwater and drawdown conditions to obtain the respective stage–fall–discharge relation for the base gauge.
6. Discharges, Q_m (data required for the USGS rating method), are plotted against the corresponding stages to obtain the stage–discharge relation for the base gauge.

After plotting stage versus rating fall (F_r), stage versus rating discharge (Q_r) and Q_m/Q_r versus F_m/F_r are established, the discharge at the gauge can be estimated directly from the observed stages at base and auxiliary gauges.

The rating procedure described above is summarized in Figure 2. In practice, the stage–fall–discharge rating method studied herein may be applied to both hydrological stations with and without stream flow gauges on natural streams. Application of the proposed method to the Keelung River in Taiwan is carried out to examine the reliability of discharge estimation at several existing stream flow gauging stations and to investigate the feasibility of the proposed rating method for other applications.

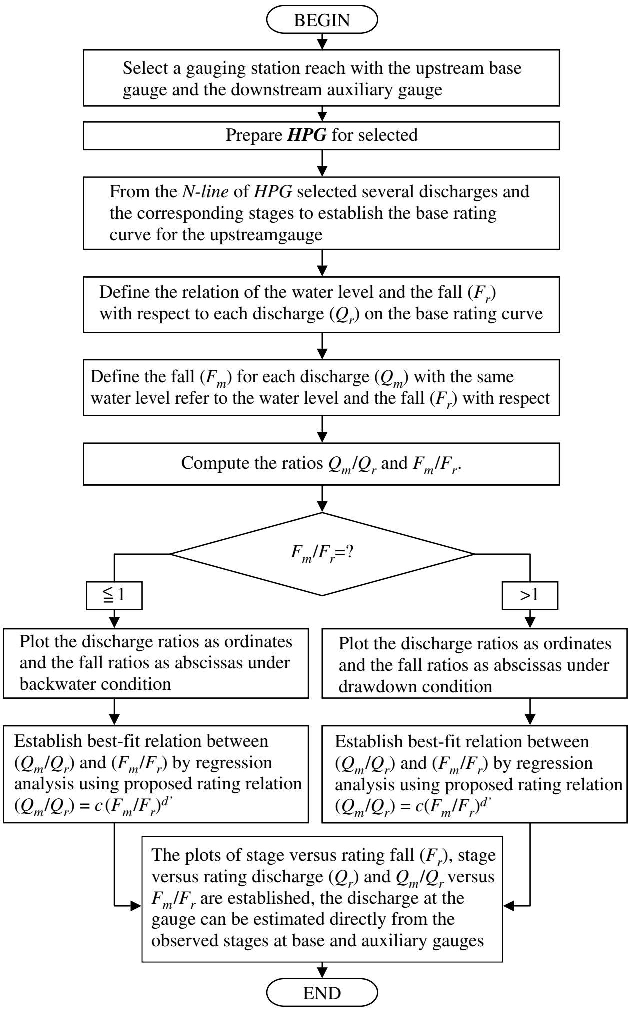

Figure 2. The process of deriving an analytical rating method

APPLICATION

Along the Keelung River in northern Taiwan, channel cross-sections at stream flow gauge stations have been regularly surveyed since 1969 and flow measurements are available since 1962. Stations with relatively more measured data information are selected in this case study.

Hydrologic stations and study reaches

There are 13 stream flow gauging stations along the Keelung River as shown in Figure 3a. No auxiliary gauge is installed at any of the gauging stations and their rating curves are established according to measured stage and discharge data. Of all the stations, Wu-Du station possesses most measurements of stage–discharge data in that the maximum observed discharge is 1150 $m^3 sec^{-1}$, which has a return period of approximately 5 years. In the case study the application of the proposed rating method to three gauging scenarios is considered:

(1) Channel reach with hydrologic station. The Wu-Du station (shown in Figure 3a) is selected. The

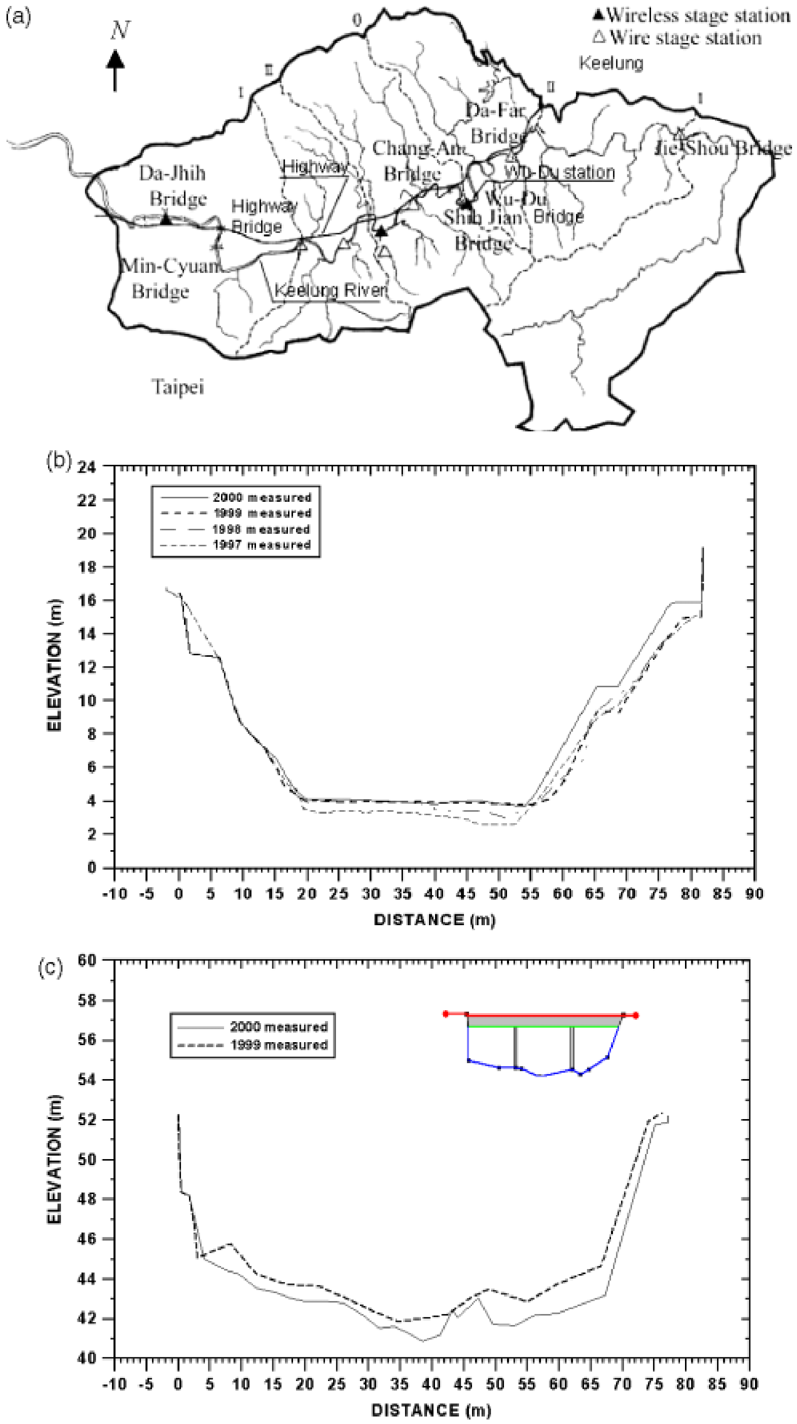

Figure 3. (a) Locations of stream flow gauging stations along the Keelung River; (b) Wu-Du station cross-section; (c) Jie-Shou Bridge station cross-section

station has been in operation since 1962. The channel cross-sections at the station from 1997 to 2000 are shown in Figure 3b. In backwater computations all flows are assumed to be confined within the channel. Since there is no specific auxiliary gauge for the Wu-Du station (base gauge), Shih-Jian Bridge, located 306 m downstream is used as the downstream auxiliary gauge. The river reach between the base gauge and auxiliary gauge has an average width of approximately 80 m and there is a Wu-Du Bridge located downstream of Wu-Du station.

(2) Channel reach with gauging station located at downstream side of bridge pier. The Jie-Shou Bridge station (Figure 3a) is located on the downstream side of the Jie-Shou Bridge pier. The station has been in operation since 1981. The cross-sections for 1999 and 2000 at the station are shown in Figure 3c. Since no auxiliary gauge exists for the station, the neighbouring downstream section, located 515 m downstream, is taken as the auxiliary gauge.

(3) Channel reach without stream-flow gauging station. In the lower reach of the Keelung River, gauges with adequate measured discharge data are lacking. To estimate discharges in the lower Keelung River during major flood events, an investigation is made to examine the applicability of the proposed rating method to a river reach without a stream flow gauging station. For this purpose, the reach between Min-Cyuan Bridge and Da-Jhih Bridge (shown in Figure 3a) is selected. At Min-Cyuan Bridge data on high-water marks during typhoon events are available and at the downstream Da-Jhih Bridge a river stage recorder is installed. This study reach is approximately 4320 m in length and there is a highway Bridge located at downstream of Min-Cyuan Bridge.

Numerical model setup, calibration, and verification

Before HPG*s* for the selected reaches are developed, Manning's roughness coefficient for the study reaches are calibrated through unsteady flow simulations for the entire river reach so that the computed water surface elevations match well with the measured stage hydrograph.

The unsteady flow model of HEC-RAS (Version 3·01) is used to simulate the flood flow during Typhoon

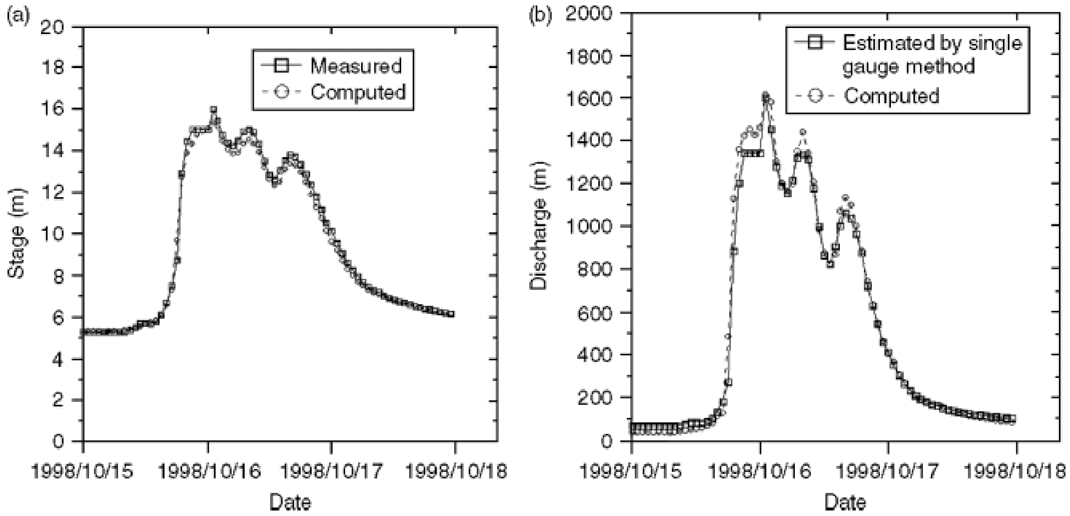

Figure 4. (a) Stage hydrograph; (b) discharge hydrograph at Wu-Du station for Typhoon Zeb

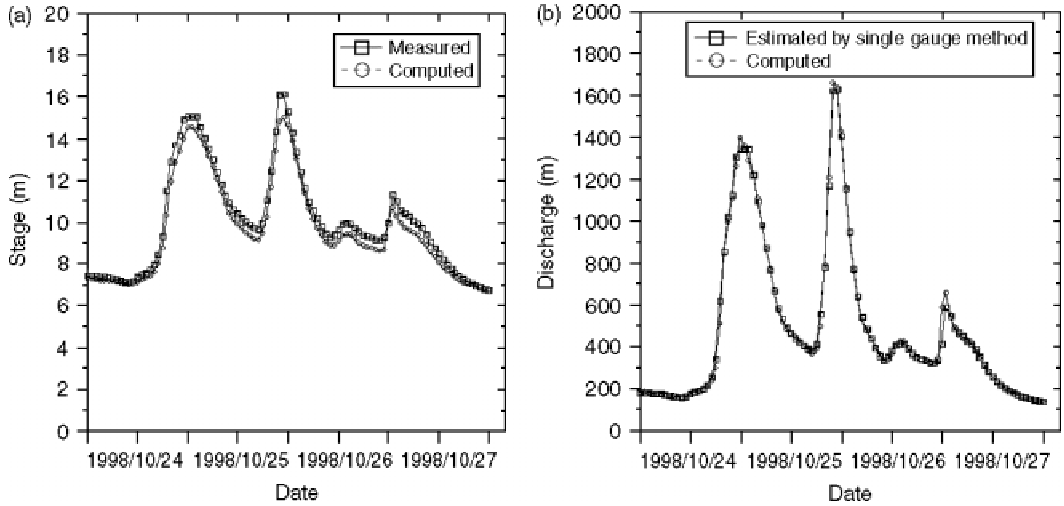

Figure 5. (a) Stage hydrograph; (b) discharge hydrograph at Wu-Du station for Typhoon Babs

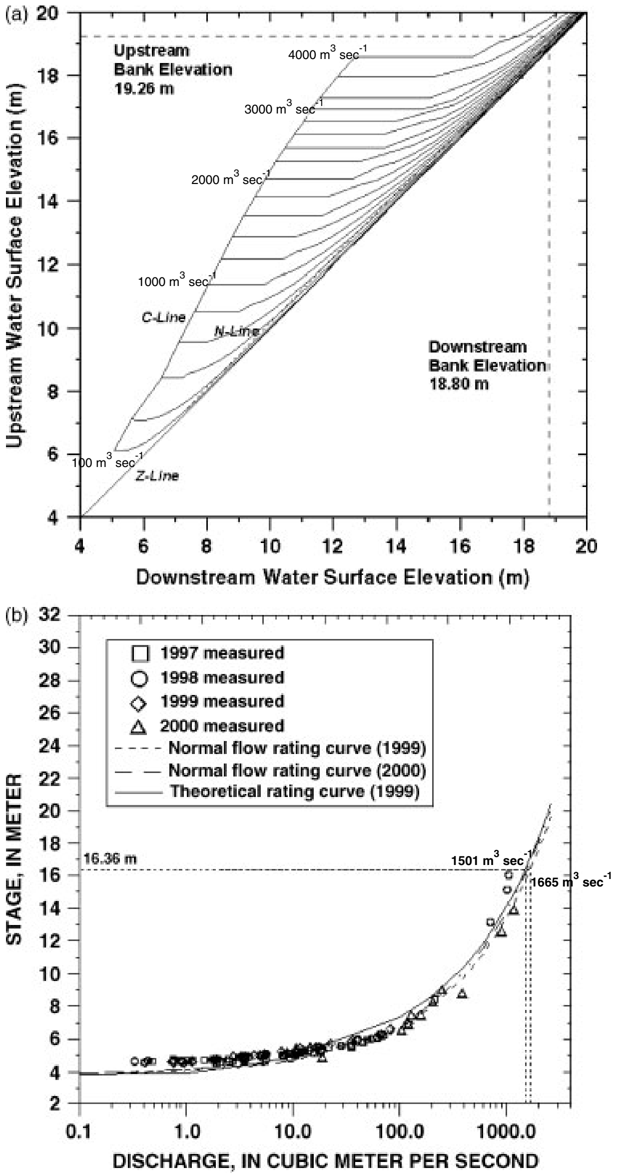

Figure 6. (a) HPG for Wu-Du station reach; (b) comparison of theoretical rating curve and measured data for Wu-Du station

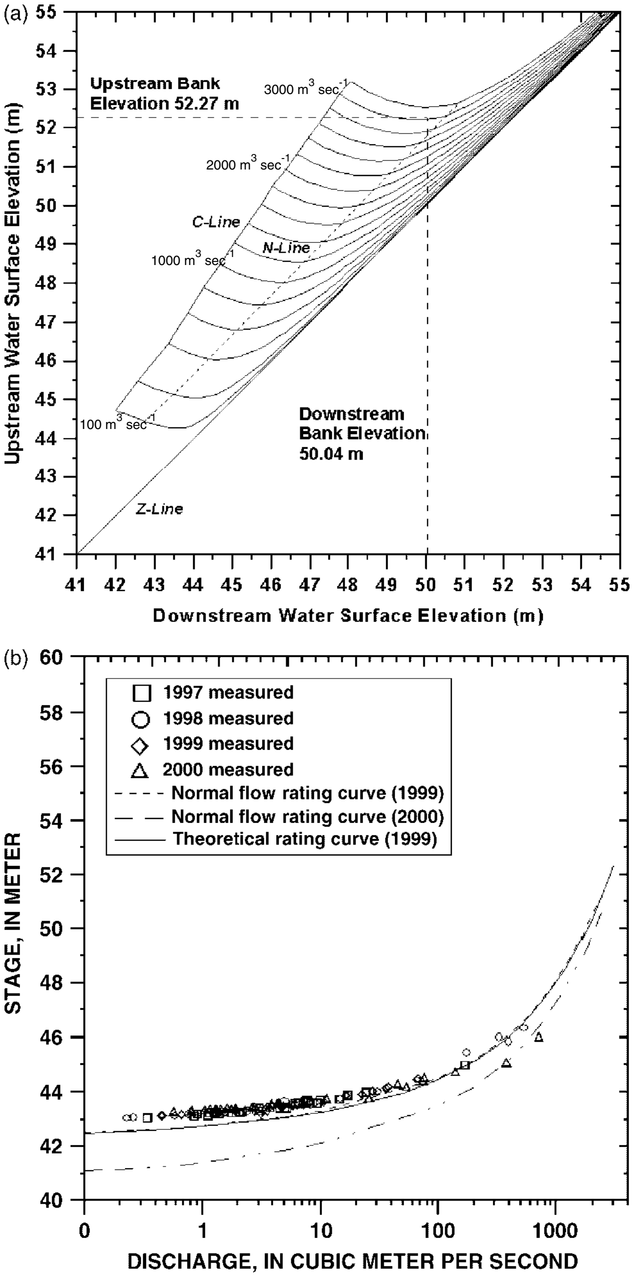

Figure 7. (a) HPG for Jie-Shou Bridge reach; (b) comparison of theoretical rating curve and measured data for Jie-Shou Bridge station

Zeb on 16 October 1998. Comparison of simulated water surface elevations is made with the measured stage hydrograph at the gauge to calibrate Manning's roughness coefficient for the reach where the gauge is located. Channel cross-sections measured in 1999 are used in the simulation. The main reason for using the Typhoon Zeb event for calibration is because relatively more complete flood information is available. After calibration, the flood event Typhoon Babs on 26 October 1998 is simulated to verify the calibrated Manning's roughness coefficient. Stage hydrographs at Jie-Shou Bridge, Da-Far Bridge, Wu-Du and Chang-An Bridge (refer to Figure 3a) are used as the basis for calibration. For the study reach where Wu-Du station is located, Manning's n values are calibrated to be 0·035 in the main channel and 0·050 on the overbanks. Simulated stage and discharge hydrographs at Wu-Du station for Typhoon Zeb are shown in Figure 4a and 4b. For verification, the simulated stage and discharge hydrographs at Wu-Du station for the Typhoon Babs event are shown in Figure 5a and 5b. Manning's n values determined through calibration and verification of unsteady flow simulations in this study are considered to be satisfactory.

However, deviations of the simulated hydrograph from the measured one at high discharges can be noted from Figures 4b and 5b. This is because the measured discharge hydrographs are derived directly from the recorded stage hydrographs on the basis of the single-gauge stage–discharge rating curve. In general, data from direct measurements on the rating curve in high stages are often inadequate or even lacking, discharges associated with high stages thus derived from the rating curve involve high uncertainties. In addition, the

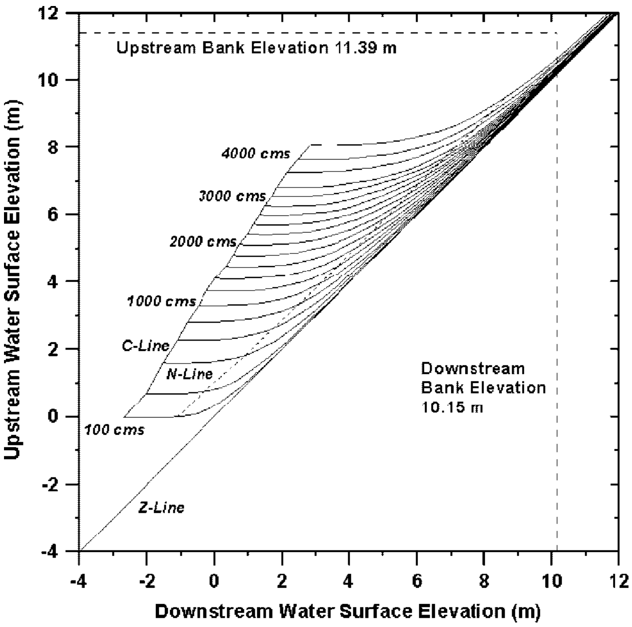

Figure 8. HPG for the reach from Min-Cyuan Bridge to Da-Jhih Bridge

stage–discharge relations derived using single-gauge measurement data often have questionable accuracy.

RESULTS AND DISCUSSIONS

Establishment of HPG and theoretical rating curve

The HPG for the reach from Wu-Du to Shih-Jian (scenario-1) is established following the procedure previously described. Backwater computations start with a critical depth to find water surface elevation at the upstream section for various discharges ranging from 100 to 4000 $m^3 sec^{-1}$. The resulting HPG for Wu-Du station is shown in Figure 6a. The theoretical rating curve as shown in Figure 6b for Wu-Du station is obtained by plotting upstream stage against discharge from the N-line. Similar results for HPG and a theoretical rating curve for Jie-Shou Bridge reach (scenario-2) are shown in Figure 7a and 7b. Regarding scenario-3, the resulting HPG for the reach between Min-Cyuan Bridge (where high-water marks are available) and Da-Jhih

Table I. Comparisons of discharge estimated using HPG and different rating curves

(a) Wu-Du station reach

Flood event Gauging station Maximum stage (m)	Typhoon Xangsane		Typhoon Zeb	
	Auxiliary 17·89	Base 17·98	Auxiliary 16·02	Base 16·36
Single-gauge rating curve ($m^3 sec^{-1}$)	1900		1600	
1999 normal-flow rating curve ($m^3 sec^{-1}$)	2100		1665	
1999 theoretical rating curve ($m^3 sec^{-1}$)	1905		1501	
HPG ($m^3 sec^{-1}$)	1200*		1630	

(b) Jie-Shou Bridge reach

Flood event Maximum stage (m)	Typhoon Xangsane		Typhoon Zeb	
	Jie-Shou Bridge reach			
	Auxiliary —	Base 49·54	Auxiliary —	Base 47·28
Single-gauge rating curve ($m^3 sec^{-1}$)	1600		678	
1999 normal-flow rating curve ($m^3 sec^{-1}$)	1500		729	
1999 theoretical rating curve ($m^3 sec^{-1}$)	1600		750	
HPG ($m^3 sec^{-1}$)	—		—	

(c) Min-Cyuan Bridge reach

Flood event Gauging station Maximum stage (m)	Typhoon Xangsane		Typhoon Zeb	
	Auxiliary 7·30	base 7·90	Auxiliary 5·50	base 6·40
Single-gauge rating curve ($m^3 sec^{-1}$)	—		—	
1999 normal-flow rating curve ($m^3 sec^{-1}$)	—		—	
1999 theoretical rating curve ($m^3 sec^{-1}$)	3108		2072	
HPG ($m^3 sec^{-1}$)	2800		2100	

* There were backwater effects due to the blockage at bridge opening and overbank flow downstream from Wu-Du during Typhoon Xangsane.

DOI: 10.1002/hyp

Bridge (where a river stage recorder is available) is shown in Figure 8. The corresponding theoretical rating curve for the Min-Cyuan Bridge crest-stage gauge can be obtained from the N-line. These HPGs and theoretical rating curves for the study reaches are used in later analyses.

Suitability of the theoretical rating curve

To investigate the suitability of using the theoretical rating curve for the stream flow gauging station, the theoretical rating curve is compared with the normal flow rating curve obtained by assuming that the energy slope at the gauging station equals the average channel slope. For Wu-Du gauging station, the average channel slope is about 0·0003 and the normal depths are computed for various discharges from 100 $m^3 sec^{-1}$ to 4000 $m^3 sec^{-1}$. The resulting stage–discharge rating curves are plotted in Figure 6b for comparison.

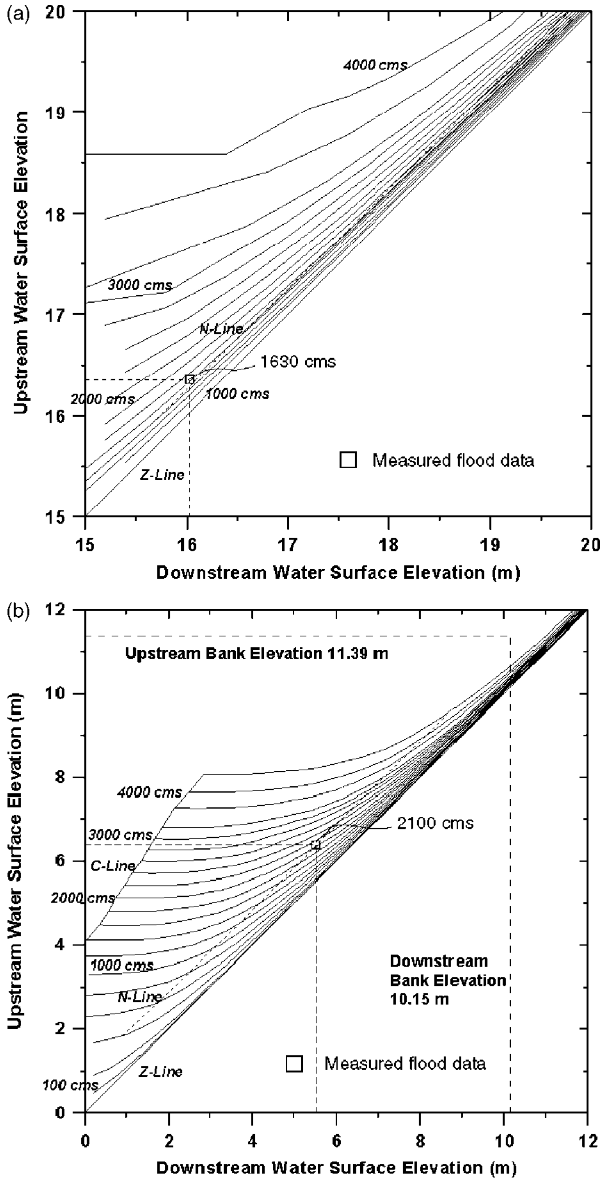

Figure 9. Typhoon flood discharge evaluated by HPG for (a) Wu-Du station reach and (b) Min-Cyuan Bridge reach

Table II. Regressional c & d' values and statistics for different flow conditions at three gauging stations

(a) Wu-Du station reach

Flow condition	c	d'	R^2	S_x
Drawdown	0·9884	0·3208	0·978	—
Backwater	0·9614	0·4956	0·982	—
Mixed (proposed)	0·9081	0·4582	0·985	0·090
Mixed (USGS)	1·0000	0·4376	—	0·254

(b) Jie-Shou Bridge reach

Flow condition	c	d'	R^2	S_x
Drawdown	—	—	—	—
Backwater	1·1114	0·4427	0·980	—
Mixed (proposed)	1·1114	0·4427	0·980	0·108
Mixed (USGS)	1·0000	0·4097	—	0·198

(c) Min-Cyuan Bridge reach

Flow condition	c	d'	R^2	S_x
Drawdown	1·0101	0·2603	0·975	—
Backwater	1·0377	0·4660	0·994	—
Mixed (proposed)	0·9788	0·4400	0·989	0·231
Mixed (USGS)	1·0000	0·4469	—	0·266

R^2 is the coefficient of determination; S_x is the standard error.

Comparisons of the theoretical rating curve (based on the base and auxiliary gauges) with the normal-flow rating curve in Figure 6b indicate that Wu-Du gauging station is indeed influenced by backwater due to varying channel cross-section geometry and the presence of the bridge. However, the effect is not so significant and, for this reason, it appears to be a good site for a stream flow gauge. It is also noted from Figure 6b that relatively higher discharges were measured in 1998 and 2000, and the measured data in 2000 are closer to the theoretical rating curve.

Regarding the Jie-Shou Bridge gauging station in scenario-2, it is noted from Figure 7b that relatively higher discharge measurements were estimated in 1998 and 2000. Actual measured data, except that of year 2000, are quite close to the theoretical rating curve, which almost coincides with the normal-flow rating curve. This implies that steady uniform flows prevail downstream from the base gauge, and that it is a good site for stream gauge. It should also be noted that the normal-flow rating curve based on year 1999 data deviates significantly from that of year 2000 data. This is the result of great changes in cross-section at Jie-Shou Bridge gauge (Figure 3c).

The results for Wu-Du and Jie-Shou Bridge gauging stations indicate that the theoretical rating curve derived on the basis of normal flows at the auxiliary gauge is quite reliable. However, deviation of measured data from the theoretical rating curve increases as discharge increases, indicating that the dependence of discharge on energy slope is becoming more apparent. Whenever a rating curve derived from single-gauge measurements

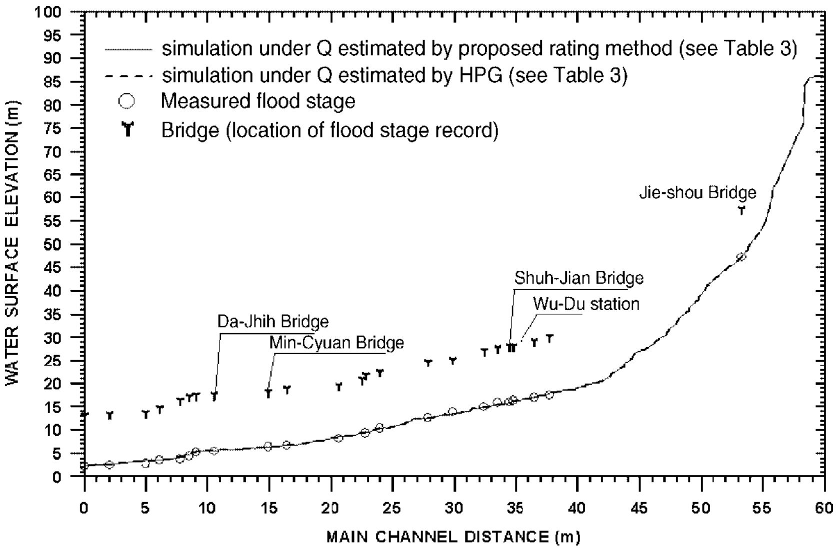

Figure 10. Comparison of simulated flood stages and measured stages for Typhoon Zeb along the Keelung River

is used to estimate discharges, appropriate corrections should be made. Also observed is that the theoretical rating curve deviates much more from the normal-flow rating curve as discharges become larger. This indicates that the energy slope is used to account for backwater effect when applying the HPG-based theoretical rating curve to estimate discharges.

Comparison of discharge estimated from different rating curves

Comparisons of discharge estimated from HPG and different rating curves are tabulated in Table I. To evaluate the stage–discharge relation for gauging stations subject to backwater effect, discharges can be estimated from the HPG based on upstream and downstream stages. Comparison between HPG-based discharge and those from the single-gauge rating curve can be made to check the reliability of the latter. Referring to Table I(a), for Wu-Du gauging station, the discharge of 1630 $m^3 sec^{-1}$ is estimated from the HPG (Figure 6a) based on the maximum stage of 16·02 m at downstream Shih-Jian Bridge and that of 16·36 m at Wu-Du during Typhoon Zeb, as shown in Figure 9a. The estimated discharge is different from the 1665 $m^3 sec^{-1}$ estimated by the 1999 normal flow rating curve and 1501 $m^3 sec^{-1}$ estimated by the 1999 theoretical rating curve at the stage of 16·36 m (Figure 6b).

For Min-Cynan Bridge, Table I(c) shows that the discharge of 2100 $m^3 sec^{-1}$ is estimated from the HPG based on the maximum stages of 5·5 m at Da-Jhih Bridge and 6·40 m at Min-Cyuan Bridge during Typhoon Zeb (Figure 9b). To verify the reliability of these discharge estimates, 750 $m^3 sec^{-1}$ at the Jie-Shou Bridge gauge, 1630 $m^3 sec^{-1}$ at Wu-Du gauge and 2100 $m^3 sec^{-1}$ at Da-Jhih Bridge are used to simulate the flood event Typhoon Zeb. Simulation results, shown as the dashed line in Figure 10, reveal that the flood stage profile based on discharges estimated from the HPG with known upstream and downstream stages, agree very well with measured data.

Examination on different stage-fall-discharge relations

Based on the computed results for the study reaches following various rating procedures, stage–fall plots indicate that fall is a function of stage for all three gauging stations, as shown in Figures 11a, 12a and 13a. The stage–fall–discharge relations shown in Figures 11b, 12b, and 13b for these stations are derived in the form $(Q_m/Q_r) = (F_m/F_r)^d$ used by the USGS and the form $(Q_m/Q_r) = c(F_m/F_r)^{d'}$ that considers mixed conditions of backwater and drawdown for the study reaches. For Wu-Du gauging station (Figure 11b), the value of d' and c were found to be 0·46 and 0·908, respectively. The same data fitted by the USGS form yield a d value of 0·44 (dashed line in Figure 11b).

To investigate the difference in rating relation under different flow conditions and the effect of rating relation on discharge corrections, the stage–fall–discharge relations are also derived separately for backwater conditions and for drawdown conditions following the rating process shown in Figure 2. The results are shown in Figures 11c, d, 12c and 13c, d, and in Table II. It can be seen from Table II that c value ranges from 0·90 to 1·10 whereas d' value falls between 0·4 and 0·6 for backwater conditions and is smaller than 0·4 for drawdown conditions. For Wu-Du gauging station, the d' value was found to be 0·50 and 0·32 under backwater and drawdown conditions, respectively, and a better correlation between discharge and fall is found for the backwater condition. At Jie-Shou Bridge station, the stage–fall–discharge relation exists only for backwater conditions. The analyses for all the study reaches indicate that application of the USGS rating relation (shown as dashed line) to natural streams does not appear to be quite adequate to reflect the relation between stage, fall and discharge for discharge correction although it has been used in practice. The discharges after correction using USGS rating relation shown in

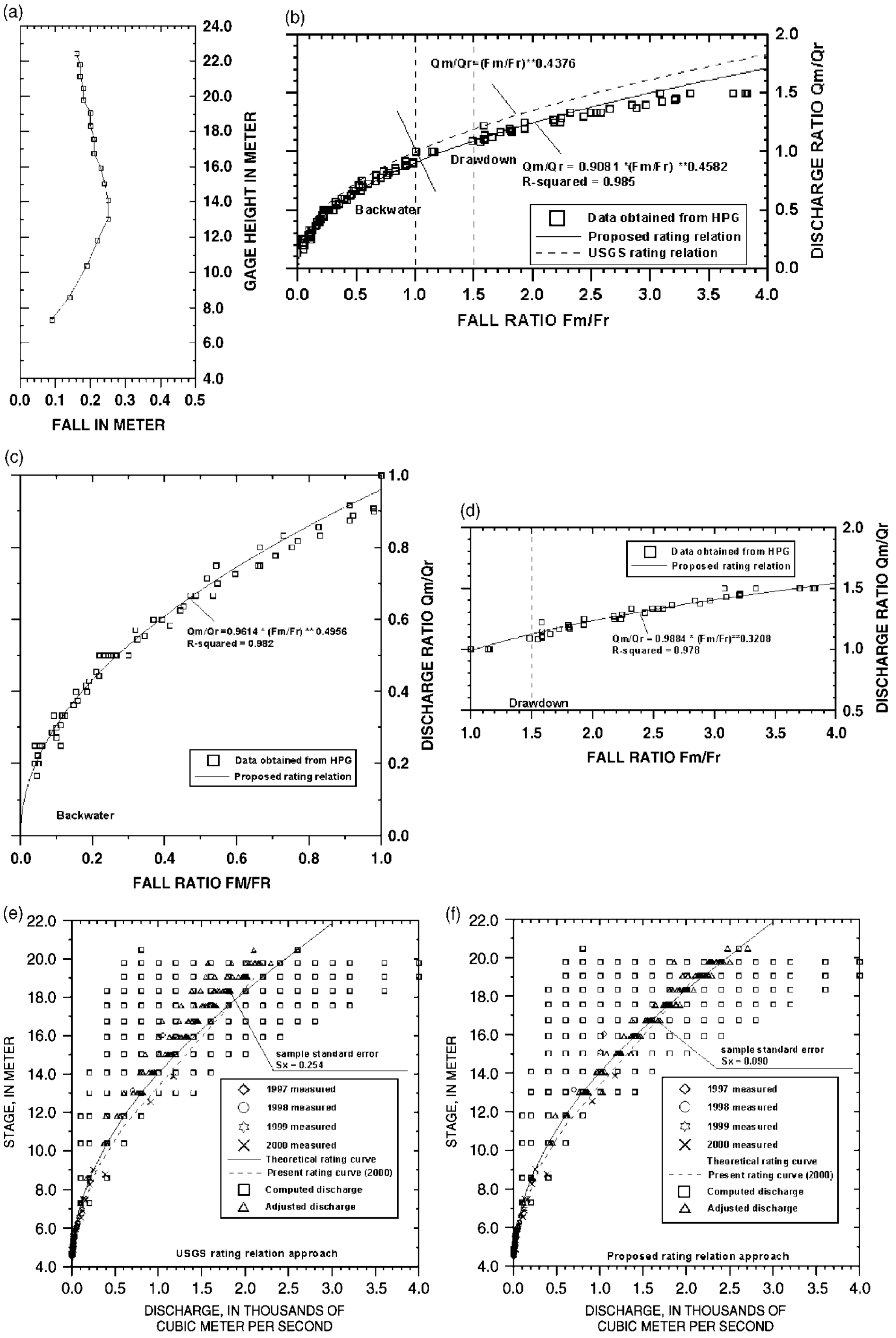

Figure 11. (a) Stage–fall relation. (b) Discharge–fall relation. (c) Discharge–fall relation under backwater conditions. (d) Discharge–fall relation under drawdown conditions. (e) Corrected stage–discharge relation using USGS rating method. (f) Corrected stage–discharge relation using proposed rating method at Wu-Du station

Figures 11b, 12b and 13b and the corresponding stages are plotted in Figures 11e, 12d and 13e along with the HPG-based theoretical rating curve, and correction using the proposed rating relation obtained following step 6 of the proposed rating process shown in Figures 11b, 12b and 13b, and the corresponding stages are plotted in Figures 11f, 12e and 13f. Their correction statistics are given in Table II. Comparisons of standard errors in the difference between adjusted discharges and rating discharge from the theoretical rating curve indicate that the adjusted discharges using the proposed rating relation $(Q_m/Q_r) = c(F_m/F_r)^{d'}$ are in better agreement with the theoretical rating curve than those using the exponential form of Equation (2).

Discussions on discharge corrected by stage-fall-discharge relations

As shown in Figures 11f, 12e and 13f, computed discharges (equivalent to estimated discharges in application) after the correction appear to be closer to the theoretical rating curves, and measured data points fall

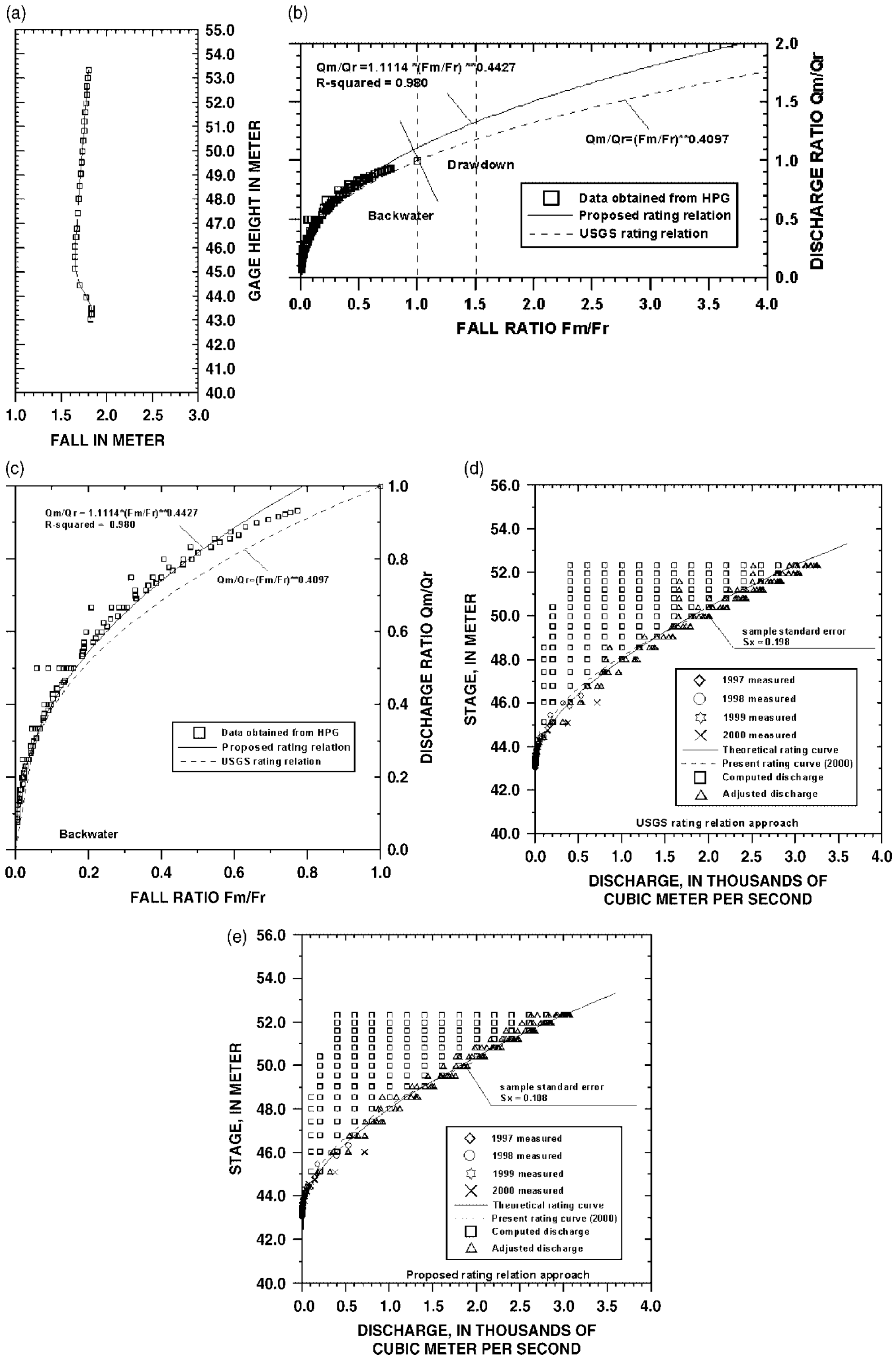

Figure 12. (a) Stage–fall relation. (b) Discharge–fall relation. (c) Discharge–fall relation under backwater conditions. (d) Corrected stage–discharge relation using USGS rating method. (e) Corrected stage–discharge relation using proposed rating method at Jie-Shou Bridge

within the spread of computed data. Hence, the HPG-based theoretical rating curve in conjunction with the Q_m/Q_r versus F_m/F_r relation provides acceptable rating curves for natural streams. Owing to the lack of measured high discharges and thus requiring extension of the rating curves beyond the data range covered by the conventional rating method, discharges corresponding to high stages are underestimated at Wu-Du station (Figures 6a and 11f) and overestimated at the Jie-Shou Bridge station (Figures 7a and 12e), compared to the results from the theoretical rating curves.

Results obtained in the previous section are applied to estimate flood discharge at Wu-Du station and Min-Cynan Bridge during Typhoon Zeb. For Wu-Du station, the maximum stage is 16·02 m at Shih-Jian Bridge and 16·36 m at Wu-Du station, hence the stage

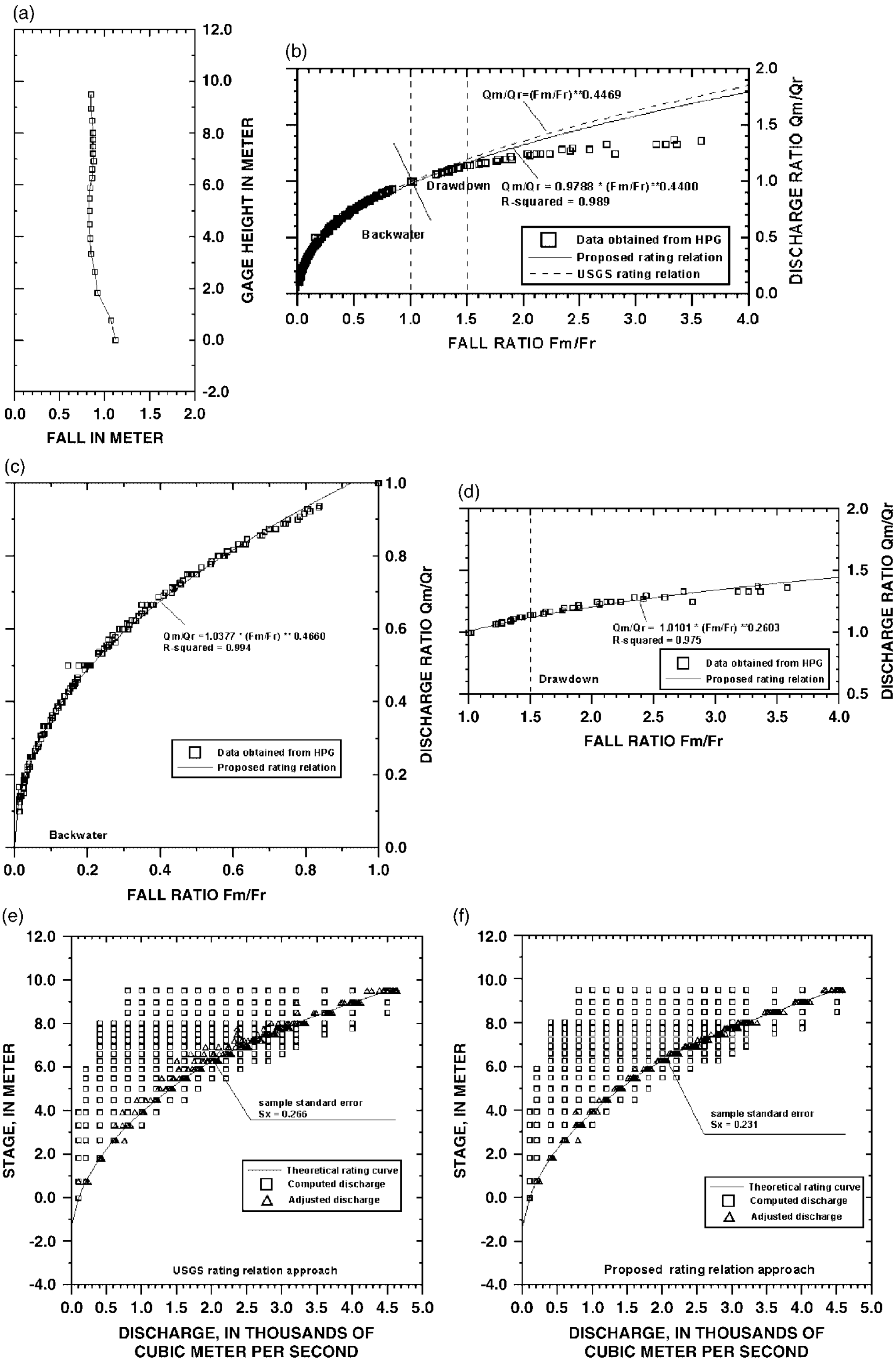

Figure 13. (a) Stage–fall relation. (b) Discharge–fall relation. (c) Discharge–fall relation under backwater conditions. (d) Discharge–fall relation under drawdown conditions. (e) Corrected stage–discharge relation using USGS rating method. (f) Corrected stage–discharge relation using proposed rating method at Min-Cyuan Bridge

fall, F_m, is 0·34 m. The rating fall, F_r, is found to be 0·22 from the stage versus rating fall plot in Figure 11a. The rating discharge, Q_r, is found to be 1501 m^3 sec^{-1} from the theoretical rating curve in Figure 11e. The stage–fall–discharge relation $Q_m/Q_r = 0{\cdot}9884(F_m/F_r)^{0{\cdot}3208}$ for the drawdown condition shown in Figure 11d (since $F_m/F_r = 0{\cdot}34/0{\cdot}22 > 1{\cdot}0$) is used to obtain the estimated discharge of 1706 m^3 sec^{-1}. At Min-Cynan Bridge, the maximum stage is 5·50 m at Da-Jhih Bridge and 6·40 m at Min-Cynan Bridge, hence the measured fall, F_m, is 0·90 m. The rating fall, F_r, is found to be 0·89 from the stage versus rating fall

Table III. Comparisons of discharge estimated using various rating methods

(a) Wu-Du station reach

Flood event	Typhoon Xangsane		Typhoon Zeb	
Gauging station Maximum stage (m)	auxiliary 17·89	base 17·98	auxiliary 16·02	base 16·36
Single-gauge rating curve ($m^3\,sec^{-1}$)	1900		1600	
HPG ($m^3\,sec^{-1}$)	1200*		1630	
USGS rating method	1329*		1816	
Proposed rating method	1218*		1706	

(b) Min-Cyuan Bridge reach

Flood event	Typhoon Xangsane		Typhoon Zeb	
Gauging station Maximum stage (m)	auxiliary 7·30	base 7·90	Auxiliary 5·50	base 6·40
Single-gauge rating curve ($m^3\,sec^{-1}$)	—		—	
HPG ($m^3\,sec^{-1}$)	2800		2100	
USGS rating method	2606		2093	
Proposed rating method	2684		2105	

* There were backwater effects due to the blockage at bridge opening and overbank flow downstream from Wu-Du during Typhoon Xangsane.

plot in Figure 13a. The rating discharge, Q_r, is found to be 2072 $m^3\,sec^{-1}$ from the theoretical rating curve in Figure 13e. The stage–fall–discharge relation $Q_m/Q_r = 1{\cdot}0101(F_m/F_r)^{0{\cdot}2603}$ for the drawdown conditions shown in Figure 13d (since $F_m/F_r = 0{\cdot}90/0{\cdot}89 > 1{\cdot}0$) is used to obtain the estimated discharge of 2105 $m^3\,sec^{-1}$ (the discharge estimated previously from the HPG is 2,100 $m^3\,sec^{-1}$). Comparisons of discharges estimated by the various rating methods are listed in Table III. Although the estimated discharges are somewhat higher than the discharges estimated previously using the HPG, the method is more convenient to use and less prone to human error. Meanwhile, the analytical rating method and its practical applications proposed herein appear to be highly reliable, as evidenced by the verification results using measured data shown in Figure 10 (solid line). Since no auxiliary gauge exists at Wu-Du gauging station, use of high-water marks at Shih-Jian Bridge as the auxiliary gauge may not provide an adequate stage reading downstream. It is hoped that streamflow gauging stations will have base gauge and auxiliary gauge installed so that better quality measurements can be made available to improve the accuracy of the stage–discharge relation.

CONCLUDING REMARKS

A HPG is developed for natural streams. The stages at the gauging station and the discharges read from the intersection of the N-line and the hydraulic performance curves in the HPG at different discharges are used to establish the theoretical rating curve for the gauging station, where rating fall is a function of stage. All computed data for the HPG are plotted and used to derive charts and functional forms of stage–fall–discharge relations. A case study on the Keelung River is carried out to examine the reliability of discharge rating results for the existing stream flow gauging stations and to investigate the feasibility of the proposed rating methods for other scenarios. The following conclusions may be drawn:

1. This study used the HPG to derive rating curves as the basis for establishing a stage–fall–discharge relation following the USGS rating procedure.
2. Unlike conventional rating methods, which rely on limited measured data and empirical processes for corrections, the proposed analytical rating method uses a hydraulic modelling tool to obtain a wider range of data to establish the rating curve.
3. Owing to backwater effects and spatial variations in the channel cross-section, the proposed stage–fall–dis-charge relation for natural streams takes the form $(Q_m/Q_r) = c(F_m/F_r)^{d'}$ where the value of d' varies between 0·4 and 0·6 under backwater conditions and is lower than 0·4 under drawdown conditions, and c falls between 0·90 and 1·10.
4. From the case study on the Keelung River, comparisons of estimated discharges using the HPG-based stage–fall–discharge relation $(Q_m/Q_r) = c(F_m/F_r)^{d'}$, fitting actual measured data, and the single-gauging rating curves at existing gauging stations indicates that the proposed rating method is capable of supplementing shortcomings of the conventional stage–discharge rating procedure. Hence, it enhances the reliability of flood estimates and forecasting.
5. The stage–fall–discharge relation depends upon the geometries of the channel and cross-section, channel

bed slope, and Manning's roughness coefficient. Further studies on the influences and uncertainties of each of these parameters are necessary.

ACKNOWLEDGEMENTS

The authors would like to express their sincere gratitude to Dr J. N. Hsieh and Professor Y. K. Tung for generously sharing their time and knowledge to advise and help with this paper. The financial support from Water Resources Agency, Taiwan is greatly appreciated.

REFERENCES

Chow VT. 1959. *Open Channel Hydraulics*, McGraw-Hill: New York.

Corbett DM, *et al*. 1943. *Stream gauging procedure*. US Geological Survey, Water–Supply Paper 888, pp. 130–167.

Eisenlohr WS Jr. 1964. *Discharge ratings for streams at submerged section controls*. US Geological Survey, Water–Supply Paper 1779-L.

Gonzalez-Castro JA, Yen BC. 2000. *Applicability of hydraulic performance graph for unsteady flow routing*. Civil Engineering Studies, Hydraulic Engineering Series, No. 64, University of Illinois at Urbana-Champaign.

Mitchell WD. 1954. *Stage–fall–discharge relations for steady flow in prismatic channels.* US Geological Survey, Water–Supply Paper 1164.

Rantz SE, *et al*. 1982. *Measurement and computation of streamflow*. US Geological Survey, Water–Supply Paper 2175.

Schmidt AR. 2002. *Analysis of stage–discharge relations for open-channel flows and their associated uncertainties*. PhD dissertation, Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign.

Tsai CW-S, Yen BC. 2001. On noninertia wave versus diffusion wave in flood routing. *Journal of Hydrology* **244**: 97–104.

US Army Corps of Engineers, Hydrologic Engineering Center. 2001. *HEC-RAS, River Analysis System, Hydraulic Reference Manual*, Version 3·0. Davis, California.

Wu RB, Yang JC, Yen BC. 2003. Evaluation of theoretical rating curve for natural channel. *Journal of the Chineses Institute of Civil and Hydraulic Engineering* **15**(2): 241–252 (in Chinese).

Yen BC, Gonzalez JA. 1994. *Determination of Boneyard Creek flow capacity by hydraulic performance graph*. Research Report No. 219, Water Resources Center, University of Illinois at Urbana-Champaign.

Yen BC, Gonzalez JA. 1995. *Bottleneck analysis and channel capacity improvement alternatives for UIUC Campus portion of Boneyard Creek*, Report No. 46, Hydraulic Engineering Series, Department of Civil Engineering, University of Illinois at Urbana-Champaign.

Yen BC, Gonzalez-Castro JA. 2000. Open-channel capacity determination using hydraulic performance graph. *Journal of Hydraulic Engineering, American Society of Civil Engineers* **126**(2): 112–122.