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High-energy massive string scatterings from orientifold planes

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Abstract

We calculate bosonic massive closed string states at arbitrary mass levels scattered from orientifold planes in the high-energy, fixed angle limit. For the case of O-particle scatterings, we obtain infinite linear relations among high-energy scattering amplitudes of different string states. We also confirm that there exist only closed string Regge poles in the form factor of the O-particle amplitudes as expected. For the case of O-domain-wall scatterings, we find that, like the well-known D-instanton scatterings, the amplitudes behave like field theory scatterings, namely UV power-law without infinite Regge poles. In addition, we discover that there exist only finite number of t-channel closed string poles in the form factor of O-domain-wall scatterings, and the masses of the poles are bounded by the masses of the external legs. We thus confirm that all massive closed string states do couple to the O-domain-wall.

1. Introduction

Being a consistent theory of quantum gravity, string theory is remarkable for its soft ultraviolet structure. This is mainly due to two closely related fundamental characteristics of high-energy string scattering amplitudes. The first is the softer exponential fall-off behavior of the form factors of high-energy string scatterings in contrast to the power-law (hard) behavior of point particle field theory scatterings. The second is the existence of infinite Regge poles in the form factor of string scattering amplitudes. Recently high-energy, fixed angle string scattering amplitudes [1–3] was re-investigated for massive string states at arbitrary mass levels [4–12]. An infinite number

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of linear relations among string scattering amplitudes were obtained. The most important new ingredient of these calculations is the zero-norm states (ZNS) [13–15] in the old covariant first quantized (OCFQ) string spectrum. The existence of these infinite linear relations constitutes the *third* fundamental characteristics of high energy string scatterings. Other approaches related to this development can be found in [16],

These linear relations persist [17] for string scattered from generic Dp-brane [18] except D-instanton and D-domain-wall. For the scattering of D-instanton, the form factor exhibits the well-known power-law behavior without Regge pole structure, and thus resembles a field theory amplitude. For the special case of D-domain-wall scattering [19], it was discovered [20] that its form factor behaves as *power-law* with infinite *open* Regge pole structure at high energies. This discovery makes D-domain-wall scatterings an unique example of a hybrid of string and field theory scatterings. Moreover, it was shown [20] that the linear relations break down for the D-domain-wall scattering due to this unusual power-law behavior. This result seems to imply the coexistence of linear relations and soft UV structure of string scatterings. Recent study of high-energy scatterings of compactified closed string justified this conjecture [21]. In order to further uncover the mysterious relations among these three fundamental characteristics of string scatterings, namely, the soft UV structure, the existence of infinite Regge poles and the newly discovered linear relations stated above, it will be important to study more string scatterings, which exhibit the unusual behaviors in the high energy limit.

In this paper, we calculate massive closed string states at arbitrary mass levels scattered from orientifold planes in the high-energy, fixed angle limit. The scatterings of massless states from orientifold planes were calculated previously by using the boundary states formalism [22,23], and more recently [24] on the worldsheet of real projected plane RP_2. Many speculations were made about the scatterings of *massive* string states, in particular, for the case of O-domain-wall scatterings. It is one of the purposes of this paper to clarify these speculations and to discuss their relations with the three fundamental characteristics of high-energy string scatterings stated above. For the generic Op-planes with $p \geqslant 0$, one expects to get the infinite linear relations except O-domain-wall scatterings. For simplicity, we consider only the case of O-particle scatterings. For the case of O-particle scatterings, we obtain infinite linear relations among high-energy scattering amplitudes of different string states. We also confirm that there exist only t-channel closed string Regge poles in the form factor of the O-particle scatterings amplitudes as expected. For the case of O-domain-wall scatterings, we find that, like the well-known D-instanton scatterings, the amplitudes behave like field theory scatterings, namely UV power-law without Regge pole. In addition, we discover that there exist only finite number of t-channel closed string poles in the form factor of O-domain-wall scatterings, and the masses of the poles are bounded by the masses of the external legs. We thus confirm that all massive closed string states do couple to the O-domain-wall as was conjectured previously [19,24]. This is also consistent with the boundary state descriptions of O-planes. For both cases of O-particle and O-domain-wall scatterings, we confirm that there exist no s-channel open string Regge poles in the form factor of the amplitudes as O-planes were known to be not dynamical. However, the usual claim that there is a thinkness of order $\sqrt{\alpha'}$ for the O-domain-wall is misleading as the UV behavior of its scatterings is power-law instead of exponential fall-off. This paper is organized as following. In Section 2, we write down a class of high-energy vertex operators at general mass levels for the scatterings of orientifold planes. We then calculate the scattering from O-particle. In Section 3, we calculate the scatterings from O-domain-wall and discuss the pole structure in the form factor. A brief conclusion and discussion are given in Section 4.

2. High-energy O-particle scatterings

We will use the real projected plane RP_2 as the worldsheet diagram for the scatterings of orientifold planes. The standard propagators of the left and right moving fields are

$$\langle X^\mu(z)X^\nu(w)\rangle = -\eta^{\mu\nu}\log(z-w), \tag{2.1}$$

$$\langle \tilde{X}^\mu(\bar{z})\tilde{X}^\nu(\bar{w})\rangle = -\eta^{\mu\nu}\log(\bar{z}-\bar{w}). \tag{2.2}$$

In addition, there are also nontrivial correlator between the right and left moving fields as well

$$\langle X^\mu(z)\tilde{X}^\nu(\bar{w})\rangle = -D^{\mu\nu}\ln(1+z\bar{w}). \tag{2.3}$$

As in the usual convention [18], the matrix D reverses the sign for fields satisfying Dirichlet boundary condition. The wave functions of a tensor at general mass level can be written as

$$T_{\mu_1\cdots\mu_n} = \frac{1}{2}\left[\varepsilon_{\mu_1\cdots\mu_n}e^{ik\cdot x} + (D\cdot\varepsilon)_{\mu_1}\cdots(D\cdot\varepsilon)_{\mu_n}e^{iD\cdot k\cdot x}\right] \tag{2.4}$$

where

$$\varepsilon_{\mu_1\cdots\mu_n} \equiv \varepsilon_{\mu_1}\cdots\varepsilon_{\mu_n}. \tag{2.5}$$

The vertex operators corresponding to the above wave functions are

$$\begin{aligned} &V(\varepsilon,k,z,\bar{z}) \\ &\quad = \frac{1}{2}\left[\varepsilon_{\mu_1\cdots\mu_n}V^{\mu_1\cdots\mu_n}(k,z,\bar{z}) + (D\cdot\varepsilon)_{\mu_1}\cdots(D\cdot\varepsilon)_{\mu_n}V^{\mu_1\cdots\mu_n}(D\cdot k,z,\bar{z})\right]. \end{aligned} \tag{2.6}$$

For simplicity, we are going to calculate one tachyon and one massive closed string state scattered from the O-particle in the high-energy limit. One expects to get similar results for the generic Op-plane scatterings with $p \geqslant 0$ except O-domain-wall scatterings, which will be discussed in Section 3. For this case $D_{\mu\nu} = -\delta_{\mu\nu}$, and the kinematic setup are

$$e^P = \frac{1}{M}(-E,-\mathrm{k}_2,0) = \frac{k_2}{M}, \tag{2.7}$$

$$e^L = \frac{1}{M}(-\mathrm{k}_2,-E,0), \tag{2.8}$$

$$e^T = (0,0,1), \tag{2.9}$$

$$k_1 = (E,\mathrm{k}_1\cos\phi,-\mathrm{k}_1\sin\phi), \tag{2.10}$$

$$k_2 = (-E,-\mathrm{k}_2,0) \tag{2.11}$$

where e^P, e^L and e^T are polarization vectors of the tensor state k_2 on the high-energy scattering plane. One can easily calculate the following kinematic relations in the high-energy limit

$$e^T\cdot k_2 = e^L\cdot k_2 = 0, \tag{2.12}$$

$$e^T\cdot k_1 = -\mathrm{k}_1\sin\phi \sim -E\sin\phi, \tag{2.13}$$

$$e^T\cdot D\cdot k_1 = \mathrm{k}_1\sin\phi \sim E\sin\phi, \tag{2.14}$$

$$e^T\cdot D\cdot k_2 = 0, \tag{2.15}$$

$$e^L\cdot k_1 = \frac{1}{M}[\mathrm{k}_2E - \mathrm{k}_1E\cos\phi] \sim \frac{E^2}{M}(1-\cos\phi), \tag{2.16}$$

$$e^L \cdot D \cdot k_1 = \frac{1}{M}[\mathrm{k}_2 E + \mathrm{k}_1 E \cos\phi] \sim \frac{E^2}{M}(1+\cos\phi), \tag{2.17}$$

$$e^L \cdot D \cdot k_2 = \frac{1}{M}[-\mathrm{k}_2 E - \mathrm{k}_2 E] \sim -\frac{2E^2}{M}. \tag{2.18}$$

We define

$$a_0 \equiv k_1 \cdot D \cdot k_1 = -E^2 - \mathrm{k}_1^2 \sim -2E^2, \tag{2.19}$$

$$a_0' \equiv k_2 \cdot D \cdot k_2 = -E^2 - \mathrm{k}_2^2 \sim -2E^2, \tag{2.20}$$

$$b_0 \equiv k_1 \cdot k_2 = \left(E^2 - \mathrm{k}_1\mathrm{k}_2\cos\phi\right) \sim E^2(1-\cos\phi), \tag{2.21}$$

$$c_0 \equiv k_1 \cdot D \cdot k_2 = \left(E^2 + \mathrm{k}_1\mathrm{k}_2\cos\phi\right) \sim E^2(1+\cos\phi), \tag{2.22}$$

and the Mandelstam variables can be calculated to be

$$t \equiv -(k_1+k_2)^2 = M_1^2 + M_2^2 - 2k_1\cdot k_2 = M_2^2 - 2(1+b_0), \tag{2.23}$$

$$s \equiv \frac{1}{2}k_1 \cdot D \cdot k_1 = \frac{1}{2}a_0, \tag{2.24}$$

$$u = -2k_1 \cdot D \cdot k_2 = -2c_0. \tag{2.25}$$

In the high-energy limit, we will consider an incoming tachyon state k_1 and an outgoing tensor state k_2 of the following form

$$\left(\alpha_{-1}^T\right)^{n-m-2q}\left(\alpha_{-2}^L\right)^q \otimes \left(\tilde{\alpha}_{-1}^T\right)^{n-m'-2q'}\left(\tilde{\alpha}_{-2}^L\right)^{q'}|0\rangle. \tag{2.26}$$

For simplicity, we have omitted above a possible high-energy vertex $(\alpha_{-1}^L)^r \otimes (\tilde{\alpha}_{-1}^L)^{r'}$ [17,21]. For this case, with momentum conservation on the O-planes, we have

$$a_0 + b_0 + c_0 = M_1^2 = -2. \tag{2.27}$$

The high-energy scattering amplitude can then be written as

$$\begin{aligned} A^{RP_2} &= \int d^2z_1\, d^2z_2 \frac{1}{2}\left[V(k_1,z_1)\tilde{V}(k_1,\bar{z}_1) + V(D\cdot k_1,z_1)\tilde{V}(D\cdot k_1,\bar{z}_1)\right] \\ &\quad \times \frac{1}{2}\varepsilon_{T^{n-2q}L^q,T^{n-2q'}L^{q'}} V^{T^{n-2q}L^q}(k_2,z_2)\tilde{V}^{T^{n-2q'}L^{q'}}(k_2,\bar{z}_2) \\ &\quad + (D\cdot\varepsilon_T)^{n-2q}(D\cdot\varepsilon_L)^q(D\cdot\tilde{\varepsilon}_T)^{n-2q'}(D\cdot\tilde{\varepsilon}_L)^{q'} V^{T^{n-2q}L^q}(D\cdot k_2,z_2) \\ &\quad \times \tilde{V}^{T^{n-2q'}L^{q'}}(D\cdot k_2,\bar{z}_2) \\ &= A_1 + A_2 + A_3 + A_4, \end{aligned}$$

where

$$\begin{aligned} A_1 &= \frac{1}{4}\varepsilon_{T^{n-2q}L^q,T^{n-2q'}L^{q'}} \int d^2z_1\, d^2z_2 \left\langle V(k_1,z_1)\tilde{V}(k_1,\bar{z}_1)V^{T^{n-2q}L^q}(k_2,z_2)\right. \\ &\quad \left. \times \tilde{V}^{T^{n-2q'}L^{q'}}(k_2,\bar{z}_2)\right\rangle, \end{aligned} \tag{2.28}$$

$$\begin{aligned} A_2 &= \frac{1}{4}\varepsilon_{T^{n-2q}L^q,T^{n-2q'}L^{q'}} \int d^2z_1\, d^2z_2 \left\langle V(D\cdot k_1,z_1)\tilde{V}(D\cdot k_1,\bar{z}_1)V^{T^{n-2q}L^q}(k_2,z_2)\right. \\ &\quad \left. \times \tilde{V}^{T^{n-2q'}L^{q'}}(k_2,\bar{z}_2)\right\rangle, \end{aligned} \tag{2.29}$$

$$A_3 = \frac{1}{4}(D\cdot\varepsilon_T)^{n-2q}(D\cdot\varepsilon_L)^q(D\cdot\tilde{\varepsilon}_T)^{n-2q'}(D\cdot\tilde{\varepsilon}_L)^{q'} \times \int d^2z_1\, d^2z_2 \big\langle V(k_1,z_1)\tilde{V}(k_1,\bar{z}_1)V^{T^{n-2q}L^q}(D\cdot k_2,z_2)\tilde{V}^{T^{n-2q'}L^{q'}}(D\cdot k_2,\bar{z}_2)\big\rangle, \tag{2.30}$$

$$A_4 = \frac{1}{4}(D\cdot\varepsilon_T)^{n-2q}(D\cdot\varepsilon_L)^q(D\cdot\tilde{\varepsilon}_T)^{n-2q'}(D\cdot\tilde{\varepsilon}_L)^{q'} \times \int d^2z_1\, d^2z_2 \big\langle V(D\cdot k_1,z_1)\tilde{V}(D\cdot k_1,\bar{z}_1)V^{T^{n-2q}L^q}(D\cdot k_2,z_2) \times \tilde{V}^{T^{n-2q'}L^{q'}}(D\cdot k_2,\bar{z}_2)\big\rangle. \tag{2.31}$$

One can easily see that

$$A_1 = A_4, \qquad A_2 = A_3. \tag{2.32}$$

We will choose to calculate A_1 and A_2. For the case of A_1, we have

$$\begin{aligned}
4A_1 &= \varepsilon_{T^{n-2q}L^q,T^{n-2q'}L^{q'}} \int d^2z_1\, d^2z_2 \big\langle e^{ik_1X}(z_1)e^{ik_1\tilde{X}}(\bar{z}_1)\big(\partial X^T\big)^{n-2q}\big(i\partial^2X^L\big)^q \\
&\quad\times e^{ik_2X}(z_2)\big(\bar{\partial}\tilde{X}^T\big)^{n-2q'}\big(i\bar{\partial}^2\tilde{X}^L\big)^{q'}e^{ik_2\tilde{X}}(\bar{z}_2)\big\rangle \\
&= (-1)^{q+q'}\int d^2z_1\, d^2z_2\,(1+z_1\bar{z}_1)^{a_0}(1+z_2\bar{z}_2)^{a_0'}|z_1-z_2|^{2b_0}|1+z_1\bar{z}_2|^{2c_0} \\
&\quad\times\left[\frac{ie^T\cdot k_1}{z_1-z_2}-\frac{ie^T\cdot D\cdot k_1}{1+\bar{z}_1z_2}\bar{z}_1-\frac{ie^T\cdot D\cdot k_2}{1+\bar{z}_2z_2}\bar{z}_2\right]^{n-2q} \\
&\quad\times\left[-\frac{ie^T\cdot D\cdot k_1}{1+z_1\bar{z}_2}z_1+\frac{ie^T\cdot k_1}{\bar{z}_1-\bar{z}_2}-\frac{ie^T\cdot D\cdot k_2}{1+z_2\bar{z}_2}z_2\right]^{n-2q'} \\
&\quad\times\left[\frac{e^L\cdot k_1}{(z_1-z_2)^2}+\frac{e^L\cdot D\cdot k_1}{(1+\bar{z}_1z_2)^2}\bar{z}_1^2+\frac{e^L\cdot D\cdot k_2}{(1+\bar{z}_2z_2)^2}\bar{z}_2^2\right]^q \\
&\quad\times\left[\frac{e^L\cdot D\cdot k_1}{(1+z_1\bar{z}_2)^2}z_1^2+\frac{e^L\cdot k_1}{(\bar{z}_1-\bar{z}_2)^2}+\frac{e^L\cdot D\cdot k_2}{(1+z_2\bar{z}_2)^2}z_2^2\right]^{q'}.
\end{aligned} \tag{2.33}$$

To fix the modulus group on RP_2, choosing $z_1 = r$ and $z_2 = 0$ and we have

$$\begin{aligned}
4A_1 &= (-1)^n\int_0^1 dr^2\,\big(1+r^2\big)^{a_0}r^{2b_0} \\
&\quad\times\left[\frac{e^T\cdot k_1}{r}-\frac{e^T\cdot D\cdot k_1}{1}r\right]^{n-2q}\cdot\left[-\frac{e^T\cdot D\cdot k_1}{1}r+\frac{e^T\cdot k_1}{r}\right]^{n-2q'} \\
&\quad\times\left[\frac{e^L\cdot k_1}{r^2}+\frac{e^L\cdot D\cdot k_1}{1}r^2\right]^q\cdot\left[\frac{e^L\cdot D\cdot k_1}{1}r^2+\frac{e^L\cdot k_1}{r^2}\right]^{q'} \\
&= (-1)^n(E\sin\phi)^{2n}\left(\frac{2\cos^2\frac{\phi}{2}}{M\sin^2\phi}\right)^{q+q'}\sum_{i=0}^{q+q'}\binom{q+q'}{i}\left(\frac{\sin^2\frac{\phi}{2}}{\cos^2\frac{\phi}{2}}\right)^i \\
&\quad\times\int_0^1 dr^2\,\big(1+r^2\big)^{a_0+2n-2(q+q')}\cdot\big(r^2\big)^{b_0-n+2(q+q')-2i}.
\end{aligned} \tag{2.34}$$

Similarly, for the case of A_2, we have

$$4A_2 = (-1)^n \int_0^1 dr^2 (1+r^2)^{a_0} r^{2c_0} \times \left[\frac{e^T \cdot D \cdot k_1}{r} - \frac{e^T \cdot k_1}{1} r\right]^{n-2q} \cdot \left[-\frac{e^T \cdot k_1}{1} r + \frac{e^T \cdot D \cdot k_1}{r}\right]^{n-2q'} \times \left[\frac{e^L \cdot D \cdot k_1}{r^2} + \frac{e^L \cdot k_1}{1} r^2\right]^{q} \cdot \left[\frac{e^L \cdot k_1}{1} r^2 + \frac{e^L \cdot D \cdot k_1}{r^2}\right]^{q'}$$
$$= (-1)^n (E\sin\phi)^{2n} \left(\frac{2\cos^2\frac{\phi}{2}}{M\sin^2\phi}\right)^{q+q'} \sum_{i=0}^{q+q'} \binom{q+q'}{i} \left(\frac{\sin^2\frac{\phi}{2}}{\cos^2\frac{\phi}{2}}\right)^{i} \times \int_0^1 dr^2 \left(1+r^2\right)^{a_0+2n-2(q+q')} \left(r^2\right)^{c_0-n+2i}. \tag{2.35}$$

The scattering amplitude on RP_2 can therefore be calculated to be

$$A^{RP_2} = A_1 + A_2 + A_3 + A_4 = \frac{1}{2}(-1)^n (E\sin\phi)^{2n} \left(\frac{2\cos^2\frac{\phi}{2}}{M\sin^2\phi}\right)^{q+q'} \sum_{i=0}^{q+q'} \binom{q+q'}{i} \left(\frac{\sin^2\frac{\phi}{2}}{\cos^2\frac{\phi}{2}}\right)^{i} \times \int_0^1 dr^2 \left(1+r^2\right)^{a_0+2n-2(q+q')} \cdot \left[\left(r^2\right)^{b_0-n+2(q+q')-2i} + \left(r^2\right)^{c_0-n+2i}\right]. \tag{2.36}$$

The integral in Eq. (2.36) can be calculated as following

$$\int_0^1 dr^2 \left(1+r^2\right)^{a_0+2n-2(q+q')} \cdot \left[\left(r^2\right)^{b_0-n+2(q+q')-2i} + \left(r^2\right)^{c_0-n+2i}\right]$$
$$= \left[\frac{2^{1+a_0+2n-2(q+q')}}{1+b_0-n+2(q+q')-2i}\right] \times F(2+a_0+b_0+n-2i, 1, 2+b_0-n+2(q+q')-2i, -1) + \left[\frac{2^{1+a_0+2n-2(q+q')}}{1+c_0-n+2i}\right] F(2+a_0+c_0+n-2(q+q') + 2i, 1, 2+c_0-n+2i, -1), \tag{2.37}$$

where we have used the following identities of the hypergeometric function $F(\alpha,\beta,\gamma,x)$

$$F(\alpha,\beta,\gamma;x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^1 dy\, y^{\beta-1}(1-y)^{\gamma-\beta-1}(1-yx)^{-\alpha}, \tag{2.38}$$

$$F(\alpha,\beta,\gamma,x) = 2^{\gamma-\alpha-\beta} F(\gamma-\alpha, \gamma-\beta, \gamma, x). \tag{2.39}$$

To further reduce the scattering amplitude into beta function, we use the momentum conservation in Eq. (2.27) and the identity

$$(1+\alpha)F(-\alpha,1,2+\beta,-1)+(1+\beta)F(-\beta,1,2+\alpha,-1) \\ = 2^{1+\alpha+\beta}\frac{\Gamma(\alpha+2)\Gamma(\beta+2)}{\Gamma(\alpha+\beta+2)} \tag{2.40}$$

to get

$$\begin{aligned}
&\left[\frac{2^{1+a_0+2n-2(q+q')}}{1+b_0-n+2(q+q')-2i}\right]F\big(-c_0+n-2i,1,2+b_0-n+2(q+q')-2i,-1\big) \\
&\quad+\left[\frac{2^{1+a_0+2n-2(q+q')}}{1+c_0-n+2i}\right]F\big(-b_0+n-2(q+q')+2i,1,2+c_0-n+2i,-1\big) \\
&\quad=\frac{\Gamma(1+c_0-n+2i)\Gamma(1+b_0-n+2(q+q')-2i)}{\Gamma(2+b_0+c_0-2n+2(q+q'))} \\
&\quad\sim B(1+b_0,1+c_0)\frac{(1+c_0)^{-n+2i}(1+b_0)^{-n+2(q+q')-2i}}{(2+b_0+c_0)^{-2n+2(q+q')}} \\
&\quad\sim B(1+b_0,1+c_0)\left(\cos^2\frac{\phi}{2}\right)^{-n+2i}\left(\sin^2\frac{\phi}{2}\right)^{-n+2(q+q')-2i}.
\end{aligned} \tag{2.41}$$

We finally end up with

$$\begin{aligned}
A^{RP_2} &= A_1+A_2+A_3+A_4 \\
&=\frac{1}{2}(-1)^n(E\sin\phi)^{2n}\left(\frac{2\cos^2\frac{\phi}{2}}{M\sin^2\phi}\right)^{q+q'}\sum_{i=0}^{q+q'}\binom{q+q'}{i}\left(\frac{\sin^2\frac{\phi}{2}}{\cos^2\frac{\phi}{2}}\right)^i \\
&\quad\times B(1+b_0,1+c_0)\left(\cos^2\frac{\phi}{2}\right)^{-n+2i}\left(\sin^2\frac{\phi}{2}\right)^{-n+2(q+q')-2i} \\
&=\frac{1}{2}(-1)^n(2E)^{2n}\left(\frac{\sin^2\frac{\phi}{2}}{2M}\right)^{q+q'}B(1+b_0,1+c_0)\sum_{i=0}^{q+q'}\binom{q+q'}{i}\left(\frac{\cos^2\frac{\phi}{2}}{\sin^2\frac{\phi}{2}}\right)^i \\
&=\frac{1}{2}(-1)^n(2E)^{2n}\left(\frac{1}{2M}\right)^{q+q'}B(1+b_0,1+c_0) \\
&\sim\frac{1}{2}(-1)^n(2E)^{2n}\left(\frac{1}{2M}\right)^{q+q'}B\left(-\frac{t}{2},-\frac{u}{2}\right).
\end{aligned} \tag{2.42}$$

From Eq. (2.42) we see that the UV behavior of O-particle scatterings is exponential fall-off and one gets infinite linear relations among string scattering amplitudes of different string states at each fixed mass level. Note that both t and u correspond to the closed string channel poles, while s corresponds to the open string channel poles. It can be seen from Eq. (2.42) that an infinite closed string Regge poles exist in the form factor of O-particle scatterings. Furthermore, there are no s-channel open string Regge poles as expected since O-planes are not dynamical. This is in contrast to the D-particle scatterings [17] where both infinite s-channel open string Regge poles and t-channel closed string Regge poles exist in the form factor. We will see that the fundamental characteristics of O-domain-wall scatterings are very different from those of O-particle scatterings as we will now discuss in the next section.

3. High-energy O-domain-wall scatterings

For this case the kinematic setup is

$$e^P = \frac{1}{M}(-E, \mathrm{k}_2 \cos\theta, -\mathrm{k}_2 \sin\theta) = \frac{k_2}{M}, \tag{3.1}$$

$$e^L = \frac{1}{M}(-\mathrm{k}_2, E\cos\theta, -E\sin\theta), \tag{3.2}$$

$$e^T = (0, \sin\theta, \cos\theta), \tag{3.3}$$

$$k_1 = (E, -\mathrm{k}_1 \cos\phi, -\mathrm{k}_1 \sin\phi), \tag{3.4}$$

$$k_2 = (-E, \mathrm{k}_2 \cos\theta, -\mathrm{k}_2 \sin\theta). \tag{3.5}$$

In the high-energy limit, the angle of incidence ϕ is identical to the angle of reflection θ and Diag $D_{\mu\nu} = (-1, 1, -1)$. The following kinematic relations can be easily calculated

$$e^T \cdot k_2 = e^L \cdot k_2 = 0, \tag{3.6}$$

$$e^T \cdot k_1 = -2\mathrm{k}_1 \sin\phi\cos\phi \sim -E\sin 2\phi, \tag{3.7}$$

$$e^T \cdot D \cdot k_1 = 0, \tag{3.8}$$

$$e^T \cdot D \cdot k_2 = 2\mathrm{k}_2 \sin\phi\cos\phi \sim E\sin 2\phi, \tag{3.9}$$

$$e^L \cdot k_1 = \frac{1}{M}\left[\mathrm{k}_2 E - \mathrm{k}_1 E\left(\cos^2\phi - \sin^2\phi\right)\right] \sim \frac{2E^2}{M}\sin^2\phi, \tag{3.10}$$

$$e^L \cdot D \cdot k_1 = 0, \tag{3.11}$$

$$e^L \cdot D \cdot k_2 = \frac{1}{M}\left[-\mathrm{k}_2 E + \mathrm{k}_2 E\left(\cos^2\phi - \sin^2\phi\right)\right] \sim -\frac{2E^2}{M}\sin^2\phi. \tag{3.12}$$

We define

$$a_0 \equiv k_1 \cdot D \cdot k_1 \sim -2E^2\sin^2\phi - 2M_1^2\cos^2\phi + M_1^2, \tag{3.13}$$

$$a_0' \equiv k_2 \cdot D \cdot k_2 = -E^2 - \mathrm{k}_2^2 \sim -2E^2, \tag{3.14}$$

$$b_0 \equiv k_1 \cdot k_2 \sim 2E^2\sin^2\phi + 2M_1^2\cos^2\phi - \frac{1}{2}\left(M_1^2 + M^2\right), \tag{3.15}$$

$$c_0 \equiv k_1 \cdot D \cdot k_2 = E^2 - \mathrm{k}_1\mathrm{k}_2 \sim \frac{1}{2}\left(M_1^2 + M^2\right), \tag{3.16}$$

and the Mandelstam variables can be calculated to be

$$t \equiv -(k_1 + k_2)^2 = M_1^2 + M_2^2 - 2k_1 \cdot k_2 = M_2^2 - 2(1 + b_0), \tag{3.17}$$

$$s \equiv \frac{1}{2}k_1 \cdot D \cdot k_1 = \frac{1}{2}a_0, \tag{3.18}$$

$$u = -2k_1 \cdot D \cdot k_2 = -2c_0. \tag{3.19}$$

The first term of high-energy scatterings from O-domain-wall is

$$4A_1 = (-1)^n \int_0^1 dr^2 \left(1 + r^2\right)^{a_0} r^{2b_0}$$
$$\times \left[\frac{e^T \cdot k_1}{r} - \frac{e^T \cdot D \cdot k_1}{1} r\right]^{n-2q} \cdot \left[-\frac{e^T \cdot D \cdot k_1}{1} r + \frac{e^T \cdot k_1}{r}\right]^{n-2q'}$$

$$\times\left[\frac{e^L\cdot k_1}{r^2}+\frac{e^L\cdot D\cdot k_1}{1}r^2\right]^q\cdot\left[\frac{e^L\cdot D\cdot k_1}{1}r^2+\frac{e^L\cdot k_1}{r^2}\right]^{q'}$$
$$\sim(-1)^n(E\sin 2\phi)^{2n}\left(\frac{1}{2M\cos^2\phi}\right)^{q+q'}\int_0^1 dr^2\,\left(1+r^2\right)^{a_0}\left(r^2\right)^{b_0-n}. \tag{3.20}$$

The second term can be similarly calculated to be

$$4A_2=(-1)^n\int_0^1 dr^2\,\left(1+r^2\right)^{a_0}r^{2c_0}$$
$$\times\left[\frac{e^T\cdot D\cdot k_1}{r}-\frac{e^T\cdot k_1}{1}r\right]^{n-2q}\cdot\left[-\frac{e^T\cdot k_1}{1}r+\frac{e^T\cdot D\cdot k_1}{r}\right]^{n-2q'}$$
$$\times\left[\frac{e^L\cdot D\cdot k_1}{r^2}+\frac{e^L\cdot k_1}{1}r^2\right]^q\cdot\left[\frac{e^L\cdot k_1}{1}r^2+\frac{e^L\cdot D\cdot k_1}{r^2}\right]^{q'}$$
$$\sim(-1)^n(E\sin 2\phi)^{2n-2(q+q')}\left(\frac{2E^2}{M}\sin^2\phi\right)^{q+q'}\int_0^1 dr^2\,\left(1+r^2\right)^{a_0}\left(r^2\right)^{c_0+n}. \tag{3.21}$$

The scattering amplitudes of O-domain-wall on RP_2 can therefore be calculated to be

$$\begin{aligned}A^{RP_2}&=A_1+A_2+A_3+A_4\\&=\frac{1}{2}(-1)^n(E\sin 2\phi)^{2n}\left(\frac{1}{2M\cos^2\phi}\right)^{q+q'}\\&\quad\times\int_0^1 dr^2\,\left(1+r^2\right)^{a_0}\left[\left(r^2\right)^{b_0-n}+\left(r^2\right)^{c_0+n}\right].\end{aligned} \tag{3.22}$$

By using the similar technique for the case of O-particle scatterings, the integral above can be calculated to be

$$\begin{aligned}&\int dr^2\left(1+r^2\right)^{a_0}\left[\left(r^2\right)^{b_0-n}+\left(r^2\right)^{c_0+n}\right]\\&\quad=\frac{F(-a_0,1+b_0-n,2+b_0-n,-1)}{1+b_0-n}+\frac{F(-a_0,1+c_0+n,2+c_0+n,-1)}{1+c_0+n}\\&\quad=\frac{2^{2+a_0+b_0+c_0}}{(1+b_0-n)(1+c_0+n)}\frac{\Gamma(2+c_0+n)\Gamma(2+b_0-n)}{\Gamma(2+b_0+c_0)}\\&\quad=\frac{\Gamma(1+c_0+n)\Gamma(1+b_0-n)}{\Gamma(2+b_0+c_0)}.\end{aligned} \tag{3.23}$$

One thus ends up with

$$\begin{aligned}A^{RP_2}&=A_1+A_2+A_3+A_4\\&=\frac{1}{2}(-1)^n(E\sin 2\phi)^{2n}\left(\frac{1}{2M\cos^2\phi}\right)^{q+q'}\frac{\Gamma(c_0+n+1)\Gamma(b_0-n+1)}{\Gamma(b_0+c_0+2)}.\end{aligned} \tag{3.24}$$

Some crucial points of this result are in order. First, since c_0 is a constant in the high-energy limit, the UV behavior of the O-domain-wall scatterings is power-law instead of the usual exponential fall-off in other O-plane scatterings. Second, there exist only *finite* number of closed string poles in the form factor. Note that although we only look at the high energy kinematic regime of the scattering amplitudes, it is easy to see that there exists no infinite closed string Regge poles in the scattering amplitudes for the whole kinematic regime. This is because there is only one kinematic variable for the O-domain-wall scatterings. In fact, the structure of poles in Eq. (3.24) can be calculated to be

$$\begin{aligned}\frac{\Gamma(1+c_0+n)\Gamma(1+b_0-n)}{\Gamma(2+b_0+c_0)} &= \frac{\Gamma(1+M^2)\Gamma(1+b_0-n)}{\Gamma(b_0+n)} \\ &= \Gamma\left(1+M^2\right)\frac{(b_0-n)!}{(b_0+n-1)!} \\ &= \Gamma\left(1+M^2\right)\prod_{k=1-n}^{n-1}\frac{1}{b_0-k} \qquad (3.25)\end{aligned}$$

where we have used $c_0 \equiv \frac{1}{2}(M_1^2+M^2)$ in the high-energy limit. It is easy to see that the larger the mass M of the external leg is, the more numerous the closed string poles are. We thus confirm that all massive string states do couple to the O-domain-wall as was conjectured previously [19,24]. This is also consistent with the boundary state descriptions of O-planes. However, the claim that there is a thinkness of order $\sqrt{\alpha'}$ for the O-domain-wall is misleading as the UV behavior of its scatterings is power-law instead of exponential fall-off. This concludes that, in contrast to the usual behavior of high-energy, fixed angle string scattering amplitudes, namely soft UV, linear relations and the existence of infinite Regge poles, O-domain-wall scatterings, like the well-known D-instanton scatterings, behave like field theory scatterings.

4. Conclusions and discussions

In this paper, we calculate bosonic massive closed string states at arbitrary mass levels scattered from Orientifold planes in the high-energy limit. We have concentrated on the discussions of three fundamental characteristics of high-energy, fixed angle string scattering amplitudes, namely soft UV, infinite Regge poles and infinite linear relations discovered recently. For the case of O-particle scatterings, we obtain infinite linear relations among high-energy scattering amplitudes of different string states at each fixed mass level. Moreover, the amplitude was found to be UV soft, namely, exponential fall-off behavior. We also confirm that there exist only infinite t-channel closed string Regge poles in the form factor of the O-particle scatterings amplitudes as expected. For the case of O-domain-wall scatterings, we find that, like the well-known D-instanton scatterings, the amplitudes behave like field theory scatterings, namely UV power-law without infinite Regge poles. In addition, we discover that there exist only finite number of t-channel closed string poles in the form factor, and the masses of the poles are bounded by the masses of the external legs. We thus confirm that all massive closed string states do couple to the O-domain-wall as was conjectured previously [19,24]. This is also consistent with the boundary state descriptions of O-planes. For both cases of O-particle and O-domain-wall scatterings, we confirm that there exist no open string Regge poles in the form factor of the amplitudes as O-planes were known to be not dynamical.

We summarize the Regge pole structures of closed strings states scattered from various D-branes and O-planes in Table 1. The s-channel and t-channel scatterings for both D-branes and

Table 1

	$p=-1$	$1 \leqslant p \leqslant 23$	$p=24$
Dp-branes	X	C + O	O
Op-planes	X	C	X

"C" and "O" represent infinite closed string Regge poles and open string Regge poles respectively. "X" means there are no infinite Regge poles.

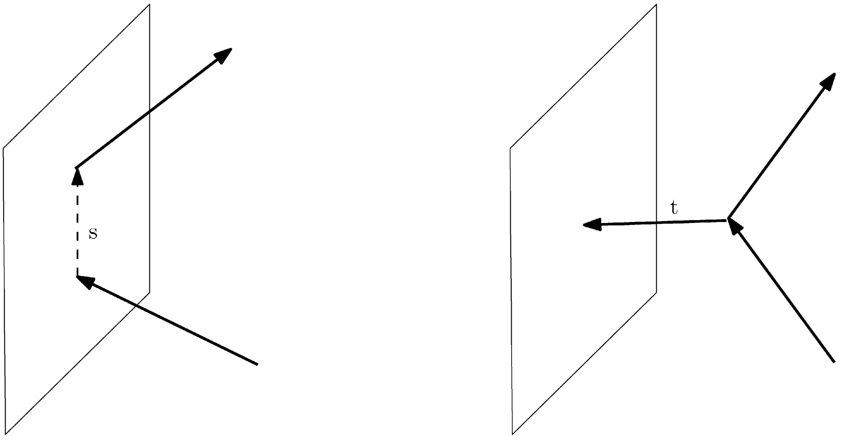

Fig. 1. There are two possible channels for closed strings scattered from D-branes/O-planes. The diagram on the left-hand side corresponds to the s-channel scatterings, and the diagram on the right-hand side is the t-channel scatterings.

O-planes are shown in Fig. 1. For O-plane scatterings, the s-channel open string Regge poles are not allowed since O-planes are not dynamical. For both cases of Domain-wall scatterings, the t-channel closed string Regge poles are not allowed since there is only one kinematic variable instead of two as in the usual cases.

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