PAPER

Observer-Based Synchronization for a Class of Unknown Chaos Systems with Adaptive Fuzzy-Neural Network

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SUMMARY This investigation applies the adaptive fuzzy-neural observer (AFNO) to synchronize a class of unknown chaotic systems via scalar transmitting signal only. The proposed method can be used in synchronization if nonlinear chaotic systems can be transformed into the canonical form of Lur'e system type by the differential geometric method. In this approach, the adaptive fuzzy-neural network (FNN) in AFNO is adopted on line to model the nonlinear term in the transmitter. Additionally, the master's unknown states can be reconstructed from one transmitted state using observer design in the slave end. Synchronization is achieved when all states are observed. The utilized scheme can adaptively estimate the transmitter states on line, even if the transmitter is changed into another chaos system. On the other hand, the robustness of AFNO can be guaranteed with respect to the modeling error, and external bounded disturbance. Simulation results confirm that the AFNO design is valid for the application of chaos synchronization.

key words: *chaos, fuzzy-neural network (FNN), adaptive fuzzy-neural observer (AFNO), synchronization, robust*

1. Introduction

The synchronization of chaotic systems has been extensively studied and given its potential application to security communications. Synchronization means that the master and slave have identical states as time goes to infinity. Pecora and Carroll first considered the synchronization of chaotic systems [18], in which the drive-response concept is introduced to achieve synchronization by a scalar transmitted signal. Perfectly identical parameters cannot be achieved in real applications. Therefore, the nonlinear robust control [22], [23] concept is employed to chaos synchronization with previous known states within the margin of synchronization error. An adaptive recurrent neural controller can be utilized to synchronize with respect to unknown systems [19], [20]. However, all states should be measurable with this algorithm. In contrast, the nonlinear observer is designed to synchronize chaotic systems [3], [8], [16]. Morgül and Solak [16] presented global synchronization is possible for a system with Brunowsky canonical form. Grassi and Mascolo [8] provided a systematic method for synchronizing using a scale transmitted signal. Message-free synchronization has been developed to permit communication with masking message in chaotic signals [14]. Messages can be extracted with message-free synchronization. Moreover, Boutayeb [3] proposed a scheme which is provided to synchronize and extract message simultaneously. Nevertheless, these systems do not consider the robustness of the state observer with respect to parameters mismatch [3], [8], [16]. Adaptive sliding observer design [2], [7] can handle parameters mismatch. Furthermore, a robust observer [13] is designed for synchronization using the Takagi-Sugeno fuzzy model and the LMI approach. Millerioux and Daafouz recently introduced the input-independent global chaos synchronization [15]. In this method, the added message does not affect the synchronization if the observer gain is appropriately designed. Other studies consider nonlinear observer designs for chaos synchronization [1], [17]. However, by the methods of previous descriptions, the chaotic systems should be known previously before synchronization design. Recently, the system identification approaches [5], [6], [9] have been introduced for a scale signal identification and chaos synchronization respectively. In [6], the system identification concepts are applied to approximate the chaos signal. The proposed identification scheme assumes a Lur'e type system as a reference model. This allows us to separate the identification process into two parts, adjusting alternatively the parameters of the linear and the nonlinear part. For modeling the linear system, the autoregressive moving average (ARMA) approach is utilized. On the other hand, the genetic algorithm is applied to optimize the break points parameters of nonlinear static functions to approximate nonlinear mapping. However, this approach is based on off-line identification, and it is not an on-line tuning scheme. Furthermore, the order in linear part identification should be by trial and error. The identification results just imitate the transmission signal and the other states in the master end cannot be achieved to synchronize simultaneously. In addition, the simulation results of this approach seem not very well. According to [5], the recursive identification is applied for chaos synchronization when the slave has exactly identical structure to the master system, but its parameters are unknown. It is shown that the unknown slave system parameters can be found by the concepts of adaptive synchronization. In other words, when the unknown slave system parameters are found, the synchronization is achieved. However, the structures in the master and slave ends should be known previously and exactly the same, although the pa-

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rameters in the slave end can be estimated by recursive identification. The discussion of robustness is not included too. More recently, an alternative indirect Takagi–Sugeno fuzzy model based adaptive fuzzy observer design has been applied to chaos synchronization under assumptions that states are unmeasurable and parameters are unknown [9]. The adaptive law is designed to estimate the unknown parameters in the T-S fuzzy model of the slave end. When the unknown parameters are estimated correctly, the synchronization is achieved. However, the form of the T-S fuzzy model should be known first, and then the adaptive fuzzy observer is designed by the T-S fuzzy model. In addition, the discussion of robustness is not included.

This investigation achieves synchronization with respect to a class of unknown master chaotic systems by introducing the concepts of AFNO [11], Brunowsky canonical form [16] and Lur'e systems [21]. The proposed system includes a chaotic master with canonical form and the slave with AFNO. The AFNO combines a FNN and a linear observer. In this design, the slave should synchronize with the master by a scale transmitted signal. This approach employs adaptive FNN to model the nonlinear term of the master end. The output of the adaptive FNN, robust input and a transmitted state are sent to the linear observer to estimate the states of the slave. The master and slave achieve synchronization when all states are estimated at the slave. Additionally, the adaptive laws are needed to update the weights of the FNN, when the reconstructed and transmitted states differ from each other.

The benefits of provided AFNO for synchronization can be stated as follows. AFNO is first applied to chaotic synchronization with only one transmitted signal. Since AFNO is on line learning at the slave, the synchronization can be achieved respect to a switched unknown chaotic system with the Lur'e type. Additionally, the adaptability for parameters change or even system switched in the mater and the robustness for modeling error and external bounded disturbance are also given. AFNO also has FNN's inherent properties of fault-tolerance, parallelism learning, linguistic information and logic control. By comparing with [5], [6], [9], our presentation provides the on-line, robust and adaptive synchronization for a class of chaos systems. The form of nonlinear functions in the master end cannot be known in previous due to soft computing with FNN for fitting it in the slave end.

The paper is organized as follows. Section 2 describes the overall structure of adaptive synchronization with the AFNO design. Section 3 then introduces the AFNO design. Next, Sect. 4 includes the simulation results, including two examples to demonstrate the effectiveness of this application. Conclusions are finally are made in Sect. 5.

2. Overall Structure of Adaptive Synchronization with Fuzzy-Neural Observer Design

2.1 Introduction of Overall Structure

Assume that the master and slave are all Lur'e type. Figure 1 illustrates the overall structure of adaptive synchronization with AFNO, which is synthesized with an FNN and a linear observer. In this design, only a scalar transmitted signal x_{M1} is sent to the slave from the master. By the observed state $\underline{\hat{x}}_S$, $f_S(\underline{\hat{x}}_S)$ can be computed to approximate $f_M(\underline{x}_M)$ with FNN. The adaptive laws update the weights in FNN when the error exists between x_{M1} and $\hat{x}_{s1}$. The linear observer inputs are $u_S = f_S(\underline{\hat{x}}_S)$, the transmission signal x_{M1}, and the robust input u_r. The synchronization is achieved when $\underline{x}_M = \underline{\hat{x}}_S$.

2.2 Dynamics of the Master and Slave Ends

Master End:

$$\begin{aligned} \underline{\dot{x}}_M &= A_M \underline{x}_M + B_M(f_M(\underline{x}_M) + d) \\ y_{M1} &= x_{M1} = C_M \underline{x}_M, \end{aligned} \tag{1}$$

Slave End: [11], [12]

$$\begin{aligned} \underline{\dot{\hat{x}}}_S &= A_S \underline{\hat{x}}_S + B_S(\hat{f}_S(\underline{\hat{x}}_S) - u_r) + K_o e_o \\ y_{S1} &= \hat{x}_{S1} = C_S \underline{\hat{x}}_S, \end{aligned} \tag{2}$$

where

$$A_M = A_S = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}; B_M = B_S = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix};$$

$C_M = C_S = [1\,0 \cdots 0\,0]$; d denotes an bounded external disturbance;

$$\underline{x}_M = [x_M\ \dot{x}_M \cdots x_M^{(n-1)}]^T = [x_{M1}\ x_{M2} \cdots x_{Mn}]^T \in \mathcal{R}^n,$$

and $\underline{x}_S = [\underline{x}_S\ \dot{x}_S \cdots x_S^{(n-1)}]^T = [x_{S1}\ x_{S2} \cdots x_{Sn}]^T \in \mathcal{R}^n$; observer gain $K_o^T = [k_1\ k_2 \cdots k_n]$ is designed to satisfy $A_S - K_o C_S$ strictly Hurwitz, where (C_S, A_S) represents observer pair; $e_o = x_{M1} - \hat{x}_{S1}$; u_r is designed to enhance the robustness caused by d; $f_M(\underline{x}_M)$ is approximated by adaptive FNN with $\hat{f}_S(\underline{\hat{x}}_S)$. $f_M(\underline{x}_M)$ is unknown (uncertain) but bounded continuous functions [4], [25].

Synchronization Error:

The synchronization error can be defined as:

$$\underline{e}_{syn} = \underline{x}_M - \underline{\hat{x}}_S, \tag{3}$$

where

$$\underline{e}_{syn} = [e_{syn}\ \dot{e}_{syn} \cdots e_{syn}^{(n-1)}]^T = [e_{syn1}\ e_{syn2} \cdots e_{synn}]^T \in \mathcal{R}^n.$$

The master and slave achieve synchronization when all states are estimated at the slave.

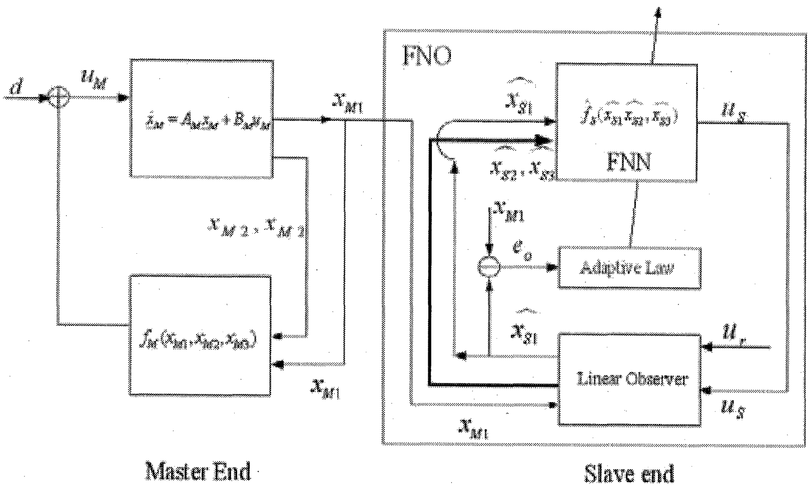

Fig. 1 The overall structure of synchronization with AFNO.

3. Adaptive Fuzzy-Neural Network Observer Design

In this section, AFNO is introduced. Under an assumption, the designed AFNO can estimate the master's states to achieve synchronization. AFNO can then be synthesized by an FNN and a linear observer.

3.1 Fuzzy-Neural Network [11], [12]

The FNN is designed to model the nonlinear function $f_M(\underline{x}_M)$ with $f_S(\hat{\underline{x}}_S)$. The FNN depicted in Fig. 2 is utilized as an approximator to model the nonlinear functions such as $f(\underline{x})$. The FNN [10], [24], which consists of fuzzy IF-THEN rules and a fuzzy inference engine, is adopted as a function approximator. The fuzzy inference engine employs the IF-THEN rules to generate a mapping from an input linguistic vector $\underline{x} = [x_1\ x_2 \cdots x_n]^T \in \mathcal{R}^n$ to an output linguistic variable $y(\underline{x}) \in \mathcal{R}$. Fuzzy IF-THEN rule ith is thus written as:

$$R^{(i)}: \text{if } x_1 \text{ is } A_1^i \text{ and} \ldots \text{and } x_n \text{ is } A_n^i, \text{ then } y \text{ is } B^i,$$

where $A_1^i, A_2^i, \cdots, A_n^i$ and B^i are fuzzy sets with membership functions $\mu_{A_j^i}(x_j)$ and $\mu_{B^i}(\bar{y}^i)$, respectively. By using product inference, center-average, and singleton fuzzifier, output $y(\underline{x})$ from the fuzzy-neural approximator can be written as

$$y(\underline{x}) = \frac{\sum_{i=1}^{h} \bar{y}^i \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j) \right)}{\sum_{i=1}^{h} \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j) \right)} = \underline{\theta}_f^T \Gamma(\underline{x}), \tag{4}$$

where $\mu_{A_j^i}(x_j)$ denotes the membership function value of fuzzy variable x_j; h is the total number of IF-THEN rules, and $\bar{y}^i$ is the point at which $\mu_{B^i}(\bar{y}^i) = 1$. $\underline{\theta}_f = [\bar{y}^1\ \bar{y}^2 \cdots \bar{y}^h]^T$ denotes an adjustable parameter vector, and $\Gamma = [\tau^1\ \tau^2 \cdots \tau^h]^T$ represents a fuzzy basic vector, where τ^i

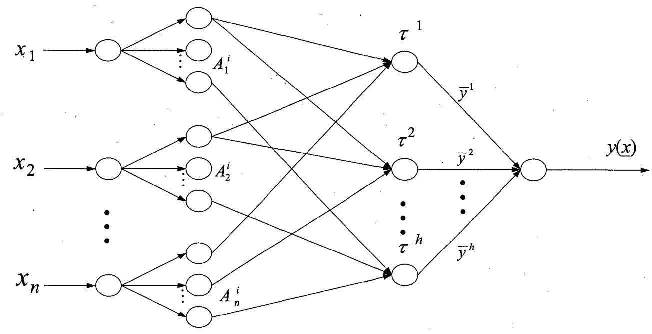

Fig. 2 The fuzzy-neural approximator [11], [12].

is given by

$$\tau^i(\underline{x}) = \frac{\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j) \right)}{\sum_{i=1}^{h} \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j) \right)}. \tag{5}$$

By adjusting the parameter vector $\underline{\theta}_f$ in (4) with adaptive laws, the uncertain nonlinear function $f(\underline{x})$ can be approximated by $\hat{f}(\underline{x})$ generated in (6). By using the fuzzy-neural approximator, the estimated functions $\hat{f}(\underline{x})$ can be determined from the outputs of the fuzzy-neural approximator, which is defined as follows:

$$\hat{f}(\underline{x}|\underline{\theta}_f) = \underline{\theta}_f^T \Gamma(\underline{x}), \tag{6}$$

where $\underline{\theta}_f$ is an adjustable parameter vector.

In summary, (6) can describe the input-output relation of the FNN. The overall structure of the FNN is divided into four layers as shown in Fig. 2. The physical meanings of (6) can be interpreted by Fig. 2 in the following. The input nodes in Layer I represent input linguistic vectors. Nodes in Layer II denote values of the membership function of total linguistic variables. Each node in Layer III excuses a fuzzy rule. The output of Layer IV is the output signal modeling the nonlinear function. The connection parameters between layer III and layer IV are adjusted by using adaptive laws. The number of fuzzy rules can be dependent on complex level of nonlinear systems. In general, the more complex the systems are, the more numerous rules are demand. Of course, the computing load is heavy with more numerous rules. On the other hands, when the rules are less, the computing load is slight. This is a trade off problem.

3.2 Adaptive Fuzzy-Neural Network Observer

Assumption 1 [11], [12]:

The master state vector $\underline{x}_M$ and the slave state vector $\hat{\underline{x}}_S$ belong to compact sets S_M and S_S respectively, where

$$S_M = \{\underline{x}_M \in \mathcal{R}^n : \|\underline{x}_M\| \le \varepsilon_{\underline{x}_M} < \infty\}, \tag{7}$$
$$S_S = \{\hat{\underline{x}}_S \in \mathcal{R}^n : \|\hat{\underline{x}}_S\| \le \varepsilon_{\hat{\underline{x}}_S} < \infty\}, \tag{8}$$

and $\varepsilon_{\underline{x}_M}$ and $\varepsilon_{\hat{\underline{x}}_S}$ are designed parameters.

The optimal parameter vector $\underline{\theta}_f^*$ falls in some convex region with constant radius $\varepsilon\underline{\theta}_f$. The convex region can be specified as shown in (9).

$$R_{\underline{\theta}_f} = \left\{\underline{\theta}_f \in \mathcal{R}^n : \|\underline{\theta}_f\| \le \varepsilon_{\underline{\theta}_f}\right\}. \tag{9}$$

The optimal parameter vector $\underline{\theta}_f^*$ can be described as:

$$\underline{\theta}_f^* = \arg \min_{\underline{\theta}_f \in R_{\underline{\theta}_f}} \left\{ \sup_{\underline{x}_M \in S_M, \hat{\underline{x}}_S \in S_S} |f_M(x_M) - \hat{f}_S(\hat{\underline{x}}_S|\underline{\theta}_f)| \right\}. \tag{10}$$

Remark 1. The optimal $\underline{\theta}_f^*$ is possible in an ideal situation. In our applications, the adaptive laws will be applied to tune $\underline{\theta}_f$ to approach $\underline{\theta}_f^*$.

The adaptive fuzzy-neural nonlinear observer with respect to a class of nonlinear systems (1) can be designed under assumption 1. AFNO can be designed [11], [12]:

$$\begin{aligned} \dot{\hat{\underline{x}}}_S &= A_S \hat{\underline{x}}_S + B_S(\underline{\theta}_f^T \Gamma(\hat{\underline{x}}_S) - u_r) + K_o e_o \\ y_{S1} &= \hat{x}_{S1} = C_S \hat{\underline{x}}_S, \end{aligned} \tag{11}$$

where $\underline{\theta}_f^T \Gamma(\hat{\underline{x}}_S)$ is calculated by FNN to approximate the nonlinear functions $f_M(\underline{x}_M)$ in dynamical systems, and u_r denotes the robust input to compensate the effect due to external disturbance and the approximated modeling error by FNN. Based on [11], [12], u_r can be designed as follows:

$$u_r = -\frac{1}{\gamma} \lambda_{\min}(Q) e_o, \tag{12}$$

where $Q = Q^T > 0$, and γ is a positive constant. In general, γ should be proper designed. The small gamma will cause large u_r to attenuate the effect of disturbance. Indeed, the better attenuation performance will be obtained when the small γ is chosen. Additionally, $Q = Q^T > 0$ will make the Riccati-like equation satisfied in stability and adaptive law derivation with Lyapunove function [12].

The adaptive laws in FNN are as follows:

$$\dot{\underline{\theta}}_f = \begin{cases} \gamma_1 e_o \phi(\hat{\underline{x}}_S), & \text{if } \left\|\underline{\theta}_f\right\| < \varepsilon_{\underline{\theta}_f} \text{ or } \left(\left\|\underline{\theta}_f\right\| = \varepsilon_{\underline{\theta}_f}, \right. \\ & \left. \text{and } e_o \underline{\theta}_f^T \phi(\hat{\underline{x}}_S) \le 0\right) \\ \Pr_f(\gamma_1 e_o \phi(\hat{\underline{x}}_S)), & \text{if } \left\|\underline{\theta}_f\right\| = \varepsilon_{\underline{\theta}_f} \text{ and } e_o \underline{\theta}_f^T \phi(\hat{\underline{x}}_S) > 0, \end{cases} \tag{13}$$

where $\phi(\hat{\underline{x}}_S) = L^{-1}(s)\Gamma(\hat{\underline{x}}_S)$; $L^{-1}(s)$ denotes a proper stable transfer function to transform $H(s)L(s)$ into a proper strictly-positive real (SPR) transfer function, and γ_1 denotes the designed parameter. The function $H(s)$ is represented as follows:

$$H(s) = C_S(sI - (A_S - K_o C_S))^{-1} B_S. \tag{14}$$

$\Pr_f(\gamma_1 e_o \phi(\hat{\underline{x}}_S))$ in (15) is the operator of projection for achieving minimal modeling error for $f_M(\underline{x}_M)$.

$$\Pr_f(\gamma_1 e_o \phi(\hat{\underline{x}}_S)) = \gamma_1 e_o \phi(\hat{\underline{x}}_S) - \gamma_1 \frac{e_o \underline{\theta}_f^T(\hat{\underline{x}}_S)\phi(\hat{\underline{x}}_S)}{\left\|\underline{\theta}_f\right\|^2} \underline{\theta}_f. \tag{15}$$

The design procedure, stability proof and adaptive laws (13) can be referred in [11], [12]

4. Simulation Results

This section verifies the feasibility of AFNO for synchronization using two examples.

4.1 Example 1

In this example, AFNO is applied to synchronize a master Chua's circuit under modeling error, different initial conditions and external bounded disturbances. The results will demonstrate the adaptability and robustness of AFNO.

The master Chua's circuit is reformed as a canonical form [26].

$$\begin{bmatrix} \dot{x}_{M1} \\ \dot{x}_{M2} \\ \dot{x}_{M3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{M1} \\ x_{M2} \\ x_{M3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (f_M(\underline{x}_M) + d), \tag{16}$$

where

$$\begin{aligned} f_M(\underline{x}_M) = &\frac{14}{1805} x_{M1} - \frac{168}{9025} x_{M2} + \frac{1}{38} x_{M3} \\ &- \frac{2}{45} \times \left(\frac{28}{361} x_{M1} + \frac{7}{95} x_{M2} + x_{M3}\right)^3 \end{aligned}$$

The adaptive laws tune FNN to approach $f_S(\hat{\underline{x}}_S)$. The observer is designed to place poles of $A_S - K_o C_S$ in -30 i.e. linear observer gain vector is $K_o^T = [90 \quad 2700 \quad 27000]$.

Other parameters of AFNO are $\gamma = 10$, $\gamma_1 = 0.01$, Q is 3×3 identity matrix, and $L^{-1} = \frac{1}{s+2}$. The membership functions for $\hat{x}_{Si}$, $i = 1, 2, 3$ in FNN are given as follows:

$$\begin{aligned} \mu_{A_j^1}(\hat{x}_{Si}) &= 1/(1 + \exp(5 \times (\hat{x}_{Si} + 0.75))), \\ \mu_{A_j^2}(\hat{x}_{Si}) &= \exp(-(\hat{x}_{Si} + 0.5)^2), \\ \mu_{A_j^3}(\hat{x}_{Si}) &= \exp(-(\hat{x}_{Si} + 0.25)^2), \\ \mu_{A_j^4}(\hat{x}_{Si}) &= \exp(-(\hat{x}_{Si})^2), \\ \mu_{A_j^5}(\hat{x}_{Si}) &= \exp(-(\hat{x}_{Si} - 0.25)^2), \\ \mu_{A_j^6}(\hat{x}_{Si}) &= \exp(-(\hat{x}_{Si} - 0.5)^2), \\ \mu_{A_j^7}(\hat{x}_{Si}) &= 1/(1 + \exp(-5 \times (\hat{x}_{Si} - 0.75))). \end{aligned} \tag{17}$$

Table 1 Three cases of the initial conditions.

Cases	Initial conditions
Case 1	$\underline{x}_M(0)=[0\ \ 0\ \ 0]^T$, and $\underline{x}_S(0)=[1\ \ 1\ \ 1]^T$
Case 2	$\underline{x}_M(0)=[0\ \ 0\ \ 0]^T$, and $\underline{x}_S(0)=[2\ \ 2\ \ 2]^T$
Case3	$\underline{x}_M(0)=[0\ \ 0\ \ 0]^T$, and $\underline{x}_S(0)=[3\ \ 3\ \ 3]^T$

Note: In the simulations, the disturbances in the master end are set as Case 1 in Table 2 in three cases.
Note: In the simulations, the disturbances in the master end are set as Case 1 in Table 2 in three cases.

Table 2 Three cases of the disturbances.

Cases	Disturbance (d)
Case 1	± 0.5 with period 2π
Case 2	± 0.8 with period 2π
Case 3	± 1 with period 2π

Note: In the simulations, the initial conditions are chosen as Casel in Table 1 in three cases.
Note: In the simulations, the initial conditions are chosen as Casel in Table 1 in three cases.

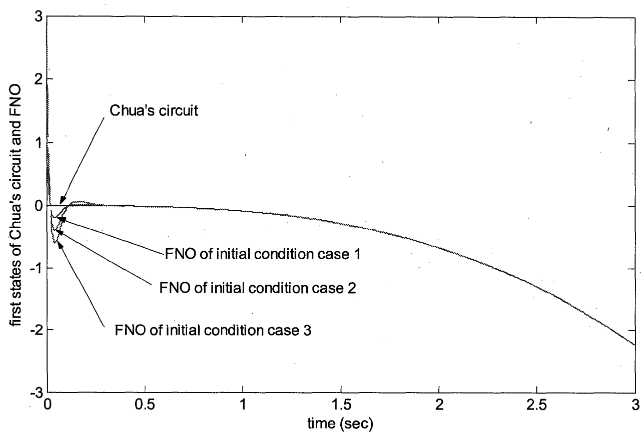

Fig. 3 The first states x_{M1} and $\hat{x}_{S1}$ in Chua's circuit and AFNO under different initial conditions.

In this example, three states should be estimated, accounting for why the fuzzy rules in process are 343. The initially adjustable parameters in adaptive FNN are chosen to be $\underline{\theta}_f(0) = 0$ to demonstrating modeling error. The weights of FNN are turned by the adaptive laws to form $f_M(\underline{x}_M)$.

Different initial conditions of the master and slave are listed in Table 1. Furthermore, the distinct disturbances are listed in Table 2.

Figures 3–5 summarize the simulation results of different initial conditions for three states in AFNO. In Figs. 3–5, the distinct initial conditions for each state in AFNO are listed in Table 1 and a type of disturbance in the master end is set as Case 1 in Table 2. Figure 3 illustrates that the first state $\hat{x}_{S1}$ in AFNO with three different initial conditions synchronizes x_{M1} in Chua's circuit. Figures 4 and 5 illustrate that $\hat{x}_{S2}$ and $\hat{x}_{S3}$ synchronize x_{M2} and x_{M3}, respectively. Although the initial conditions differ from each other, AFNO synchronizes with Chua's circuit quickly,

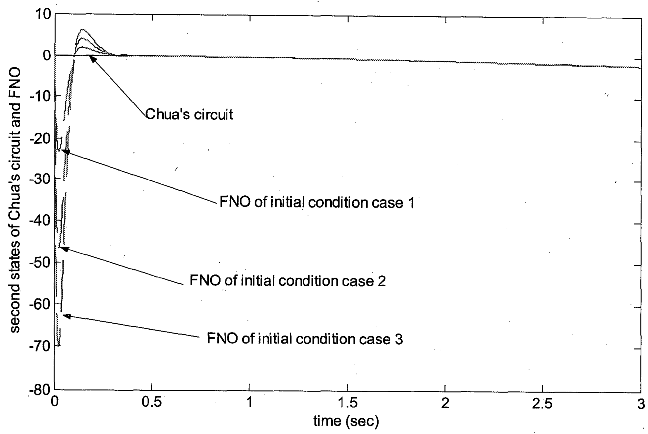

Fig. 4 The second states x_{M2} and $\hat{x}_{S2}$ in Chua's circuit and AFNO under different initial conditions.

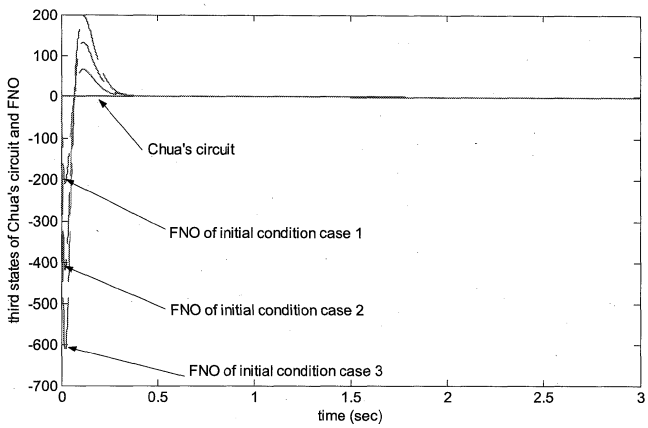

Fig. 5 The third states x_{M3} and $\hat{x}_{S3}$ in Chua's circuit and AFNO under different initial conditions.

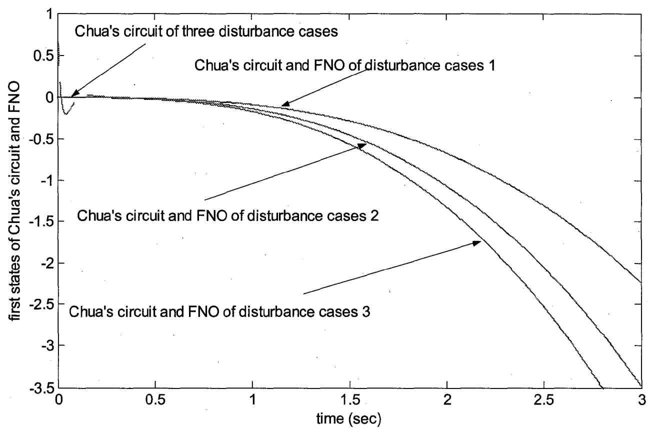

Fig. 6 The first states x_{M1} and $\hat{x}_{S1}$ in Chua's circuit and AFNO under different disturbances.

well, and adaptively. Moreover, the synchronization error approaches zero as time goes to infinity. The robustness of AFNO can be also specified from Figs. 6–8 with various intensity disturbances in the master end. In Figs. 6–8, the initial conditions of three states are selected as Casel in

Table 1 and the different disturbances are chosen as Table 2. Figure 6 demonstrates that the first state $\hat{x}_{S1}$ in the slave synchronizes x_{M1} in the master end immediately and well under three different disturbances. Figures 7 and 8 reveal that $\hat{x}_{S2}$ and $\hat{x}_{S3}$ synchronize x_{M2} and x_{M3}, individually. Even if the different disturbances are added in the master Chua's circuit, AFNO synchronizes with the master robustly.

4.2 Example 2

Example 2 demonstrates the adaptability of the utilized method by switched master between Chua's circuit and Rössler system as shown in Fig. 9. When the master is switched to another system, the slave follows to synchronize another chaotic system soon and well. The similar different initial conditions and disturbances listed in Tables 1 and 2 are considered in simulations for demonstrating the robustness of AFNO.

The original Rössler system can be presented as [16]:

$$\begin{aligned} \dot{z}_1 &= z_2 + a z_1 \\ \dot{z}_2 &= -z_1 - z_3 \\ \dot{z}_3 &= b - c z_3 + z_2 z_3, \end{aligned} \tag{18}$$

where $\underline{z} = [z_1\ z_2\ z_3]^T$.

Let

$$\underline{x}_M = T^{-1}\underline{z}, \tag{19}$$

where $T = \begin{bmatrix} -1 & 0 & 0 \\ a & -1 & 0 \\ 1 & -a & 1 \end{bmatrix}$.

The Rössler system is reformed as the canonical form with

$$f_M(\underline{x}_M) = -c x_{M1} + (ac-1)x_{M2} + (a-c)x_{M3} + a x_{M1}^2 - (a^2+1)x_{M1}x_{M2} + a x_{M1}x_{M3} + a x_{M2}^2 - x_{M2}x_{M3} + b,$$

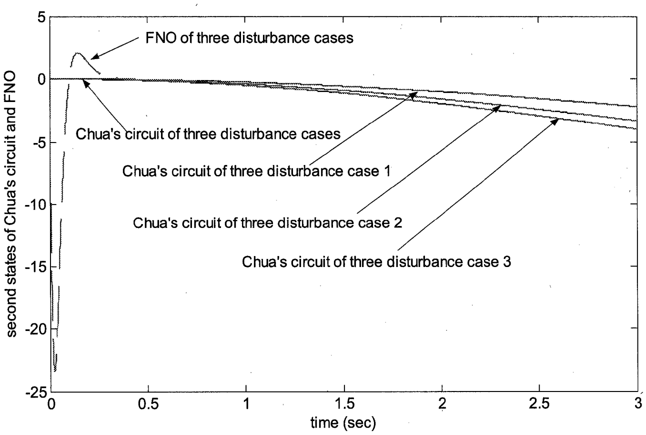

Fig. 7 The second states x_{M2} and $\hat{x}_{S2}$ in Chua's circuit and AFNO under different disturbances.

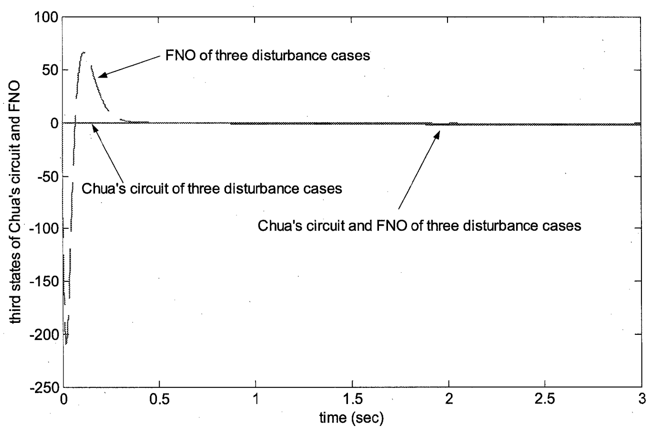

Fig. 8 The third states x_{M3} and $\hat{x}_{S3}$ in Chua's circuit and AFNO under different disturbances.

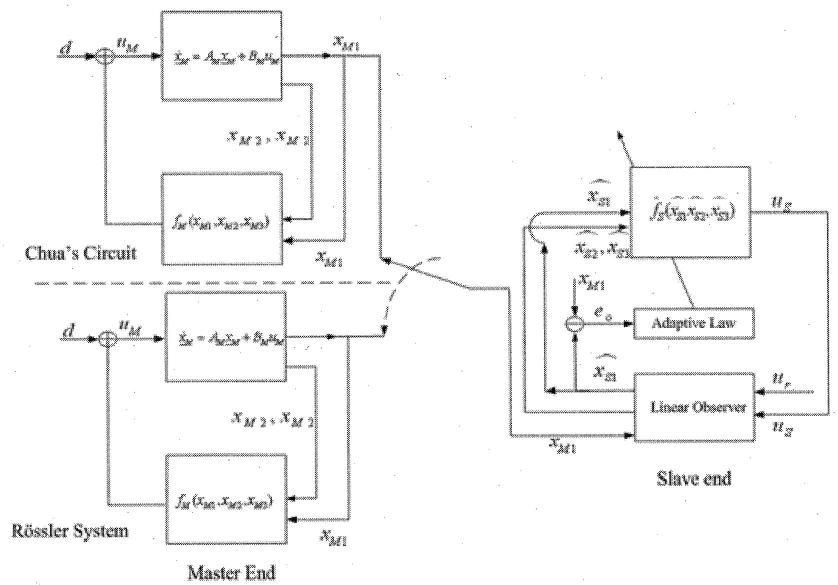

Fig. 9 The structure of synchronization with the switched masters.

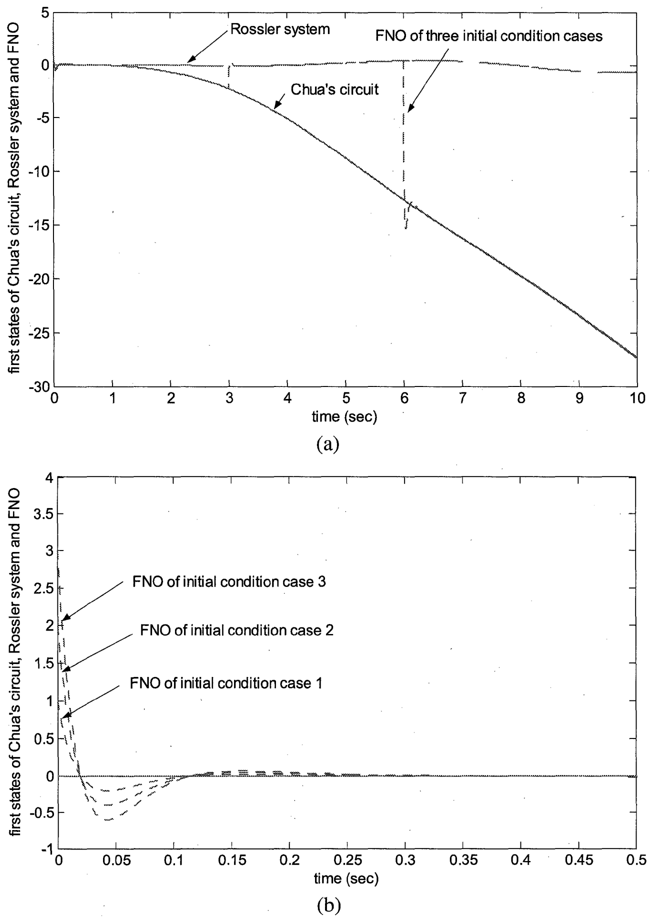

Fig. 10 The first states in Chua's circuit, Rössler system and AFNO under different initial conditions and switched masters: (a) actual figure size (b) enlarged figure size of local region.

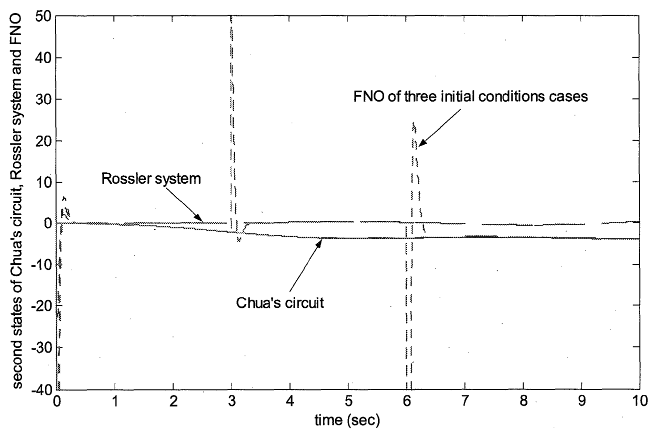

Fig. 11 The second states in Chua's circuit, Rössler system and AFNO under different initial conditions and switched masters.

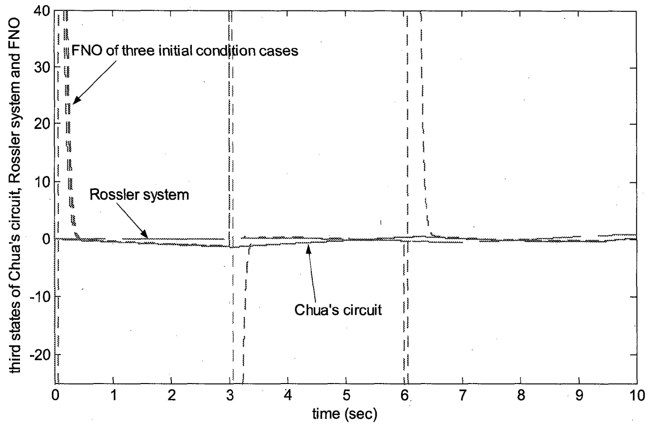

Fig. 12 The third states in Chua's circuit, Rössler system and AFNO under different initial conditions and switched masters.

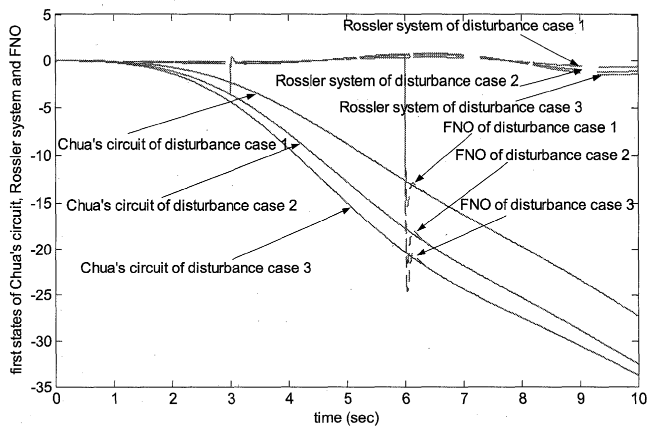

Fig. 13 The first states in Chua's circuit, Rössler system and AFNO under different disturbances and switched masters.

where $a = 0.2$, $b = 0.2$, and $c = 6.3$. Notably, $f_M(\underline{x}_M)$ is revised from [16].

The parameters of AFNO at the slave resemble those in Example 1. The initial condition of Rössler system is set $[0\ 0\ 0]^T$.

Figures 10–12 indicate the simulation results with respect to each state for diverse initial conditions in AFNO and switched masters. The distinct initial conditions for each state in AFNO are shown in Table 1 and a kind of disturbance in the master end is set as Case 1 in Table 2. Figure 10 illustrates that the first state $\hat{x}_{S1}$ in AFNO with three different initial conditions synchronizes x_{M1} in the master end, even if the switched masters exist at the third second (Chua's circuit to Rössler system) and the sixth second (Rössler system to Chua's circuit). Figures 11 and 12 exhibit that $\hat{x}_{S2}$ and $\hat{x}_{S3}$ synchronize x_{M2} and x_{M3}, respectively. Although the initial conditions differ from each other and the switched masters exist, AFNO synchronizes with the switched masters fast, well, and adaptively. On the other hand, simulation results in Figs. 13–15 verify the robustness of AFNO for the different disturbances and the switched systems in the master end. In Figs. 13–15, the initial conditions of three states are chosen as Case1 in Table 1 and the different disturbances are selected as Table 2. Figure 13 displays that the first state $\hat{x}_{S1}$ synchronizes x_{M1} immediately and well under three different disturbances, even thought the switched masters exist at the third second (Chua's circuit to Rössler system) and the sixth second (Rössler system to Chua's circuit). Fig-

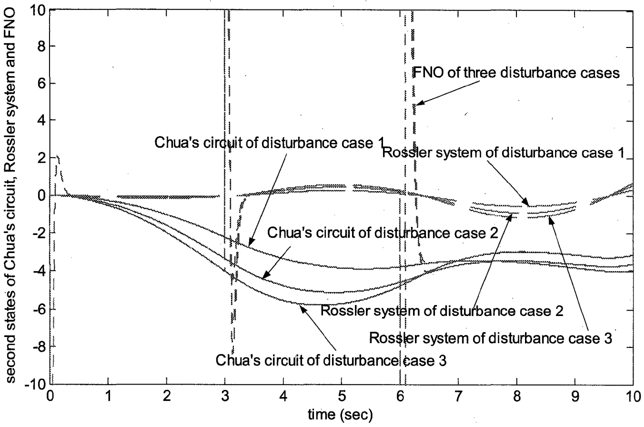

Fig. 14 The second states in Chua's circuit, Rössler system and AFNO under different disturbances and switched masters.

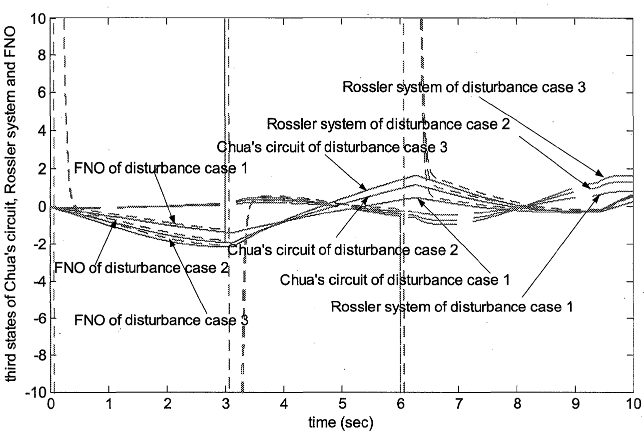

Fig. 15 The third states in Chua's circuit, Rössler system and AFNO under different disturbances and switched masters.

ures 14 and 15 reveal that $\hat{x}_{S2}$ and $\hat{x}_{S3}$ synchronize x_{M2} and x_{M3}, separately. In spite of the different disturbances and the switched systems are considered in the master end, AFNO synchronizes with the master robustly.

It is noted that Figs. 10–15 display the simulation results indicating AFNO synchronizes with Chua's circuit at 0–3 sec. The Rössler system also runs dynamically from the initial condition. AFNO synchronizes with Rössler at 3–6 sec, while Chua's circuit runs simultaneously.

From these simulation results, AFNO can synchronize with a class of unknown chaotic systems adaptively and robustly.

5. Conclusions

This work has applied AFNO for synchronization with respect to a class of unknown chaos systems via a scalar transmitted signal only. Once the nonlinear chaotic systems could be transformed into the canonical form of Lur'e system type by the differential geometric method, the AFNO method can be utilized for synchronization. In this approach, the nonlinear term in the master end was modeled by the adaptive fuzzy-neural network (FNN) in AFNO on line. Furthermore, the states in the master end were observed from a scale transmitted signal by observer design. When states in the master and slave ends were identical, we said the synchronization was reached. By this scheme, the AFNO could estimate the unknown master's states adaptively, even though the master was altered into another chaos system. On the other hand, AFNO could deal with the modeling error, and external bounded disturbance to demonstrate its robustness advantage. Simulation results showed that the adaptive and robust soft AFNO was suitable for chaos synchronization with respect to a class unknown chaos systems.

Acknowledgments

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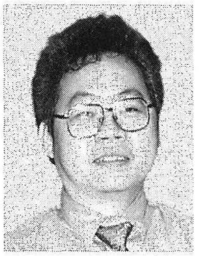

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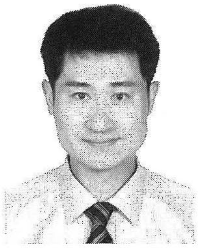

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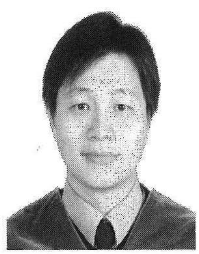

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