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Three-intensity measurement technique and its measurement in elliptical retarder

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Abstract

A three-intensity technique is applied in polarimetry for measuring the physical parameters of an elliptical retarder. In this study, quartz and mica quarter wave plates are distinguished by two models: linear retarder and elliptical retarder. By considering the twist nematic liquid crystal cell as an elliptical retarder, we are able to relate the effective optic axis to its rubbing direction and twist angle. All its physical parameters can be deduced from the model.

Keywords: Elliptical retarder; TN-LC cell; Polarimetry

1. Introduction

A retarder is a widely used optical component in polarization optics. It is commonly made of birefringence materials such as mica, calcite, and quartz. Although the quartz crystal is an optically active birefringence material [1], its optically active property is significantly weaker [2] than its linear birefringence property when it is used as a wave plate. Therefore, the optical activity has been neglected in most wave plate related experiments, but its existence has been observed by Chao [3] through the three-intensity measurement technique. By measuring the physical parameters of an elliptical retarder, we can prove that the quartz wave plate is an elliptical retarder instead of a pure linear retarder. The elliptical retarder has been characterized by an optically equivalent system that comprises a linear retarder followed by a pure rotator [4,6,7]. By matching the phase of the Babinet–Soleil compensator and rotating the analyzer, Sarma [4] sequentially located two minimum intensities to obtain the ellipse angle and phase retardation of the elliptical retarder. Instead of locating the minimum intensity, we extend the three-intensity method [3] to directly and simultaneously measure the optic axis, ellipse angle, and phase retardation of the elliptical retarder.

The physical parameters of the liquid crystal have been measured by the following methods: (1) curve fitting [5,6] the intensity distribution while the TN-LC cell is sandwiched between a polarizer and an analyzer, which are rotated simultaneously either perpendicular or parallel; and (2) converting the physical parameters of an optically equivalent system, a linear retarder followed by a pure rotator, by using the measurement of [7], a commercial Stokesmeter. The main disadvantage of this technique is that it cannot be used to determine the rubbing direction of the TN-LC cell. In this study, the TN-LC cell is considered to be an elliptical retarder whose parameters can be determined by the extended three-intensity method. We can convert the physical parameters (azimuth angle of optic axis, elliptical angle, and phase retardation) of the elliptical retarder to the physical parameters (twist angle, rubbing direction, and retardance) of the TN-LC cell by comparing the Jones matrix of the TN-LC cell [8] with that of an elliptical retarder. After introducing the effective optic axis of the TN-LC and relating it to its twist angle and rubbing direction through the parametric comparison,

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we can not only measure the rubbing direction by the three-intensity method instead of the conventional crossed polarizers' technique but also measure the twist angle and the retardance simultaneously.

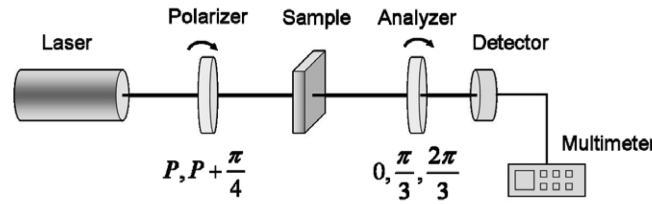

Fig. 1. Schematic setup of the polarimeter.

2. Theoretical background

The Jones matrix of an elliptical retarder can be written as [9]

$$J_{\mathrm{EB}}(\delta,\varepsilon,\theta)=\begin{pmatrix}\cos\frac{\delta}{2}+j\cos2\varepsilon\sin\frac{\delta}{2}\cos2\theta & \sin\frac{\delta}{2}\sin2\varepsilon+j\sin\frac{\delta}{2}\cos2\varepsilon\sin2\theta\\ -\sin\frac{\delta}{2}\sin2\varepsilon+j\sin\frac{\delta}{2}\cos2\varepsilon\sin2\theta & \cos\frac{\delta}{2}-j\cos2\varepsilon\sin\frac{\delta}{2}\cos2\theta\end{pmatrix}\tag{1}$$

where δ is the retardation, θ and ε are the azimuth angle and ellipticity angle of the elliptical retarder, respectively. It is easy to prove that the Jones matrix represents a linear retarder when $\varepsilon = 0$ and a circular retarder when $\varepsilon = \frac{\pi}{4}$. In a polarimetric setup, such as shown in Fig. 1, the measured intensity can be written as

$$\begin{aligned}I(\mathrm{A},\mathrm{P}) = I_0\Big\{&1+\cos2(A-P)\left[\cos^2\left(\frac{\delta}{2}\right)-\sin^2\left(\frac{\delta}{2}\right)\sin^2 2\varepsilon\right]+\cos2(A+P)\\ &\times\cos^2 2\varepsilon\cos4\theta\sin^2\left(\frac{\delta}{2}\right)+\sin2(A-P)\\ &\times\sin\delta\sin2\varepsilon+\cos^2 2\varepsilon\sin2(A+P)\sin^2\frac{\delta}{2}\sin4\theta\Big\}\end{aligned}\tag{2}$$

where A and P are the transmission axes of the analyzer and polarizer, respectively. One can prove that the total intensity measured at $A = 0°$, $60°$, and $120°$ is constant and is denoted by I_o, i.e., $I_o = [I(0°,\ p) + I(60°,\ p) + I(120°,\ p)]/3$ at any p, which can be used for intensity normalization. Using the same three-intensity measurements, one can prove the followings:

$$\tan(4\theta-2P)=\frac{Ia_2+Ib_1}{Ia_1-Ib_2}\tag{3}$$

$$\begin{aligned}\cos\delta = -\frac{1}{3}[&(Ia_1\sin2\theta-Ib_1\cos2\theta)\sin2(P-\theta)\\ &+(Ia_2\sin2\theta-Ib_2\cos2\theta)\cos2(P-\theta)]\end{aligned}\tag{4}$$

$$\begin{aligned}\sin\delta\sin2\varepsilon = -\frac{1}{3}[&(Ia_1\sin2\theta-Ib_1\cos2\theta)\cos2(P-\theta)\\ &-(Ia_2\sin2\theta-Ib_2\cos2\theta)\sin2(P-\theta)]\end{aligned}$$

$$Ia_i=\left[2I(0,p)-I\left(\frac{\pi}{3},p\right)-I\left(\frac{2\pi}{3},p\right)\right]\Big/I_o;\tag{5}$$

where

$$Ib_i=\sqrt{3}\left[I\left(\frac{\pi}{3},p\right)-I\left(\frac{2\pi}{3},p\right)\right]\Big/I_o,$$

when $i = 1$ and 2 are the intensities measured at $p = P$ and $p = P + \pi/2$, respectively. The phase retardation, azimuth angle, and ellipticity angle of the elliptical retarder can be measured by these two sets of this three-intensity measurements technique.

If the TN-LC cell is considered to be as an elliptic retarder, then its physical parameters can also be measured by the three-intensity measurement technique via the parametric comparison between the Jones matrices of elliptical retarder and TN-LC cell. The Jones matrix of the TN-LC cell is expressed as [4]

$$M_{TN\text{-}LC}=\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix}\cdot\begin{bmatrix}\cos\chi-i\dfrac{\Gamma\sin\chi}{2\chi} & \phi\dfrac{\sin\chi}{\chi}\\ -\phi\dfrac{\sin\chi}{\chi} & \cos\chi+i\dfrac{\Gamma\sin\chi}{2\chi}\end{bmatrix},$$

where the twist angle (ϕ) and phase retardation (Γ) are related by χ which is defined as $\chi=\sqrt{\phi^2+\left(\frac{\Gamma}{2}\right)^2}$, and the phase retardation is given by $\Gamma=\frac{2\pi}{\lambda}(n_e-n_o)d$ for the TN-LC cell with birefringence $(n_e - n_o)$ and cell gap d. The rubbing direction of the TN-LC cell is at c, such as shown in Fig. 2; the Jones matrix of the rotated TN-LC cell can be modified as

$$J_{\text{TN-LC}}=R(c)\cdot M_{\text{TN-LC}}\cdot R(-c)=\begin{bmatrix}\cos\phi\cos\chi+\frac{[-i\Gamma\cos(2c+\phi)+2\phi\sin(\phi)]\sin\chi}{2\chi} & -\sin\phi\cos\chi-\frac{[i\Gamma\sin(2c+\phi)-2\phi\cos(\phi)]\sin\chi}{2\chi}\\ \sin\phi\cos\chi-\frac{[i\Gamma\sin(2c+\phi)+2\phi\cos(\phi)]\sin\chi}{2\chi} & \cos\phi\cos\chi+\frac{[i\Gamma\cos(2c+\phi)+2\phi\sin(\phi)]\sin\chi}{2\chi}\end{bmatrix}\tag{6}$$

Let θ_{LC}, δ_{LC} and ε_{LC} be the effective optic axis' azimuth angle, phase retardation and ellipse angle of the TN-LC cell, respectively, we can obtain the followings:

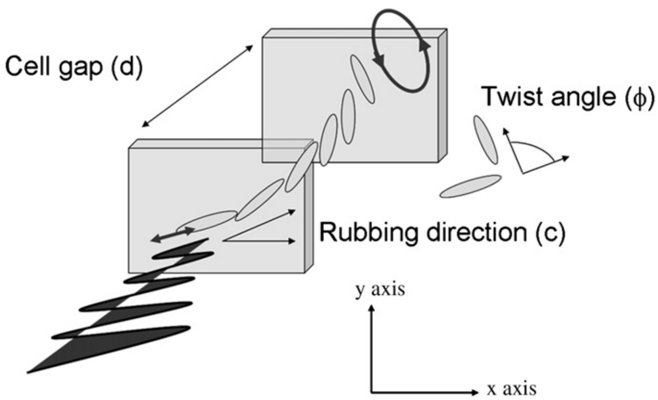

Fig. 2. Schematic drawing of a TN-LC.

$$\theta_{LC} = c + \phi/2;$$
$$\cos\left(\frac{\delta_{LC}}{2}\right) = \cos\phi\cos\chi + \frac{\phi\sin\phi\sin\chi}{\chi};$$
$$\sin\left(\frac{\delta_{LC}}{2}\right)\sin 2\varepsilon = -\cos\chi\sin\phi + \frac{\phi\cos\phi\sin\chi}{\chi} \tag{7}$$

by comparing Eqs. (1) and (6). We can calculate the twist angle, phase retardation, and rubbing direction of the TN-LC cell by substituting its measured physical parameters of elliptic retarder.

3. Experiment

The optical properties of quarter-wave plates are investigated by using the PSA photometric ellipsometer. Fig. 1 depicts the experimental setup. Light (L, from a He–Ne laser) passes through a polarizer (P) whose azimuthal angle is set to P with respect to the horizontal axis; the azimuth P can be an arbitrary value, which is measured before inserting the wave plate. The analyzer (A) is mounted on a rotator controlled by a stepping motor, and three measurements of the intensities at the azimuth of the analyzer at 0°, 60°, and 120° are performed to find the parameters of the linear retarder. After rotating the azimuth of the polarizer by 45°, the same experimental procedures are repeated to find parameters of the elliptic retarder. A TN-LC cell is designed with a twist angle of 90° and a retardance of 400 nm for the He–Ne laser. For understanding its rubbing direction, we align it in a pair of crossed polarizers and then rotate it by 15° in this polarimetric system. Three consecutive experiments are conducted by rotating the TN-LC cell in steps of 3° to measure its physical parameters.

4. Results and discussion

The three-intensity measurements can be used to find the parameters of the linear retarder [3] and elliptic retarder by deducing them from Eqs. (3)–(5). In theory, the phase retardation and azimuth angle of the wave plate should be independent of the azimuthal setting of the polarizer. From the comparison of the measurements, as shown in Table 1, under the model of the linear retarder, we can conclude that a mica wave plate is a linear retarder. This is because its measured physical parameters are almost independent of the azimuthal setting of the polarizer, while the phase retardation and azimuth angle of the quartz wave plate are different under different azimuth angles of the polarizer. However, under the model of the elliptical retarder of both wave plates, all the measured parameters (θ, δ, and ε) are the same (within the measurement error) under different azimuth angles of the polarizer. The slightly elliptical property of mica ($\varepsilon \cong 0.06^{\circ}$) can be regarded as a measurement error; however, the significant elliptical property of quartz ($\varepsilon \cong 2.02^{\circ}$) can confirm the existence of its optical activity.

By considering the TN-LC cell as an elliptical retarder, we can measure the azimuth angle of its effective optical axis, ellipticity, and retardation by the three-intensity measurement technique. To verify that the TN-LC cell is an elliptical retarder, we not only change the polarization state of the incident light but also rotate the cell in this polarimetry to measure its physical parameters. The results are summarized in Table 2: the variations in the azimuth angle of its effective optical axis are the same as those rotated angles. Its measured ellipticity is

Table 1
Optical parameters of quartz and mica under various azimuth positions of the polarizer

P (°)	Model				
	Elliptical retarder			Linear retarder	
Parameters:	θ (°)	δ (°)	ε (°)	θ (°)	δ (°)
Quartz					
15	−3.57	88.48	2.06	−2.47	98.22
30	−3.53	88.43	1.98	−1.19	93.58
45	−3.49	88.51	2.03	−0.01	89.26
Mica					
15	0.19	87.79	0.054	0.83	88.87
30	0.12	87.44	0.058	0.56	88.92
45	0.10	87.34	0.056	0.21	89.19

Table 2
Optical parameters of the TN-LC cell under various conditions

Experiments	P (°)	θ_{LC} (°)	δ_{LC} (°)	ε_{LC} (°)
TN-LC (azimuth angle: θ_{LC})				
1	−2.87	30.99	137.54	30.11
2	−0.06	31.02	137.34	30.09
3	3.00	31.09	137.16	29.93
TN-LC (azimuth angle: θ_{LC} + 3°)				
4	−2.87	28.07	137.46	30.02
5	−0.06	27.97	137.33	29.85
6	3.00	27.98	137.09	29.84
TN-LC (azimuth angle: θ_{LC} + 6°)				
7	−2.87	25.20	137.46	29.81
8	−0.06	−24.99	137.26	29.53
9	3.00	25.02	137.30	29.32
Mean			137.32 (0.14)	29.83 (0.26)

$29.83 \pm 0.26°$, which is significantly higher than that of the quartz wave plate. We can also convert the three-intensity measurements to its twist angle, retardance and rubbing direction from Eq. (7) and obtain $89.17 \pm 0.48°$, 392.96 ± 2.01 nm and 14.4°, respectively, which are close to the designed values.

5. Conclusion

The three-intensity measurement technique can be used to measure the physical parameters of a generalized retarder, i.e., the elliptical retarder. By considering the TN-LC cell as an elliptical retarder, we can measure the azimuth angle of its effective optical axis, ellipticity, and retardation. According to the derived relationships between the parameters of the TN-LC cell and the elliptic retarder, we can obtain the phase retardation, twist angle and rubbing direction of TN-LC cell simultaneously. Since only three intensities are needed to determine the physical properties of the elliptical retarder, the CCD camera can be used as a detector for performing imaging polarimetry to analyze the surface structure of the TN-LC cell.

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