



An integrated toll and ramp control methodology for dynamic freeway congestion management

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Abstract

This paper investigates an integrated freeway traffic management system, which coordinates both dynamic toll pricing and ramp control strategies for the purpose of dynamic freeway congestion management. The proposed integrated dynamic toll-ramp control methodology is built mainly on the principles of stochastic optimal control approaches, involving two developmental procedures. First, through detector configurations and system specification, a discrete-time nonlinear stochastic system is formulated to characterize the time-varying relationships of system states, control variables, and traffic data. Then, by employing the extended Kalman filtering technology, a stochastic optimal control based algorithm is proposed to execute the integrated dynamic toll and ramp control mechanism. With the aid of the Paramics microscopic traffic simulator, numerical studies under various simulated freeway congestion scenarios are conducted. Corresponding numerical results demonstrate the applicability of the proposed methodology in response to diverse freeway traffic congestion phenomena, and its relative advantages in improving both the average travel time and hourly throughputs by 16.4% and 16.5%, respectively.

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1. Introduction

Despite the fact that numerous advanced technologies have emerged in the form of intelligent transportation systems (ITS) to solve freeway traffic congestion problems, freeway traffic congestion management remains a critical issue in ITS. Here congestion management refers to a comprehensive decision-making process to identify and control traffic congestion via traffic control and management strategies with the goal of enhancing traffic safety and mobility. As is increasingly recognized, the difficulties of managing freeway traffic congestion are mainly rooted in the dynamics of congestion patterns, including recurrent and non-recurrent congestion patterns on mainline segments, and limited information in terms of en-route drivers' maneuvers responding, in real time, to both traffic congestion conditions and corresponding freeway traffic management strategies when approaching on-ramps. As a result, freeway

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traffic flows and congestion patterns turn out to be more complicated and uncontrollable in the ITS operational environment. For instance, provision of the real-time toll rates and corresponding traffic congestion information via ITS technologies seems to influence drivers' decisions of route choice to a certain extent. However, our perception of drivers' instantaneous responses to the corresponding traffic operational environment is quite limited, thus contributing to the difficulty in determining appropriate congestion tolls in real time, as well as ramp control strategies to alleviate freeway traffic congestion, efficiently and effectively.

Accordingly, to regulate traffic entering a freeway in response to the variability of freeway traffic congestion, both dynamic on-ramp metering control and toll collection appear to be two promising measures to manage freeway congestion. Nevertheless, there seem to be some arguments in terms of the limitations of the existing technologies used for freeway congestion management. Some typical examples are illustrated below for discussions.

Ramp control arises from the idea of controlling on-ramp traffic flows entering mainline segments of freeways via ramp metering so as to achieve given objectives for freeway traffic management [31,20]. More recently, a variety of sophisticated methodologies, including optimal control theories [1,39], artificial intelligence [2,38], and hybrid traffic flow-control approaches [40,41] have been proposed to improve the system performance of either local ramp control or multiple ramp control. Despite the distinctive features of these existing ramp control models, apparently, there is a developmental trend that the mechanisms of ramp control seem to be more responsive to short-term changes of traffic demands in the temporal domain, and more adaptable to integrate with other external methodologies for large-scale congestion management in the spatial domain.

Although there have been a certain advances in exploring ramp control methodologies for freeway traffic control, there seem to be some arguments remaining over the performance of ramp control under high congestion conditions. This argument holds particularly in queue-overflowing and lane-blocking incident cases. In reality, such phenomena are comprehensible due to the fact that unlike signal control on surface streets, the effect of ramp control is mainly on the entry traffic volume rather than on the mainline volume of a freeway. Without other traffic management strategies to divert either on-ramp or mainline traffic flows, the performance of ramp control may turn out to be insignificant in case where the mainline traffic loads are sufficiently greater than the on-ramp traffic loads in a given segment.

In contrast, road congestion pricing including freeway electronic toll collection (ETC) systems has been increasingly recognized as an effective traffic management strategy to manage regional peak-hour road congestion [29,12,8,15]. As pointed out in [7], considering the trade-off between out-of-pocket travel costs and delays, road users may respond to the instantaneous toll rates before entering the instrumented toll collection systems by modifying their travel behavior including diverting routes, departing at a different time and switching modes of transport. Under such a postulation, a great many researchers have made efforts on the investigation of congestion pricing theories and toll collection strategies [30,9,26,10,37,33,13]. Therein, numerous pioneer researchers advocate utilizing the principle of marginal-cost pricing to deal with the optimal congestion pricing problem and its potential impacts on traffic flows. Three fundamental elements, including the speed-flow relationship, the demand function, and the generalized cost, which affect the implementation of congestion pricing systems, were investigated [5,11,34,35,32,4,3,14,17]. Furthermore, considering the impracticability of searching for the first-best pricing solutions, diverse second-best pricing regimes, including the cordon-based second-best pricing, were proposed for real-world applications [15,28,33,40,41].

Accordingly, the existing congestion pricing theories appear promising in addressing freeway congestion management issues. Nevertheless, the use of ETC for dynamic freeway congestion management may still warrant more research efforts due to the following concerns. First, the applicability of the existing marginal-cost congestion pricing theories may hold only under certain traffic flow conditions, e.g., low-volume and medium-volume conditions. It can be seen in most of the previous literature, the corresponding average social cost curve is assumed to be strictly increasing with traffic flow without considering the link capacity issue. Consequently, a downhill-shape speed-flow curve is extensively used to determine the optimal congestion toll as well as the effects of congestion pricing on traffic flows. Obviously, the aforementioned postulation may not be applicable for highly congested cases where the speed-flow curve becomes backward bending after the maximum traffic flow point, as shown in any fundamental diagrams of traffic flow-speed-density. Similar arguments can also be found elsewhere [11]. Second, the use of aggregate demand functions in characterizing individual drivers' maneuvers responding to toll rates may remain problematic in the dynamic congestion management context. From an aggregate economics point of view, it seems more agreeable that those toll rates derived from the marginal-cost congestion pricing theories may apply merely for long-term and large-scale congestion pricing cases, where the features of system-wide road users and the

corresponding responses to congestion tolls are assumed to be consistent over a long time period. In contrast, in a dynamic congestion management system, the corresponding traffic control and management mechanisms including the embedded traffic forecasting models may need to be more adaptive to short-term changes of local road users' decisions, including route choice and driving behavior. In view of this, the postulation of a uniform demand function may no longer meet the functional requirements of dynamic congestion management systems. Thirdly, and most importantly, pure toll collection strategies, including cordon-based toll collection measures, may not have significant effects on system entry flows in the case where the estimated toll rates are widely acceptable to system road users. This may induce two related issues, i.e., toll boundaries and sensitivity of traveling costs to road users for either rerouting or rescheduling.

Arguably, regulating on-ramp traffic arrivals by integrating dynamic ramp control and toll pricing strategies appears promising for effective freeway traffic congestion management. Such an integrated traffic management strategy arises from the idea that similar to an isolated parking facility with charges, freeways can be regarded as a traffic service facility paid by the corresponding road users. Given limited service capacity, such a traffic service facility can charge the corresponding freeway users with different levels of tolls coupled with appropriate ramp control strategies under diverse traffic congestion conditions for the purpose of freeway congestion management. From an individual customer point of view, those pre-trip potential freeway users, e.g., vehicles approaching to on-ramps, may assess the corresponding trade-off effects between out-of-pocket costs and travel time saving prior to entering the freeway system. Meanwhile, the en-route freeway users, i.e., vehicles moving on tolled freeways, are supposed to pay different levels of tolls for traveling with different levels of freeway service, which also depend on the respective service time on them. From the supply side, the freeway system operator can use ramp metering as the gate-traffic controller to maintain given levels of service on freeways. Accordingly, the aim of dynamic freeway traffic congestion management seems possible through the integrated utilization of dynamic toll and ramp control strategies.

The purpose of this study is to investigate an integrated ramp control and dynamic toll pricing methodology proposed for freeway congestion management. Our research idea is to ensure the magnitude of freeway traffic congestion is adjustable under any given traffic flow conditions, thus the target level of service on freeways is achievable through the integration of dynamic toll pricing and ramp control strategies. Herein, the dynamic ramp metering control is a straightforward and efficient measure to maintain the service level of the freeway system so as to ensure that all the tolled freeway users can be served at the target service level while driving on the tolled freeway. In the proposed integrated toll-ramp control scheme, the freeway system may no longer be free to the public road users. Instead, everybody who wants to use the freeway system should be charged with the dynamic toll rates. That is, the dynamic toll rates apply to not only the en-route freeway users but also all the drivers who are arriving at on-ramps, and waiting entering into the freeway system. Note that such tolled freeway systems have been increasingly adopted in Asia, including Taiwan, China, and Japan. Furthermore, the dynamic toll rates can be used to influence those pre-travelers' decisions to determine whether or not and when to use the tolled freeway system so as to regulate the new traffic arrivals at the on-ramps of the tolled freeway system, with the ultimate goal of freeway congestion management. Motivated by the above basic ideas, an integrated dynamic toll-ramp control methodology is proposed, where the optimal values of time-varying ramp metering and toll rates are determined and updated dynamically, according to the preset target freeway service levels, as well as time-varying traffic flow conditions. The architecture of the proposed method is constructed based on the principles of stochastic optimal control approaches, together with the extended Kalman filtering technology, involving three major developmental procedures: (1) system specification, (2) stochastic modeling, and (3) control algorithm. The details of methodology development and preliminary tests are described in the following sections.

2. System specification

The system scope studied here considers any given freeway corridor, which is composed geometrically of multiple ramps and freeway mainline segments, as shown in Fig. 1. To collect real-time raw traffic data used as the input of the proposed method, specific point detector layouts are proposed, mainly involving two types of detector stations: (1) mainline detection stations implemented sequentially at mainline segments upstream from on-ramps, and (2) gate detectors implemented at the corresponding on-ramps and off-ramps of the target freeway system. Here, a service zone is defined as the area composed of a given mainline segment that is bounded by a pair of upstream and downstream mainline detector stations, including the corresponding on-ramps and off-ramps involved in the service zone.

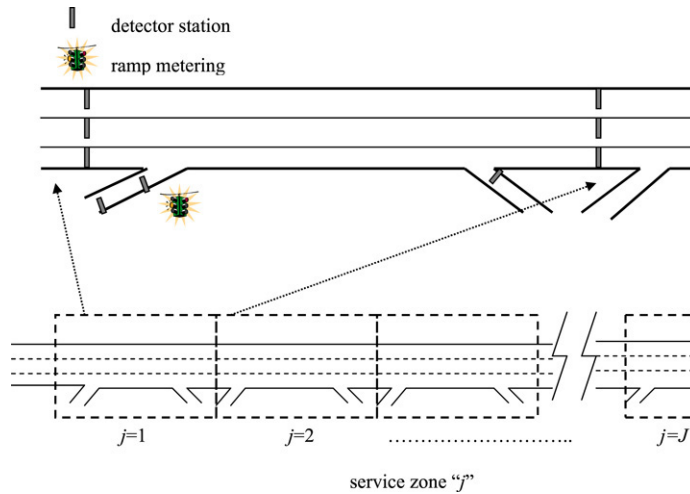


Fig. 1. Geographic specification of system scope.

Accordingly, the target freeway corridor can be regarded as a sequential traffic-transporting system with multiple service zones supervised automatically through the integration of both dynamic toll collection and ramp control devices. Here, each given service zone is regarded as a respective traffic facility serving traffic flows to move smoothly with a given level of service. For the purpose of freeway congestion management, freeway system operators, e.g., freeway traffic management center (freeway-TMC), may preset appropriate levels of service associated with these sequential service zones for daily operations of the target freeway system, according to time of day as well as historical traffic flow patterns. Given the target freeway service level guiding freeway congestion management, on-ramp metering is then used as a gate controller to regulate traffic flows moving in each service zone, followed by the determination of time-varying toll rates employing the estimates of traffic state variables associated with each zone, with both the goals of minimizing the system-wide traffic service time and maximizing the system-wide traffic service volume.

In addition, the corresponding toll effect on pre-trip drivers' decisions in choosing routes is also considered in the process of estimating dynamic toll rates. Apparently, time-varying toll rates may influence the determination of potential freeway users in route choice before they approach on-ramps of the freeway system, thus contributing to the variations in freeway entry volumes. For instance, in case of a freeway with high congestion, the strategy of high toll rates may have relatively significant effect on decreasing on-ramp entry volumes because those pre-route potential freeway users may re-assess the corresponding traveling costs and benefits under such a high congestion condition before approaching the freeway system. As such, high-toll strategies integrated with ramp metering rates can be appropriately used to regulate system-wide freeway entry volumes, resulting in the alleviation of freeway congestion to a certain extent. In contrast, low-toll strategies including the free-toll strategy, appears to encourage more road users to travel via freeways under low-volume conditions, and thus may enhance the serviceability of the freeway system in off-peak periods. Accordingly, the induced effect of toll rates on the variation of on-ramp traffic arrivals is considered in the proposed model.

With the system scope and detector layouts specified above, the conceptual model of the system investigated in this study is presented in Fig. 2, which represents a sequential traffic flow transporting system with " J " service zones. Note that all the definitions and notations of the parameters and variables presented in the text are summarized in the Appendix for clarity. For each given service zone j , the corresponding inter-zone and intra-zone traffic flow relationships can be further characterized with four types of time-varying traffic operational status, including: (1) the number of vehicles ($I_{j-1,j}(k)$) entering from the upstream service zone $j - 1$ to service zone j in time interval k , (2) the number of vehicles ($S_j(k)$) remaining in the given service zone j at the beginning of time interval k , (3) the number of vehicles ($O_{j,j+1}(k)$) moving from service zone j to the downstream service zone $j + 1$ in time interval k , and (4) the number of on-ramp vehicles entering into service zone j ($\bar{R}_j(k)$), and the number of off-ramp vehicles exiting from service zone j ($R_j(k)$) in time interval k .

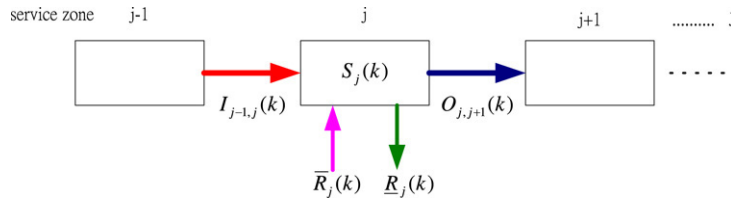


Fig. 2. Conceptual model of the proposed freeway system.

Given the target service level (\tilde{s}) predetermined by freeway-TMC, the ideal zone-based traffic flow condition, shown as Eq. (1), is then postulated as

$$0 \leq [I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)] - [O_{j,j+1}(k) + \underline{R}_j(k)] \leq S_j^{\tilde{s}} \tag{1}$$

where $S_j^{\tilde{s}}$ represents the maximum number of vehicles permitted in a given service zone j with the target service level \tilde{s} . Correspondingly, under the ideal traffic flow condition, the time-varying traffic flows remaining in any given service zone j at the end of any given time interval should not exceed the corresponding service capacity associated with \tilde{s} . Note that in reality, $S_j(k)$ can be regarded as a time-varying state-dependent variable given by

$$S_j(k) = [I_{j-1,j}(k-1) + S_j(k-1) + \bar{R}_j(k-1)] - [O_{j,j+1}(k-1) + \underline{R}_j(k-1)]. \tag{2}$$

Despite the aforementioned ideal zone-based traffic flow condition that the specified freeway system seeks for any given service zones, it may not be achievable at will without any assistance from the sophisticated traffic control models due to two major concerns. First, there exist some system states, which are not directly controllable by ramp metering control. It is noteworthy that among the traffic states shown in Eq. (1), only the number of entry vehicles $\bar{R}_j(k)$ is determined directly by ramp control. In contrast, system states, including $I_{j-1,j}(k)$, $O_{j,j+1}(k)$, and $\underline{R}_j(k)$, may be influenced more significantly by time-varying traffic flow conditions and origin-destination patterns on freeways, thus leading to some challenges in fully controlling these system states to achieve the aforementioned ideal zone-based traffic flow condition in any time interval. Second, although $\bar{R}_j(k)$ is controllable via ramp control, the corresponding optimal ramp metering rates should be predetermined at the beginning of any given time interval under which the time-varying states of $I_{j-1,j}(k)$, $O_{j,j+1}(k)$, and $\underline{R}_j(k)$ occurring in the given time interval are unknown.

Obviously, the aforementioned issues have led the proposed freeway system to be a typical dynamic system prediction and control problem in the process of searching for the ideal multi-zone traffic flow condition. That is, given initial traffic flow conditions, including $I_{j-1,j}(0)$, $O_{j,j+1}(0)$, $S_j(0)$, $\bar{R}_j(0)$ and $\underline{R}_j(0)$ in each given service zone j in the initial time interval $k = 0$, the optimal solutions of zone-based system states as well as control variables should be predicted and updated in each given time interval using traffic measurements to ensure that the goal of ideal multi-zone traffic flow condition is achievable.

To facilitate characterizing the operational problem of the proposed multi-zone freeway service system with a stochastic optimal control system, three groups of system variables, including (1) basic state variables, (2) measurement variables, and (3) control variables, are specified with the following definitions.

Basic state variables² refer to the critical informative elements that can be used to derive other time-varying system states characterizing the traffic operational conditions of the specified multi-zone freeway service system. In this study, three types of basic state variable are specified as follows.

- (1) $r_{j,j+1}(k)$ represents the time-varying proportion of vehicles leaving from a given service zone j to the following service zone $j + 1$ in a given time interval k .

² Ideally, $S_j(k)$ (i.e., the number of vehicles present in a given service zone j in a given time interval k) can be used to characterize the time-varying freeway traffic flow conditions. However, due to the difficulty of obtaining the real-time value of $S_j(k)$ directly from traffic detectors, we propose to estimate the real-time basic traffic state variables, and then use them to derive $S_j(k)$ and the other traffic state variables. Nevertheless, the basic state variables should be estimated in real time using the proposed stochastic optimal control model as they can not be measured directly from detectors. This may also clarify why we specify the basic state variables for the use in the proposed integrated toll-ramp control methodology.

- (2) $r_{j,\underline{R}_j}(k)$ represents the time-varying proportion of vehicles leaving from a given service zone j to the corresponding off-ramp \underline{R}_j in a given time interval k .
- (3) $r_{\bar{R}_j,j}(k)$ represents the time-varying proportion of on-ramp vehicles entering from a given on-ramp \bar{R}_j to the corresponding service zone j in a given time interval k .

Measurement variables correspond to the observable traffic measurements collected from specific detector stations, which can be used to update the prior predictions of basic state variables in each given time interval. Herein, three types of measurement variables are involved: $O_{j,j+1}(k)$, $\bar{R}_j(k)$, and $\underline{R}_j(k)$.

Control variables, determined fully by system operators, serve to regulate, either directly or indirectly, inter-zone and intra-zone traffic flow conditions so as to accomplish the preset level of service. In this study, two types of control variables are defined: $c_j(k)$ and $p_j(k)$, which represent the time-varying on-ramp metering and toll rates associated with a given service zone j in a given time interval k , respectively. Their corresponding units are veh/s and \$/s, standing for the number of vehicles released from the corresponding on-ramp and the time-varying unit toll rate for traveling in a given service zone j per second during a given time interval k , respectively.

Utilizing the aforementioned specified variables, we further introduce an on-ramp traffic equilibrium condition to infer the time-varying interrelationships among tolls, ramp metering rates and on-ramp traffic flows under the integrated dynamic toll-ramp control condition. For each given on-ramp j , the following on-ramp traffic flow condition must hold in each given time interval k .

$$S_{\bar{R}_j}(k+1) = A_{\bar{R}_j}(k) + S_{\bar{R}_j}(k) - \bar{R}_j(k) \leq \tilde{S}_{\bar{R}_j} \quad (3)$$

where $A_{\bar{R}_j}(k)$ represents the measured number of vehicles arriving at the on-ramp (\bar{R}_j) of a given service zone j in a given time interval k ; $S_{\bar{R}_j}(k)$ is the number of vehicles remaining in the corresponding on-ramp \bar{R}_j associated with a given service zone j at the beginning of a given time interval k ; and $\tilde{S}_{\bar{R}_j}$ represents the corresponding on-ramp capacity. In addition, it is assumed that $p_j(k)$ is a function of $(A_{\bar{R}_j}(k))$, i.e., $p_j(k) = f[A_{\bar{R}_j}(k), k]$, and has a negative effect on $A_{\bar{R}_j}(k)$, as widely postulated in the previous literature on road congestion pricing. Accordingly, the on-ramp traffic equilibrium condition³ is defined as an ideal on-ramp traffic flow condition where there is no vehicle remaining in any given on-ramp at the end of each time interval. Under the aforementioned ideal on-ramp traffic flow condition, each vehicle arriving at a given on-ramp is supposed to be served in the same time interval, and thus there is no vehicle remaining in any given on-ramp at the end of each time interval. To achieve such an ideal on-ramp traffic flow condition using the proposed integrated dynamic toll-ramp control, the following condition must hold.

$$A_{\bar{R}_j}(k) = \bar{R}_j(k) \Rightarrow f^{-1}[p_j(k)] = c_j(k) \times t \quad (4)$$

where $f^{-1}[p_j(k)]$ represents the inverse function of $p_j(k)$; and t represents the unit length of a time interval.

Note that Eq. (4) can also be used to assess whether the present toll pricing strategy is compatible with the corresponding ramp control strategy or not. Here the on-ramp traffic demands represented by the left-hand side of Eq. (4) are driven by the time-varying toll rates; and in contrast, the corresponding on-ramp traffic service rates presented on the right-hand side of Eq. (4) are regulated by ramp control. Given the existing ramp control and toll rates estimated in the previous time interval using the proposed method, there could be three potential conditions occurring in the present time interval, i.e., (1) $A_{\bar{R}_j}(k) > \bar{R}_j(k)$, (2) $A_{\bar{R}_j}(k) = \bar{R}_j(k)$, and (3) $A_{\bar{R}_j}(k) < \bar{R}_j(k)$, implying that the existing toll rate is lower, consistent with, or higher than the equilibrium toll rate, respectively, thus contributing to the above on-ramp traffic demand–supply relationships. However, in most cases, ramp control aims to regulate on-ramp entry traffic flows so as to achieve the target service level on mainline segments of freeways, and thus, it may have relatively higher priority than the toll rates to be determined in the process of freeway traffic

³ Here, the on-ramp traffic equilibrium condition is proposed to avoid the phenomenon of queue-overflows occurring on ramps under the integrated toll-ramp control. Note that the phenomena of queue-overflows have been noticed particularly in the area of traffic control of surface streets [16]. Briefly, if there are vehicles queued at an intersection from one cycle to the next, it may result in traffic overflowing which may further lead to a bottleneck where traffic arrival rate is greater than its service rate on the spot. Therefore, we apply this concept in the study case, particularly for controlling the on-ramp entry flows due to the concerns of limited on-ramp vehicle queuing space and induced impact on the traffic flows of surface streets near by.

congestion management. Therefore, the aforementioned potential conditions can be used to check if the present toll rates are appropriate to facilitate managing freeway traffic congestion.

3. Stochastic modeling

Based on the above system specification, the problem is formulated with a discrete-time nonlinear stochastic model consisting of three groups of dynamic equations: (1) state equations, (2) measurement equations, and (3) state boundaries, which are detailed below.

The state equations characterize the time-varying relationships of the basic state variables under the condition of the proposed integrated dynamic toll-ramp control. Adopting the concepts of dynamic freeway traffic flow stability claimed in previous literature [25,24], it is also assumed that the specified basic state variables follow Gaussian-Markov processes. That is, if there is no disturbance, the changing patterns of basic state variables tend to be identical, exhibiting the features of a Markov-based deterministic system; otherwise, the next-time-step system states may oscillate around the current-time-step system states in a Gaussian-based domain. Accordingly, the generalized form of the proposed state equations is formulated as

$$\mathbf{X}(k + 1) = \mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k] + \mathbf{L}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k] \mathbf{W}(k) \tag{5}$$

where $\mathbf{X}(k + 1)$ is a $(3J \times 1)$ time-varying basic state vector in time interval $k + 1$; $\mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]$, representing a deterministic term of the state equations, is a $(3J \times 1)$ time-varying basic state vector which consists of the sub-vectors of time-varying basic state variables ($\mathbf{X}(k)$) and the corresponding control variables ($\boldsymbol{\Omega}(k)$) in time step k ; $\mathbf{L}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]$ is a $(3J \times 3J)$ diagonal state-dependent noise matrix, and finally, $\mathbf{W}(k)$ represents a $(3J \times 1)$ state-independent Gaussian noise vector. The mathematical forms of $\mathbf{X}(k + 1)$, $\mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]$, $\mathbf{L}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]$, and $\mathbf{W}(k)$ are given as follows.

$$\mathbf{X}(k + 1) = \mathbf{col}(\mathbf{X}_j(k + 1), j = 1, 2, \dots, J)_{(3J \times 1)} \tag{6}$$

where $\mathbf{X}_j(k + 1)$ is given by

$$\mathbf{X}_j(k + 1) = \begin{bmatrix} r_{j,j+1}(k + 1) \\ r_{j,\underline{R}_j}(k + 1) \\ r_{\bar{R}_j,j}(k + 1) \end{bmatrix} \tag{7}$$

where $r_{j,j+1}(k + 1)$, $r_{j,\underline{R}_j}(k + 1)$, and $r_{\bar{R}_j,j}(k + 1)$ represent the basic state variables associated with a given service zone j in time interval $k + 1$.

$$\mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k] = \mathbf{col}(\mathbf{f}_j(k), j = 1, 2, \dots, J)_{(3J \times 1)} \tag{8}$$

where $\mathbf{f}_j(k)$ is given by

$$\mathbf{f}_j(k) = \begin{bmatrix} r_{j,j+1}(k) \\ r_{j,\underline{R}_j}(k) \\ \Omega_j [c_j(k), p_j(k)] \times r_{\bar{R}_j,j}(k) \end{bmatrix} \tag{9}$$

where $\Omega_j [c_j(k), p_j(k)]$ represents the aggregate effect of control variables on the change pattern of $r_{\bar{R}_j,j}(k)$, i.e., the proportion of on-ramp traffic flows entering into the corresponding service zone, and the mathematical form of $\Omega_j [c_j(k), p_j(k)]$ is given by

$$\begin{aligned} \Omega_j [c_j(k), p_j(k)] &= \left[1 + \frac{c_j(k) - c_j(k - 1)}{c_j(k - 1)} \right] \times \left[1 + \frac{p_j(k) - p_j(k - 1)}{p_j(k - 1)} \right] \\ &= \frac{c_j(k)}{c_j(k - 1)} \times \frac{p_j(k)}{p_j(k - 1)}. \end{aligned} \tag{10}$$

Eq. (10) indicates that the increase of either the ramp metering rate or toll rate may have a positive effect on $r_{\bar{R}_j,j}(k)$. For instance, higher ramp metering rates may facilitate on-ramp traffic flows entering the mainline segment of the

freeway system, thus contributing to an increase in $r_{\bar{R}_{j,j}}(k)$. In addition, the increase of the toll rate may lessen the willingness of en-route drivers to travel via freeways. Consequently, it is likely that traffic arrivals to on-ramps may decrease with the increase of the toll rate, thus leading to an increase in $r_{\bar{R}_{j,j}}(k)$.

Note that in case of no extrinsic disturbance sources (e.g., ramp control and toll collection), the short-term change patterns of freeway traffic states are likely to follow Markov processes. This may particularly hold true in our case of estimating the basic states $r_{j,j+1}(k)$ and $r_{j,\underline{R}_j}(k)$ as they are observed on the mainline segments of a freeway, where there are no extrinsic influential factors (e.g., ramp metering control) existing. Supporting arguments can also be found in our previous works [25,24] which aim to investigate the dynamics of lane traffic states on the mainline segments of freeways without the influence of traffic control strategies. In contrast, $r_{\bar{R}_{j,j}}(k)$ is likely to be influenced significantly by ramp control in the proposed integrated toll-ramp control system, and thus we incorporate the corresponding control effect into Eq. (9) to characterize its time-varying change patterns in a deterministic system.

$$\mathbf{L}[\mathbf{X}(k), \mathbf{\Omega}(k), k] = \text{dia} [\mathbf{1}_j(k), j = 1, 2, \dots, J]_{(3J \times 3J)} \tag{11}$$

where $\mathbf{1}_j(k)$ is given by

$$\mathbf{1}_j(k) = \begin{bmatrix} \left(\frac{c_j(k) - c_j(k-1)}{c_j(k-1)} \right) \times r_{\bar{R}_{j,j}}(k) \times [1 - r_{j,j+1}(k) - r_{j,\underline{R}_j}(k)] \\ \left(\frac{p_j(k) - p_j(k-1)}{p_j(k-1)} \right) \times r_{\bar{R}_{j,j}}(k) \times [1 - r_{j,j+1}(k) - r_{j,\underline{R}_j}(k)] \\ [1 - c_j(k) \times r_{\bar{R}_{j,j}}(k)] \times [1 - r_{j,j+1}(k) - r_{j,\underline{R}_j}(k)] \end{bmatrix}_{3 \times 1} \tag{12}$$

The elements of the state-dependent noise vector refer to the disturbance sources that may divert the corresponding basic state variables from deterministic to stochastic conditions; and the magnitudes of the disturbance effects also depend on the time-varying system states existing in the current time interval. In reality, we have introduced similar concepts concerning the state-dependent disturbance sources in certain previous works [24,23,22]. Our basic idea is rooted in the postulation that in a stochastic traffic flow system, the equilibrium conditions of time-varying traffic states could be affected by state-related disturbance sources. In the study, the state-dependent noise terms may originate from three potential disturbance sources, including (1) the amount of traffic flows being served in a given service zone, (2) the ease of traffic flowing through the corresponding downstream detector station under the integrated dynamic toll-ramp control, and (3) the amount of vehicles queuing in on-ramps. For instance, the time-varying proportion of on-ramp vehicles entering into the mainline segment in the next time interval ($r_{\bar{R}_{j,j}}(k)$) can be affected by both the time-varying proportion of vehicles queuing on the ramp and that of vehicles remaining in the mainline segment nearby in the current time interval. Accordingly, the magnitude of the corresponding disturbance effect is measured by multiplying the aforementioned two terms, as shown in the 3rd element of vector $\mathbf{1}_j(k)$ (see Eq. (12)). Employing similar concepts, the other state-dependent noise terms are then formulated to form the corresponding matrix ($\mathbf{L}[\mathbf{X}(k), \mathbf{\Omega}(k), k]$).

In contrast, $\mathbf{W}(k)$ represents a state-independent noise vector, where the elements of $\mathbf{W}(k)$ are assumed to follow Gaussian processes denoting the disturbance effects resulting from the time-varying patterns of traffic arrivals to the freeway system. Accordingly, $\mathbf{W}(k)$ is given by

$$\mathbf{W}(k) = \text{col} (\mathbf{w}_j(k), j = 1, 2, \dots, J)_{(3J \times 1)} \tag{13}$$

where $\mathbf{w}_j(k)$ is given by

$$\mathbf{w}_j(k) = \begin{bmatrix} \omega_{r_{j,j+1}}(k) \\ \omega_{r_{j,\underline{R}_j}}(k) \\ \omega_{r_{\bar{R}_{j,j}}}(k) \end{bmatrix}_{3 \times 1} \tag{14}$$

Herein $\omega_{r_{j,j+1}}(k)$, $\omega_{r_{j,\underline{R}_j}}(k)$ and $\omega_{r_{\bar{R}_{j,j}}}(k)$ represent the state-independent noises associated, respectively, with $r_{j,j+1}(k)$, $r_{j,\underline{R}_j}(k)$, and $r_{\bar{R}_{j,j}}(k)$, and follow Gaussian processes. In the proposed model, they are mainly derived from the changing patterns of traffic arrivals at the corresponding sub-areas, including the mainline segments and on-ramps, in consideration of the phenomenon that the magnitude of the aforementioned state-dependent noises shown

in $\mathbf{I}_j(k)$ may also depend to a certain extent on the diversity of new traffic arrivals. Accordingly, the Gaussian state-independent vector $\mathbf{w}_j(k)$, indicating the patterns of change for traffic arrivals, is involved in the formulation of noise terms in the proposed stochastic model.

The measurement equations characterize the time-varying relationships between traffic measurements, i.e., the collected traffic counts, and the basic state variables. Utilizing the specified relationships, the basic state variables predicted from the state equations are updated in real-time in response to the time-varying patterns of zone-based traffic flows under the proposed integrated dynamic toll-ramp control. The generalized form of the measurement equations is given by

$$\mathbf{Z}(k) = \mathbf{H}[\mathbf{X}(k), k] + \mathbf{V}(k) \tag{15}$$

where $\mathbf{Z}(k)$ is a $(3J \times 1)$ time-varying measurement vector which is composed of the time-varying zone-based outflows collected in the mainline segments and off-ramps, as well as the time-varying freeway entry flows from on-ramps in time interval k ; $\mathbf{H}[\mathbf{X}(k), k]$ is a $(3J \times 1)$ time-varying measurement-component vector in which each element is associated with a given element shown in $\mathbf{Z}(k)$, characterizing the components of the corresponding element of $\mathbf{Z}(k)$ with the state variables and collected upstream traffic arrivals in time interval k ; $\mathbf{V}(k)$ is a $(3J \times 1)$ Gaussian noise vector, representing the measurement errors of the collected traffic data in time interval k . The notations of $\mathbf{Z}(k)$, $\mathbf{H}[\mathbf{X}(k), k]$ and $\mathbf{V}(k)$ are given in the following.

$$\mathbf{Z}(k) = \mathbf{col}(\mathbf{z}_j(k), j = 1, 2, \dots, J)_{(3J \times 1)} \tag{16}$$

where $\mathbf{z}_j(k)$ is given by

$$\mathbf{z}_j(k) = \begin{bmatrix} O_{j,j+1}(k) \\ \underline{R}_j(k) \\ \bar{R}_j(k) \end{bmatrix}_{3 \times 1} \tag{17}$$

$$\mathbf{H}(k) = \mathbf{col}(\mathbf{h}_j(k), j = 1, 2, \dots, J)_{(3 \times 1)} \tag{18}$$

where $\mathbf{h}_j(k)$ is given by

$$\mathbf{h}_j(k) = \begin{bmatrix} (I_{j-1,j}(k) + S_j(k)) \times r_{j,j+1}(k) \\ (I_{j-1,j}(k) + S_j(k)) \times r_{j,\underline{R}_j}(k) \\ (A_{\bar{R}_j}(k) + S_{\bar{R}_j}(k)) \times r_{\bar{R}_j,j}(k) \end{bmatrix}_{3 \times 1} . \tag{19}$$

Herein, all the variables shown in Eq. (19) are associated with the given service zone j in time interval k , and their corresponding definitions are the same as presented previously in the description of state variables.

$$\mathbf{V}(k) = \mathbf{col}(\mathbf{v}_j(k), j = 1, 2, \dots, J)_{(3J \times 1)} \tag{20}$$

where $\mathbf{v}_j(k)$ is given by

$$\mathbf{v}_j(k) = \begin{bmatrix} v_{r_{j,j+1}}(k) \\ v_{r_{j,\underline{R}_j}}(k) \\ v_{r_{\bar{R}_j,j}}(k) \end{bmatrix}_{3 \times 1} \tag{21}$$

where $v_{r_{j,j+1}}(k)$, $v_{r_{j,j+1}}(k)$, $v_{r_{\bar{R}_j,j}}(k)$ follow Gaussian processes.

The boundary constraints of the stochastic model are postulated to ensure that the estimates of basic state and control variables yielded in the proposed integrated dynamic toll-ramp control algorithm are feasible. Herein, three groups of boundary constraints associated, respectively, with the corresponding state variables ($\mathbf{X}_j(k)$), ramp metering rates ($c_j(k)$), and toll rates ($p_j(k)$) are incorporated in the proposed model. Their generalized forms are given by

$$\mathbf{0} \leq \mathbf{X}_j(k) \leq \mathbf{1} \quad \forall j, k \tag{22}$$

$$0 \leq c_j(k) \leq 1 \quad \forall j, k \tag{23}$$

$$0 \leq p_j(k) \leq p_{\max} \quad \forall j, k. \tag{24}$$

4. Control algorithm

Utilizing extended Kalman filtering technique, a stochastic optimal control based algorithm is proposed to estimate the time-varying basic state and control variables of the stochastic model. Herein, following the theorems of stochastic optimal control for the minimum mean square estimation of the basic state variables and control variables [27,19], we attempt to minimize the differences between the estimated values of state variables and their corresponding ideal values during a given control period with K time intervals. The ideal values of state variables refer to the desired values of the state variables that facilitate moving zone-based traffic flows through the corresponding service zones with target service levels. Accordingly, we have the aggregated objective function ($\tilde{\zeta}_K$) as follows for each given K -based control period, where the number of time intervals is K .

$$\tilde{\zeta}_K = \min \sum_{jK=1}^J \zeta_{jK} \tag{25}$$

where ζ_{jK} is given by

$$\begin{aligned} \zeta_{jK} = E \left\{ \sum_{k=1}^K \left[X_j(k) - \hat{\mathbf{X}}_j^{\tilde{s}}(k) \right]^T \Phi_j^1(k) \left[X_j(k) - \hat{\mathbf{X}}_j^{\tilde{s}}(k) \right] \right. \\ \left. + \left[\Omega_j(k) - \hat{\Omega}_j^{\tilde{s}}(k) \right]^T \Phi_j^2(k) \left[\Omega_j(k) - \hat{\Omega}_j^{\tilde{s}}(k) \right] \right\} \tag{26} \end{aligned}$$

where $\hat{\mathbf{X}}_j^{\tilde{s}}(k)$ represents a (3×1) time-varying target state vector associated with $X_j(k)$, involving the corresponding target values of basic states under the proposed freeway traffic control with a given target service level \tilde{s} ; similarly, $\hat{\Omega}_j^{\tilde{s}}(k)$ represents a (2×1) target control vector, involving the corresponding target values of control variables to maintain the proposed freeway system with a given target service level \tilde{s} , and conveniently, the elements of $\hat{\Omega}_j^{\tilde{s}}(k)$ are set to be consistent with the elements of the estimated control-variable vector (i.e., $\Omega_j(k - 1)$) of the previous time interval to serve the purpose of minimizing the risks caused by the changes of control variables; $\Phi_j^1(k)$ and $\Phi_j^2(k)$ represent the (3×3) and (2×2) time-varying diagonal, positive-definite weighting matrices associated, respectively with the estimated basic state vector ($X_j(k)$) and that of the control variable vector ($\Omega_j(k)$). Herein, $\hat{\mathbf{X}}_j^{\tilde{s}}(k)$, $\Phi_j^1(k)$, and $\Phi_j^2(k)$ can be further derived as in the following.

As mentioned previously, $\hat{\mathbf{X}}_j^{\tilde{s}}(k)$ refers to the corresponding target vector of $X_j(k)$. Correspondingly, each element of $\hat{\mathbf{X}}_j^{\tilde{s}}(k)$ represents the corresponding target value associated with a given basic state under the proposed freeway traffic control in a given service level. Therefore, employing the ideal zone-based traffic flow condition shown in Eq. (1), we have

$$\hat{\mathbf{X}}_j^{\tilde{s}}(k) = \begin{bmatrix} \hat{r}_{j,j+1}(k) \\ \hat{r}_{j,R_j}(k) \\ \hat{r}_{\bar{R}_j,j}(k) \end{bmatrix}. \tag{27}$$

Here, $r_{\bar{R}_j,j}(k)$ is set to be 1 considering the aforementioned on-ramp traffic equilibrium condition. In contrast, $\hat{r}_{j,j+1}(k)$ and $\hat{r}_{j,R_j}(k)$ are determined using the ideal zone-based traffic flow condition shown in Eq. (1), and thus they are given by

$$\hat{r}_{j,j+1}(k) = \left[\frac{\sum_{\tau=0}^{k-1} O_{j,j+1}(k - \tau)}{\sum_{\tau=0}^{k-1} O_{j,j+1}(k - \tau) + R_j(k - \tau)} \right] \times \left[1 - \frac{S_j^{\tilde{s}}}{I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)} \right] \tag{28}$$

$$\hat{r}_{j,\underline{R}_j}(k) = \left[\frac{\sum_{\tau=0}^{k-1} \underline{R}_j(k-\tau)}{\sum_{\tau=0}^{k-1} O_{j,j+1}(k-\tau) + \underline{R}_j(k-\tau)} \right] \times \left[1 - \frac{S_j^{\bar{s}}}{I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)} \right]. \tag{29}$$

The above specification of $\hat{r}_{j,j+1}(k)$ and $\hat{r}_{j,\underline{R}_j}(k)$ is rooted in the postulation that the ideal zone-based outflow ratio (i.e., $\hat{r}_{j,j+1}(k) + \hat{r}_{j,\underline{R}_j}(k)$) should be equal to the time-varying net zone-based outflow ratio $(1 - \frac{S_j^{\bar{s}}}{I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)})$ in any given time interval. This can be derived by taking the sum of Eqs. (28) and (29). Then, $\hat{r}_{j,j+1}(k)$ and $\hat{r}_{j,\underline{R}_j}(k)$ can be readily specified through allocating $1 - \frac{S_j^{\bar{s}}}{I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)}$ to each of them according to their time-varying weights, i.e., $[\frac{\sum_{\tau=0}^{k-1} O_{j,j+1}(k-\tau)}{\sum_{\tau=0}^{k-1} O_{j,j+1}(k-\tau) + \underline{R}_j(k-\tau)}]$ and $\frac{\sum_{\tau=0}^{k-1} \underline{R}_j(k-\tau)}{\sum_{\tau=0}^{k-1} O_{j,j+1}(k-\tau) + \underline{R}_j(k-\tau)}$, respectively.

The elements of $\Phi_j^1(k)$ indicate the costs of state deviation under the proposed integrated dynamic toll-ramp control. To a certain extent, the proposed objective function implies that the more the time-varying basic state variables are divergent from their corresponding ideal values, the greater the penalty costs of the control system should pay for. Accordingly, $\Phi_j^1(k)$ is involved in the objective function to indicate the weights of penalty costs associated with the controlled basic states, and is given by

$$\Phi_j^1(k) = \begin{bmatrix} \phi_{r_{j,j+1}}(k) & 0 & 0 \\ 0 & \phi_{r_{j,\underline{R}_j}}(k) & 0 \\ 0 & 0 & \phi_{r_{\bar{R}_j,j}}(k) \end{bmatrix} \tag{30}$$

where $\phi_{r_{j,j+1}}(k)$, $\phi_{r_{j,\underline{R}_j}}(k)$, and $\phi_{r_{\bar{R}_j,j}}(k)$ are given by

$$\phi_{r_{j,j+1}}(k) = \frac{O_{j,j+1}(k)}{O_{j,j+1}(k) + \bar{R}_j(k) + \underline{R}_j(k)} \tag{31}$$

$$\phi_{r_{j,\underline{R}_j}}(k) = \frac{\underline{R}_j(k)}{O_{j,j+1}(k) + \bar{R}_j(k) + \underline{R}_j(k)} \tag{32}$$

$$\phi_{r_{\bar{R}_j,j}}(k) = \frac{\bar{R}_j(k)}{O_{j,j+1}(k) + \bar{R}_j(k) + \underline{R}_j(k)}. \tag{33}$$

Similarly, the elements of $\Phi_j^2(k)$ indicate the costs caused by the control variable deviation under the proposed integrated dynamic toll-ramp control. Theoretically, the more frequently the time-varying control variables switch from their corresponding target values the greater the penalty costs of the control system should pay for. Therefore, the elements of $\Phi_j^2(k)$ are specified to indicate the weights of the aforementioned penalty costs. Here $\Phi_j^2(k)$ is given by

$$\Phi_j^2(k) = \begin{bmatrix} \phi_{c_j}(k) & 0 \\ 0 & \phi_{p_j}(k) \end{bmatrix} \tag{34}$$

where $\phi_{c_j}(k)$ and $\phi_{p_j}(k)$ are given by

$$\phi_{c_j}(k) = \frac{\bar{R}_j(k)}{I_{j-1,j}(k) + \bar{R}_j(k)} \tag{35}$$

$$\phi_{p_j}(k) = 1 - \frac{O_{j,j+1}(k) + \underline{R}_j(k)}{I_{j-1,j}(k) + S_j(k) + \bar{R}_j(k)}. \tag{36}$$

It is worth mentioning that in reality, Eq. (26) is a well-known cost function (or termed quadratic loss function) representing a general objective function consisting of the minimum mean square (MMS) estimates of the state and

control variables produced by the Kalman filter fed back through optimal gains for stochastic optimal control. Such a generalized form and its fundamentals can be extensively found in the field of optimal control and related areas [27, 19,6]. Following the aforementioned fundamentals, the components of the cost function, particularly in terms of the target values of the basic state variables (through Eqs. (27)–(29)) and these penalty weight matrixes (Eqs. (30)–(36)) are postulated based on the goal of the proposed integrated toll-ramp control system. Briefly, it is expected through the specification of the cost function, these MMS state estimates and control variables can capture their target values as well as possible to avoid the estimation divergence problems during the control period.

The primary computational scenarios involved in the proposed algorithm include: (1) system initialization, (2) prior prediction of basic state variables, (3) stochastic optimal estimation of basic state variables, and (4) determination of time-varying control variables. In order to obtain the minimum mean square estimates of the state variables through the aforementioned scenarios (2) and (3), the fundamentals of an extended Kalman filter are applied. Note that the concepts and distinctive features of extended Kalman filtering technologies can be readily found elsewhere [27,19, 24,23], thus being omitted in this content. Except for the scenario of system initialization, which is conducted with preset parameters to initialize the control algorithm, the other three computational scenarios are executed in sequence in each time interval during a given K -based control period. The following summarizes the major computational steps of the proposed integrated dynamic toll-ramp control logic.

Step 0. Initialize system states, and input collected raw traffic data. Given $k = 0$, system states are initialized including: (1) the basic state vector $\mathbf{X}(0|0)$, referring to the vector of state variables estimated at the beginning of the initial time interval, i.e., $k = 0$; (2) the covariance matrix of the state estimation error $\text{Cov}_{\mathbf{x}}(0|0)$; and (3) the aggregate weighting matrices $\Phi_j^1(0)$ and $\Phi_j^2(0)$ which involve $\Phi_j^1(0)$ and $\Phi_j^2(0)$ associated with each given service zone j . In addition, determine the target service level \tilde{s}_j for each service zone, and then let the time-varying control variables be the following initial values: $p_j(0) = \rho_{\tilde{s}}$ and $c_j(0) = \frac{f^{-1}[p_j(0)]}{t}$, where $\rho_{\tilde{s}}$ is the ideal toll rate preset for the target service level \tilde{s}_j .

Note that in the proposed control algorithm, only the initial toll rates (i.e., $\rho_{\tilde{s}}$) are needed, and can be predetermined according to either toll policies predetermined by the corresponding freeway traffic management administration sectors or using the existing economic theories in congestion pricing. Here, we merely use the aforementioned formulas to determine the initial toll and ramp metering rates. Adopting the concept of elastic demand functions proposed by Yang [36] and Yang et al. [34,35], we assume $f^{-1}[p_j(k)]$ as a respective negative exponential function, i.e., $f^{-1}[p_j(k)] = \alpha_j \exp[-\beta_j(p_j(k))]$, for simplicity, where parameters α_j and β_j are calibrated using simulation data. Briefly, using Paramics, we built one simple network containing one origin and one destination connected with two respective links with the same distances, and initial traffic flow conditions. In addition, a given set of origin-destination flows was predetermined. Then, we designed a set of toll rates, including $p_j(k) = 0$ for one of these two links, and then measured the link flows going through these two links. Therein, by setting the toll rate to be zero, we could readily calibrate the value of α_j embedded in $f^{-1}[p_j(k)] = \alpha_j \exp[-\beta_j(p_j(k))]$, followed by the calibration of β_j . Once α_j and β_j were calibrated, their calibrated values were then input to the proposed algorithm for the recursive estimation of state and control variables.

Step 1. Compute the prior prediction of the time-varying state vector ($\mathbf{X}(k+1|k)$) and the covariance matrix of the state estimation error ($\text{Cov}_{\mathbf{x}}(k+1|k)$), respectively, by

$$\mathbf{X}(k+1|k) = \mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k] \quad (37)$$

$$\text{Cov}_{\mathbf{x}}(k+1|k) = \dot{\mathbf{F}}(k)\text{Cov}_{\mathbf{x}}(k|k)\dot{\mathbf{F}}^T(k) + \mathbf{L}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k] \Phi_j^1(k) \mathbf{L}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]^T \quad (38)$$

where $\dot{\mathbf{F}}^T(k)$ is the transpose matrix of $\dot{\mathbf{F}}(k)$, in which $\dot{\mathbf{F}}(k)$ is given by

$$\dot{\mathbf{F}}(k) = \left. \frac{\partial \mathbf{F}[\mathbf{X}(k), \boldsymbol{\Omega}(k), k]}{\partial \mathbf{X}(k)} \right|_{\mathbf{X}(k)=\mathbf{X}(k|k)} \quad (39)$$

Step 2. Calculate the time-varying Kalman gain ($\delta(k+1)$) by

$$\delta(k+1) = \text{Cov}_{\mathbf{x}}(k+1|k) \dot{\mathbf{H}}^T(k+1) \left[\dot{\mathbf{H}}(k+1) \text{Cov}_{\mathbf{x}}(k+1|k) \dot{\mathbf{H}}^T(k+1) + \text{Cov}_{\mathbf{v}}(k+1) \right]^{-1} \quad (40)$$

where $\text{Cov}_v(k + 1)$ is the covariance matrix of $\mathbf{V}(k + 1)$; and $\dot{\mathbf{H}}(k + 1)$ is denoted by

$$\dot{\mathbf{H}}(k + 1) = \left. \frac{\partial \mathbf{H}[\mathbf{X}(k + 1), k + 1]}{\partial \mathbf{X}(k + 1)} \right|_{\mathbf{X}(k+1)=\mathbf{X}(k+1|k)} \quad (41)$$

Step 3. Update the prior prediction of the time-varying state vector ($\mathbf{X}(k + 1|k + 1)$) by

$$\mathbf{X}(k + 1|k + 1) = \mathbf{X}(k + 1|k) + \delta(k + 1)\Delta\mathbf{Z}(k + 1|k) \quad (42)$$

where $\Delta\mathbf{Z}(k + 1|k)$ is given by

$$\Delta\mathbf{Z}(k + 1|k) = \mathbf{Z}(k + 1) - \mathbf{H}[\mathbf{X}(k + 1|k), k + 1]. \quad (43)$$

In Eq. (43), $\mathbf{H}[\mathbf{X}(k + 1|k), k + 1]$ is the prior measurement-component vector, in which each element characterizes the components of the corresponding element of $\mathbf{Z}(k + 1)$ using the prior predictions of state variables at the beginning of the given time interval $k + 1$ and the upstream traffic counts collected in the given time interval $k + 1$.

Step 4. Truncate the updated state vector ($\mathbf{X}(k + 1|k + 1)$) with the aforementioned boundary constraint, as presented in Eq. (34), such that each updated state variable is feasible, subject to the corresponding upper and lower bounds.

Step 5. Update the covariance matrix of the state estimation error ($\text{Cov}_x(k + 1|k + 1)$) as

$$\text{Cov}_x(k + 1|k + 1) = [\mathbf{I} - \delta(k + 1)\dot{\mathbf{H}}(k + 1)] \text{Cov}_x(k + 1|k). \quad (44)$$

Step 6. Update the numbers of vehicles remaining in the mainline segment and corresponding on-ramp associated with each given service zone j at the end of time interval $k + 1$ (i.e., $S_j(k + 1)$ and $S_{\bar{R}_j}(k + 1)$, respectively). The formulae for updating $S_j(k + 1)$ and $S_{\bar{R}_j}(k + 1)$ are given by

$$S_j(k + 1) = [I_{j-1,j}(k + 1) + \bar{R}_j(k + 1) + S_j(k)] - [O_{j,j+1}(k + 1) + \underline{R}_j(k + 1)] \quad (45)$$

$$S_{\bar{R}_j}(k + 1) = [A_{\bar{R}_j}(k + 1) + S_{\bar{R}_j}(k)] - \bar{R}_j(k + 1). \quad (46)$$

Note that according to Eqs. (42) and (43) presented in Step 3, the updated values of $S_j(k + 1)$ and $S_{\bar{R}_j}(k + 1)$ will be used for updating the prior prediction of the time-varying state vector in the next computational iteration.

Step 7. Calculate the control-variable vector $\mathbf{\Omega}(k + 1)$. Using the fundamentals of stochastic optimal control theories, the updated estimates of time-varying basic state variables $\mathbf{X}(k + 1|k + 1)$ are fed back through the following formulae to accomplish the goal of the pre-specified objective function (see Eqs. (5) and (6)).

$$\mathbf{\Omega}(k + 1) = -\mathbf{E}(k + 1)\mathbf{X}(k + 1|k + 1) + \mathbf{T}(k + 1). \quad (47)$$

Herein, $\mathbf{E}(k + 1)$, referring to a time-varying control gain vector, and $\mathbf{T}(k + 1)$ are given, respectively, by

$$\mathbf{E}(k + 1) = [\mathbf{B}^T(k + 1)\mathbf{S}(k + 2)\mathbf{B}(k + 1) + \Phi_J^2(k + 1)]^{-1} \mathbf{B}^T(k + 1)\mathbf{S}(k + 2)\dot{\mathbf{F}}(k + 1) \quad (48)$$

$$\mathbf{T}(k + 1) = [\mathbf{B}^T(k + 1)\mathbf{S}(k + 2)\mathbf{B}(k + 1) + \Phi_J^2(k + 1)]^{-1} [\mathbf{B}^T(k + 1)\mathbf{A}(k + 1) + \mathbf{G}(k + 1)]. \quad (49)$$

According to the principles of optimal control, the following conditions in terms of $\mathbf{A}(k + 1)$, $\mathbf{B}(k + 1)$, $\mathbf{G}(k + 1)$ and $\mathbf{S}(k + 2)$ should hold to satisfy the Riccati equation.

$$\mathbf{A}(k) = \Phi_J^1(k)\hat{\mathbf{X}}(k + 1) + [\dot{\mathbf{F}}^T(k + 1) + \mathbf{B}(k + 1)\mathbf{E}(k + 1)]^T \mathbf{A}(k + 1) \quad (50)$$

$$\mathbf{B}(k + 1) = \frac{\partial \mathbf{F}[\mathbf{X}(k), \mathbf{\Omega}(k), k]}{\partial \mathbf{\Omega}(k)} \quad (51)$$

$$\mathbf{G}(k + 1) = \Phi_J^2(k + 1)\hat{\mathbf{\Omega}}(k + 1) \quad (52)$$

$$\mathbf{S}(k + 1) = \Phi_J^1(k + 1) + \dot{\mathbf{F}}^T(k + 1)\mathbf{S}(k + 2)\dot{\mathbf{F}}(k + 1) - \dot{\mathbf{F}}^T(k + 1)\mathbf{S}(k + 2)\mathbf{B}(k + 1)\mathbf{E}(k + 1). \quad (53)$$

Step 8. Check the estimates of the time-varying control variables presented in $\mathbf{\Omega}(k + 1)$ with Eqs. (35) and (36) to satisfy the corresponding boundary conditions.

Step 9. Input the next-time-step collected traffic data; let the time step index $k = k + 1$, and then go back to *Step 1* for the next computational iteration of the proposed control algorithm until the end of the given K -based control period.

5. Numerical study

The numerical study demonstrates the relative performance of the proposed integrated dynamic toll-ramp control approach for dynamic freeway traffic management, compared to conventional freeway traffic management strategies. Here, a 4-lane mainline segment of the southbound Formosa freeway of Taiwan, located in the metropolitan area of Taipei, was selected as the study site. In addition, to investigate the potential effect of dynamic toll rate strategies on route diversion at the study site, a 4-lane toll-free suburban arterial representing another alternative for route choice is involved, where there are two lanes in each direction. In contrast with the target freeway system, traffic on the 4-lane suburban arterial must be regulated by lower speed limits and five signal controls at the five main intersections on surface streets. Presently, the target freeway system involves one manual toll collection station implemented with a fixed toll rate of US\$ 0.7/10 mile, and the corresponding traffic flows are regulated by local ramp control with fixed ramp metering rates of 0.17 veh/s, 0.25 veh/s, and 0.34 veh/s for high-volume, medium-volume, and low-volume cases, respectively. To illustrate the relative advantages of the proposed method, the numerical results were compared with results obtained using these two existing freeway traffic management strategies in the study site.

Considering the need of appropriate traffic flow data for different freeway congestion cases, simulation data generated from the Paramics microscopic traffic simulator was used for convenience. The corresponding tasks of simulator calibration and preliminary tests can be found in a previous study [21]. Particularly, in the calibration scenario, we aimed to calibrate the generalized link cost function embedded in Paramics to deal with the potential drivers' responses to the variety of dynamic toll rates estimated using the proposed model. Therein, the embedded link cost function of Paramics takes in to account three major factors of the monetary link costs including link travel time, distance and toll pricing, which allow the users to adjust via the "configuration function" so as to reproduce the drivers' route choice maneuvers [18]. For each time interval, the optimal toll and ramp metering rates were estimated using the proposed model and the simulation data collected in the previous time interval, and then were manually input to Paramics for the next-time-interval estimation. Such a recursive estimation routine was conducted in each time interval during each given simulation event. Considering the space limits of this paper, the details about the calibration and preliminary test procedures and intermediate results are not presented here.

To simulate various freeway traffic congestion cases under the ramp control and toll collection conditions in the study site, a simulation network mimicking the study site was constructed using Paramics, as illustrated in Fig. 3, which involves 2 on-ramps and 2 off-ramps for each direction in the target freeway system. Employing the Paramics simulator, a total of nine different combinations of traffic inflow patterns, including high-volume, medium-volume, and low-volume cases associated, respectively, with the target freeway system and suburban arterial, were simulated, and then the 1 min traffic data were collected during a given 1 h control period. Here, the ramp control and toll collection strategies were simulated through appropriate parameter setting in the Paramics simulation model. According to the functions embedded in Paramics [18], the input and output data including hourly throughputs and path travel times can be readily collected to generate the numerical results. In addition, Paramics also provides very friendly node-based traffic control simulation functions and link-based cost functions, which allow us to set the estimated ramp metering and toll rates in each time interval, and then measure the system performance to test the proposed model. The corresponding parameters preset for these simulation scenarios are summarized in Table 1.

In order to investigate the potential advantages of the proposed approach to dynamic freeway traffic congestion management, two evaluation measures are utilized: freeway zone-based average travel time (\overline{AT}) and average hourly throughput (\overline{TI}). In addition, for the use of comparative analyses, the average travel time ($\overline{AT}_{\text{sub}}$) and hourly traffic flow ($\overline{TI}_{\text{sub}}$) through the suburban arterial are also measured.

It is worth mentioning that the present study purpose may aim at freeway congestion management using the proposed toll-ramp control methodology. Thus, we may be more interested in knowing the effects of the proposed control strategies on the freeway-based system performance (including freeway-based travel time and throughput) and the induced impact on the alternative route (i.e., the surface street route). Such information is particularly important for the implementation of ITS-related technologies such as real-time toll and ramp control as well as route guidance because the respective path travel times and related information are needed. These may also clarify why the aforementioned four evaluation measures are proposed to be used in the numerical study.

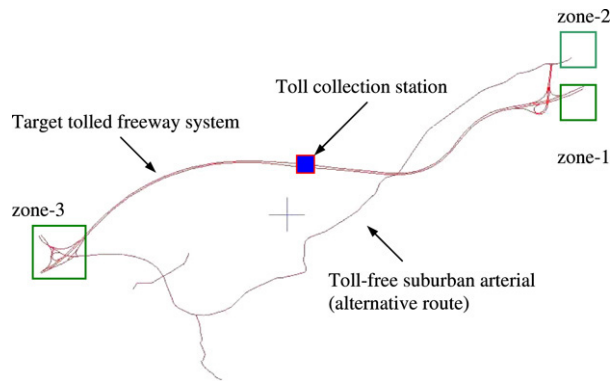


Fig. 3. Illustration of the simulated study site.

Table 1
Summary of simulation characteristics

1. Geometric characteristics		
	Target tolled freeway	Suburban arterial
Number of lanes (mainline segment)	4	2
Number of on-ramps	2	
Number of off-ramps	2	
Number of lanes (ramp segment)	1	
Geographic length	6 miles	10 miles
2. Traffic characteristics		
Inflows (freeway high-volume cases)	6000 vph	2400 vph 1200 vph 600 vph
	3000 vph	2400 vph 1200 vph 600 vph
	1500 vph	2400 vph 1200 vph 600 vph
Inflows (freeway medium-volume cases)		
Inflows (freeway low-volume cases)		
Speed limits	70 mph	40 mph
3. Control strategies		
The proposed method (alternative-1)	Dynamic ramp metering and toll rates	Conventional pre-timed signal control
The existing strategy (alternative-2)	Fixed ramp metering and toll rates	Conventional pre-timed signal control
Control-free strategy (alternative-3)	None	Conventional pre-timed signal control
4. Simulation operational characteristics		
Simulation period		60 min
Unit time interval		1 min

Utilizing the aforementioned evaluation criteria, the proposed integrated dynamic toll-ramp control method was tested with the simulation data, and compared with the aforementioned two control alternatives. Each traffic flow scenario was simulated ten times, and then the numerical results were summarized in Tables 2–4, classified by three levels of freeway traffic volume conditions.

Overall, the numerical results summarized in Tables 2–4 reveal the potential advantages of the proposed integrated dynamic toll-ramp control method for freeway traffic congestion management under various traffic flow conditions,

Table 2
Comparison of control performance (freeway high-volume cases)

Control alternatives	Criteria			
	\overline{AI} (s)	\overline{TI} (veh/h)	\overline{AI}_{sub} (s)	\overline{TI}_{sub} (veh/h)
The proposed strategy	468	4608	1469	1285
The existing strategy	508	4236	1460	1243
Control-free strategy	560	4154	1458	1237
Relative improvement of the proposed method				
Relative to existing strategy (%)	7.9	8.8	−0.6	3.4
Relative to control-free strategy (%)	16.4	10.9	−0.8	3.9

Table 3
Comparison of control performance (freeway medium-volume cases)

Control alternatives	Criteria			
	\overline{AI} (s)	\overline{TI} (veh/h)	\overline{AI}_{sub} (s)	\overline{TI}_{sub} (veh/h)
The proposed strategy	398	3631	1461	1192
The existing strategy	435	3357	1459	1184
Control-free strategy	465	3118	1455	1159
Relative improvement of the proposed method				
Relative to existing strategy (%)	8.5	8.2	−0.1	0.7
Relative to control-free strategy (%)	14.4	16.5	−0.4	2.8

Table 4
Comparison of control performance (freeway low-volume cases)

Control alternatives	Criteria			
	\overline{AI} (s)	\overline{TI} (veh/h)	\overline{AI}_{sub} (s)	\overline{TI}_{sub} (veh/h)
The proposed strategy	347	2036	1458	1074
The existing strategy	368	1987	1462	1083
Control-free strategy	376	1963	1465	1091
Relative improvement of the proposed method				
Relative to existing strategy (%)	5.7	2.5	0.3	−0.8
Relative to control-free strategy (%)	7.7	3.7	0.5	−1.1

relative either to the existing control strategy or to the control-free case. This can be demonstrated from the generalization that all the evaluation measures including \overline{AI} and \overline{TI} have been improved to a certain extent utilizing the proposed method in contrast with the other two control alternatives. Here, \overline{AI} is improved generally by 12.8%, compared to the control-free strategy; and \overline{TI} is improved by 13.6%. The corresponding improvement effects are graphically illustrated in Fig. 4.

To illustrate the effectiveness of the proposed model for dynamic freeway congestion management, Figs. 5 and 6 display typical examples of change patterns of the dynamic ramp control and toll rates output from the proposed model, respectively.

As can be seen in Fig. 5, the estimated dynamic toll rates appear to change more sharply in the high-volume scenario compared with the change patterns exhibited in either the medium-volume or low-volume scenario. It is inferred that under high-volume conditions, particularly in the over-congestion case, such an integrated toll and ramp control strategy may result significantly in drivers' route diversion, thus contributing to a significant reduction in traffic arrivals at the on-ramps, followed by the adjustment of the dynamic toll rates estimated in the following time intervals. In comparison, under low-volume conditions which also means low traffic demands, drivers may not be

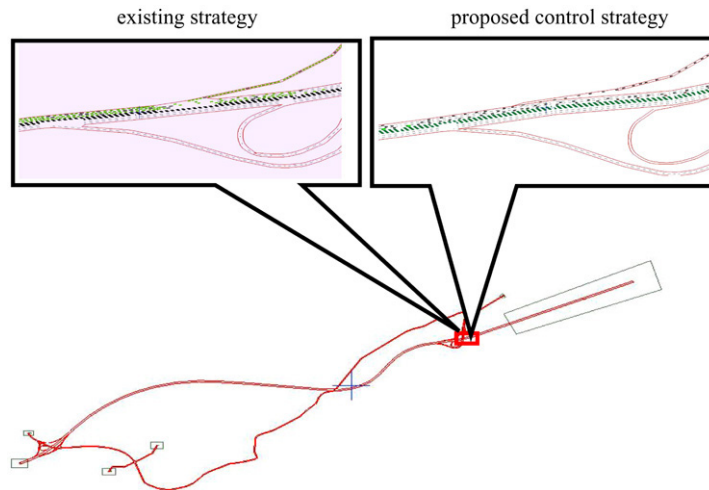


Fig. 4. Illustration of relative improvement by virtual samples of simulations.

affected significantly by the resulting low toll rates. Consequently, the estimated dynamic toll rates appear to remain stably low during the integrated control period.

In contrast, the dynamic ramp metering rates exhibited in Fig. 6 may reveal different features in change patterns. Under medium-volume and low-volume conditions, the resulting dynamic ramp metering rates appear to be rather sensitive to the dynamics of freeway traffic flows and on-ramp traffic arrivals toward the goal of maintaining the freeway system at certain target service levels, thus contributing to significant changes during the integrated control period. As such, the dynamic ramp metering mechanism may perform the primary function in place of the dynamic toll mechanism in regulating the freeway traffic flows in the low- and medium-volume scenarios. However, the resulting lower toll rates may not have significant effects on drivers' route diversion, and thus turn out to be a minor factor in low- and medium-volume scenarios.

Accordingly, it is inferred that despite the integration of dynamic toll and ramp control mechanisms used for freeway congestion management, herein dynamic tolling can be used as the main measure for high-volume traffic flow cases; and in contrast, under low- and medium-volume traffic flow conditions, dynamic ramp metering control may perform the key function aimed at maintaining the performance of the freeway system at target service levels.

In addition, several generalizations derived from the above numerical results are summarized below for discussion.

First, by comparing the measurements \overline{AT} and \overline{TI} obtained under different traffic flow conditions, the proposed integrated dynamic toll-ramp control method seems to have significant leverage under high-volume and medium-volume traffic congestion conditions, relative to low-volume cases. Particularly, the zone-based average travel time can be reduced by 1.5 min (i.e., 16.4%) per vehicle if the proposed integrated dynamic toll-ramp control strategy is implemented in high-volume cases, relative to the control-free strategy. Such a generalization is encouraging since according to our previous studies, high-volume congestion cases still remain as a critical issue in numerous existing ramp control approaches. This issue, as noted in our previous study [21], stems from the fact that the control effect of ramp metering may apply merely to the on-ramp traffic flows so as to regulate traffic merging in the weaving area. If the proportion of the number of on-ramp vehicles to the number of vehicles in the mainline segment is too small, as presented in high-volume congestion cases, the efficiency of ramp control may turn out to be rather insignificant. Nevertheless, in the numerical study, integration of dynamic freeway congestion pricing and ramp control permits alleviation of freeway high congestion efficiently.

Second, in contrast to either high-volume or medium volume cases, the improvement degree in terms of either \overline{AT} or \overline{TI} appears relatively insignificant in low-volume cases. Our inference is that under low-volume conditions, the effect of dynamic freeway congestion pricing on route diversion may not be as significant as those in high-volume and medium-volume cases, where en-route drivers may be greatly encouraged to travel through suburban arterials because of high toll and low ramp metering rates. Correspondingly, under low-volume conditions, drivers' rerouting maneuvers induced by the proposed integrated dynamic toll-ramp control approach may be unnecessary, thus contributing to a relatively lower improvement in the freeway traffic congestion management.

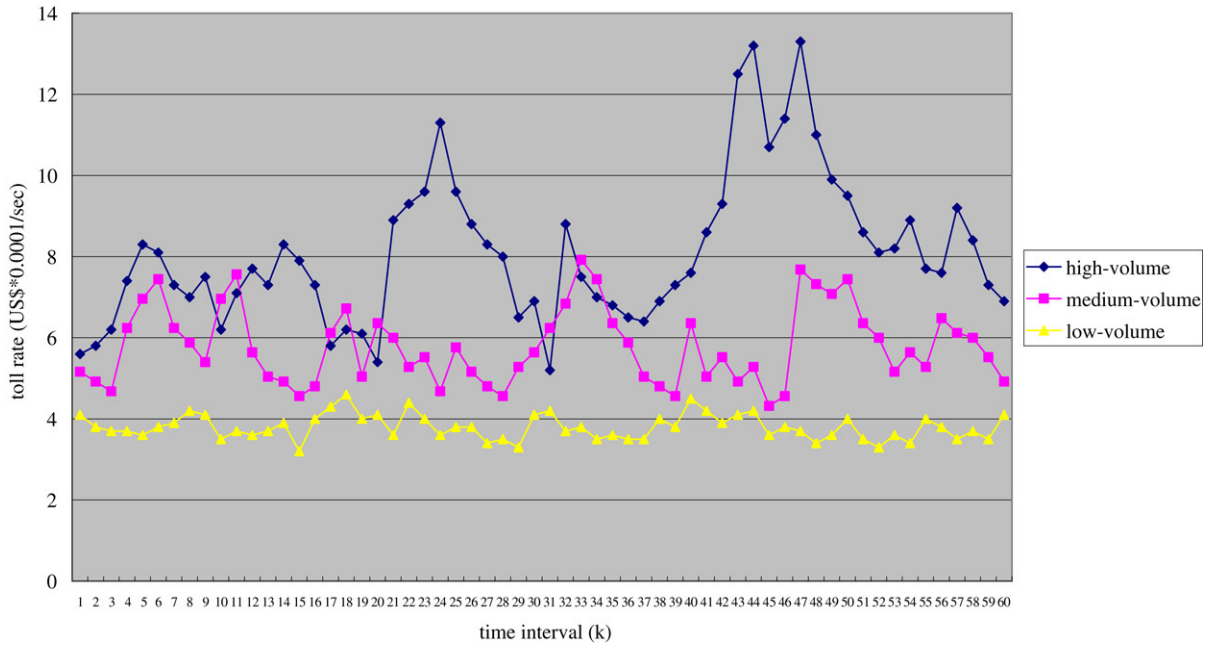


Fig. 5. Temporal variations of the estimated dynamic toll rates.

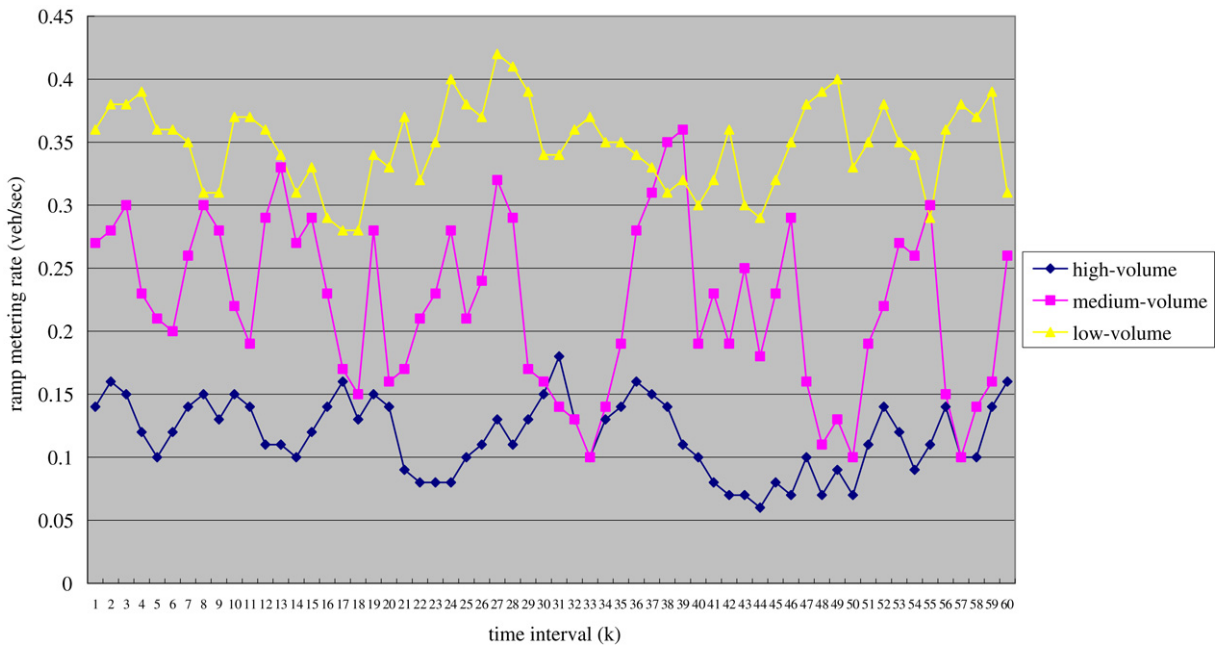


Fig. 6. Temporal variations of the estimated dynamic ramp metering rates.

Third, under the proposed integrated dynamic toll-ramp control, the traffic diversion from the freeway does not seem to have significant effects on the traffic of surface streets in the numerical study. As can be seen in the above tables, the measures in terms of the averaged travel time (\overline{AT}_{sub}) through the suburban arterial and the corresponding throughputs (\overline{TI}_{sub}) among these three control alternatives are slightly different in any traffic flow cases. This may also imply that the integration of appropriate dynamic toll collection and ramp control methodologies can be efficiently extended for further use in network-wide traffic congestion management.

In addition, the dynamic toll effect on en-route freeway traffic diversion is worth noting. It is observed that in the numerical study, under high-volume traffic congestion with high toll rate cases, a relatively greater number of vehicles traveling on freeways may leave earlier from the upstream off-ramp, traveling through the suburban arterial and then return back to the freeway via the downstream on-ramp of the study site. Such a toll-induced freeway traffic diversion phenomenon may exist practically, particularly in the ITS operational environment equipped with technologies such as route guidance and variable message signs. According to our observations from simulations, the average number of vehicles conducting the aforementioned toll-induced freeway traffic diversion maneuvers is about 108 for each high-volume simulation cases. Therefore, it is suggested that issues related to network-wide route diversion effects may still remain and warrant further research.

6. Concluding remarks

This paper has presented an integrated dynamic toll-ramp control approach to dynamic freeway traffic congestion management. Through detector configurations and system specification, a discrete-time nonlinear stochastic system, which embeds basic states and control variables, i.e., dynamic toll and ramp metering rates, is formulated. Using extended Kalman filtering technology, a stochastic optimal control based algorithm is then proposed to execute the proposed integrated dynamic toll-ramp control mechanism.

Our numerical results indicate the applicability of the proposed integrated control methodology for freeway traffic congestion management through an appropriate integration of dynamic ramp control and toll strategies used not only to regulate freeway traffic flows but also to strategically influence drivers' route diversion maneuvers before they arrive to the on-ramps. Numerical results of the study have also suggested the relative advantages of the proposed control method compared with two existing control strategies implemented in the study site. More importantly, the proposed approach reveals a new and feasible solution, which has never been exploited in previous literature to address the issues of dynamic freeway congestion management. In addition, the findings observed in the numerical study may stimulate more valuable research beneficial, not only for freeway traffic management, but also for network-wide traffic congestion management.

Nevertheless, some potential methodological concerns such as the initialization of the state and control variables as well as parameters preset in the proposed model warrant more research effort as they do influence the system performance and time taken for convergence of the dynamic system states. The diversity of driver behavior and habits in different locations and countries may also affect the model's validity. More tests as well as comparisons with other advanced integrated freeway traffic control strategies may warrant further research to verify the robustness of the proposed approach. In addition, the dynamic toll-induced route diversion effects on traffic flows, either on freeways or on surface streets, may need more investigation because of their uncertainties and difficulties in influencing drivers' route choice maneuvers. Particularly, further analysis of the network-wide traffic flow dynamics evolution over time under the integrated control may help provide information to readers for such as the use of advanced traveler information systems (ATIS). Moreover, efforts to integrate the proposed integrated control method with other advanced traffic control and management technologies, e.g., variable message signs (VMS), and dynamic traffic assignment (DTA), seem to be needed urgently for large-scale network traffic congestion management.

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Appendix. Definitions of model variables and parameters

Definitions of variables and parameters shown in the proposed method are summarized in the following.

Notation	Definition
$A_{\bar{R}_j}(k)$	the measured number of vehicles arriving at the on-ramp (\bar{R}_j) of a given service zone j in a given time interval k
$\bar{A}I$	an evaluation criterion in terms of the freeway zone-based average travel time
$\bar{A}I_{\text{sub}}$	an evaluation criterion in terms of the average travel time through the suburban arterial
$\text{Cov}_x(0 0)$	the covariance matrix of the state estimation error
$c_j(k)$	the time-varying on-ramp metering rate associated with a given service zone j in a given time interval k
$\mathbf{E}(k+1)$	a time-varying control gain vector
$\mathbf{F}[\mathbf{X}(k), \mathbf{\Omega}(k), k]$	a $(3J \times 1)$ time-varying basic state vector
$\mathbf{f}_j(k)$	a (3×1) time-varying basic state vector associated with a given control zone j in time interval k
$\mathbf{H}[\mathbf{X}(k), k]$	a $(3J \times 1)$ time-varying measurement-component vector in time interval k
$\mathbf{h}_j(k)$	a (3×1) time-varying measurement-component vector associated with a given service zone j in time step k
$I_{j-1,j}(k)$	the number of vehicles entering from the upstream service zone $j-1$ to service zone j in time interval k
J	the number of service zones involved in the proposed integrated toll-ramp control system
jK	a given service zone j which is under control in a given K -based control period
$\mathbf{L}[\mathbf{X}(k), \mathbf{\Omega}(k), k]$	a $(3J \times 3J)$ diagonal state-dependent noise matrix
$\mathbf{l}_j(k)$	a (3×1) time-varying state-dependent noise vector associated with a given service zone j in time interval k
$O_{j,j+1}(k)$	the number of vehicles moving from service zone j to the downstream service zone $j+1$ in time interval k
$p_j(k)$	the time-varying toll rates associated with a given service zone j in a given time interval k
p_{max}	the upper bound of the toll rate allowed in the proposed integrated dynamic toll-ramp control system
$\bar{R}_j(k)$	the number of on-ramp vehicles entering into service zone j
$\underline{R}_j(k)$	the number of off-ramp vehicles exiting from service zone j in time interval k
$r_{j,j+1}(k)$	the time-varying proportion of vehicles leaving from a given service zone j to the following service zone $j+1$ in a given time interval k
$\hat{r}_{j,j+1}(k)$	the corresponding ideal values associated with $r_{j,j+1}(k)$
$r_{j,\underline{R}_j}(k)$	the time-varying proportion of vehicles leaving from a given service zone j to the corresponding off-ramp \underline{R}_j in a given time interval k
$\hat{r}_{j,\underline{R}_j}(k)$	the corresponding ideal values associated with $r_{j,\underline{R}_j}(k)$
$r_{\bar{R}_j,j}(k)$	the time-varying proportion of on-ramp vehicles entering from a given on-ramp \bar{R}_j to the corresponding service zone j in a given time interval k
$\hat{r}_{\bar{R}_j,j}(k)$	the corresponding ideal values associated with $r_{\bar{R}_j,j}(k)$
\bar{s}	the target service level on a freeway
$S_j(k)$	the number of vehicles remaining in the given service zone j at the beginning of time interval k
$S_j^{\bar{s}}$	the maximum number of vehicles permitted in a given service zone j with the target service level \bar{s}
$S_{\bar{R}_j}(k)$	the number of vehicles remaining in the corresponding on-ramp \bar{R}_j associated with a given service zone j at the beginning of a given time interval k
$\tilde{S}_{\bar{R}_j}$	the capacity associated with a given on-ramp \bar{R}_j
t	the unit length of a time interval
$\bar{T}I$	an evaluation criterion in terms of the average hourly throughput
$\bar{T}I_{\text{sub}}$	An evaluation criterion in terms of the average hourly traffic flow through the suburban arterial
$\mathbf{V}(k)$	a $(3J \times 1)$ Gaussian noise vector, representing the measurement errors of the collected traffic data in time interval k
$\mathbf{v}_j(k)$	a (3×1) Gaussian noise vector referring to the measurement errors associated with a given service zone j in time interval k
$v_{r_{j,j+1}}(k)$	the measurement error associated with $O_{j,j+1}(k)$
$v_{r_{j,\underline{R}_j}}(k)$	the measurement error associated with $\underline{R}_j(k)$
$v_{r_{\bar{R}_j,j}}(k)$	the measurement error associated with $\bar{R}_j(k)$
$\mathbf{W}(k)$	a $(3J \times 1)$ state-independent Gaussian noise vector
$\mathbf{w}_j(k)$	a (3×1) time-varying state-independent noise vector associated with a given service zone j in time interval k
$\mathbf{X}(k+1)$	a $(3J \times 1)$ time-varying basic state vector in time interval $k+1$
$\mathbf{X}_j(k+1)$	a (3×1) time-varying basic state vector associated with a given service zone j in time interval $k+1$
$\mathbf{X}_j(k)$	a (3×1) time-varying basic state vector associated with a given service zone j in time interval k

$\hat{\mathbf{X}}(k + 1)$	the generalized form of the target vectors associated with the estimated basic states and control variables in given time interval $k + 1$
$\hat{\mathbf{X}}_j^{\bar{s}}(k)$	a (3×1) time-varying target state vector associated with $\mathbf{X}_j(k)$, involving the corresponding target values of basic states under the proposed freeway traffic control with a given target service level \bar{s}
$\mathbf{Z}(k)$	a $(3J \times 1)$ time-varying measurement vector in time interval k
$\mathbf{z}_j(k)$	a (3×1) time-varying measurement vector associated with a given service zone j in time interval k
α_j	a parameter of the proposed elastic demand function
β_j	a parameter of the proposed elastic demand function
$\mathbf{\Omega}(k + 1)$	a control-variable vector
$\hat{\mathbf{\Omega}}(k + 1)$	the generalized form of the target vectors associated with the estimated control variables in given time interval $k + 1$
$\Omega_j[c_j(k), p_j(k)]$	the aggregate effect of control variables on the change pattern of $r_{R_j, j}^{\bar{s}}(k)$
$\hat{\mathbf{\Omega}}_j^{\bar{s}}(k)$	a (2×1) target control vector, involving the corresponding target values of control variables to maintain the proposed freeway system with a given target service level \bar{s}
$\Phi_j^1(k)$	a $(3J \times 3J)$ time-varying diagonal, positive-definite weighting matrix involving the corresponding disaggregate weighting matrix $\Phi_j^1(k)$ associated with each given service zone j
$\Phi_j^1(k)$	a (3×3) time-varying diagonal, positive-definite weighting matrices associated with the estimated basic state vector ($\mathbf{X}_j(k)$)
$\Phi_j^2(k)$	a (2×2) time-varying diagonal, positive-definite weighting matrices associated with the control variable vector ($\mathbf{\Omega}_j(k)$)
$\tilde{\zeta}_K$	the aggregated objective function of the proposed integrated ETC-ramp control system
ζ_{jk}	the disaggregated objective function associated with jk
$\rho_{\bar{s}}$	the ideal toll rate preset for the target service level \bar{s}_j

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