Polarization-preserving angular shifter

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This study shows that by using a wedge plate the incident direction of light propagation can be rotated as desired while still preserving beam polarization. This study also deduces the basic condition of this preservation of polarization. Two typical wedge plates are analyzed for numerical demonstration. Simulation results verify that a collimated beam with a +45° linear polarization can be guided to an expected direction while preserving the state of polarization with a square of the variation of the ellipse ratio of less than 0.0001%. This study also numerically shows that the wedge vertex angle is the most critical issue and that approximately 0.1° accuracy is required to preserve the polarization state. © 2008 Optical Society of America

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1. INTRODUCTION

Preserving the polarization of a beam of light after propagation has many practical uses in a variety of applications such as biological tissue light probes or ellipsometers for refractive index and film thickness measurement. [1,2]. When light is propagated, displaced, or deflected its polarization is usually changed. For example, when light bounces off a reflective mirror the reflection introduces an additional phase that alters the polarization of the light beam. Azzam [3] and Azzam and Khan [4] suggested solutions to displace a monochromatic light beam parallel to itself without a change of polarization by utilizing a pair of parallel mirrors with a single dielectric layer coating. On the other hand, Cojocaru [5] and Wang *et al.* [6] proposed the design of coating layers with a totally reflecting prism to provide a convenient and efficient method to control the phase retardance. Galvez presented the combinations of four total reflecting prisms to preserve polarization [7]. Galvez's scheme has benefits in achromatic application and very good tolerance, which can preserve the polarization with a square of the variation of the ellipse ratio of less than 1% while 1° of component tilt occurs [8]. These studies provide very useful schemes to maintain polarization, but they generally focus on the displacement of a light beam, i.e., when the propagating direction of a displaced beam is parallel to the original beam. However, angular shifting is generally employed to reduce the optical path of beam propagation and hence the system size, which has practical value. Technically, polarization-preserving angular shifting provides more flexibility in optical engineering applications, e.g., a polarized light probe with angular incident injection could provide another useful scheme in exploring 3D structure and response. However, it is quite uncertain and difficult to deflect the beam direction while maintaining the state of polarization using this technique. This study proposes and numerically verifies a novel polarization-preserving beam angular shifter based on a wedge plate. A polarization-preserving wedge has an advantage in manufacture and has better extendibility on the applications of polarization-controlling mechanisms. A polarization-preserving wedge can provide both spatial and angular shifts of the beam propagation path, and zero vertex angle can reduce a wedge angular shifter to become a beam displacer.

This paper is organized as follows. Section 2 provides a basic formalism of polarized ray tracing in dielectric-filled material. Section 3 presents the issue of polarization conservation and its conditions. Section 4 shows the simulation verification. Section 5 gives the limitation of the polarization-preserving angular shifter. Section 6 investigates the tolerance issue of manufacture accuracy and incident beam angle. Section 7 contains conclusions.

2. BASIC PROPAGATION FORMALISM OF POLARIZED LIGHT: THE MULLER MATRIX

Let us first summarize the general properties of a polarized ray propagating in a dielectric-filled material. According to Fresnel's equations the transmitted and reflected fields of a light beam on the air–dielectric interface follow [9]:

$$R_s = \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t}E_s, \quad R_p = \frac{n\cos\theta_i - \cos\theta_t}{n\cos\theta_i + \cos\theta_t}E_p,$$

$$T_s = \frac{2\cos\theta_i}{\cos\theta_i + n\cos\theta_t}E_s, \quad T_p = \frac{2\cos\theta_i}{n\cos\theta_i + \cos\theta_t}E_p. \quad (1)$$

In Eq. (1), E, R, and T represent the incident, reflected, and refracted fields, and the subscripts s and p denote the parallel and perpendicular directions, respectively. θ_i is the incident angle, θ_t is the angle of refraction, and n is the refractive index of the dielectric medium.

Stokes parameters make it possible to derive the Mueller matrix for the reflection and refraction on the air–dielectric interface from Eq. (1). Referring to [10], the Mueller matrix of transmission on the air–dielectric interface is

$$M_T = \frac{\sin 2\theta_i \sin \theta_t}{2(\sin \Theta_+ \cos \Theta_-)^2} \begin{pmatrix} \cos^2 \Theta_- + 1 & \cos^2 \Theta_- - 1 & 0 & 0 \\ \cos^2 \Theta_- - 1 & \cos^2 \Theta_- + 1 & 0 & 0 \\ 0 & 0 & 2\cos \Theta_- & 0 \\ 0 & 0 & 0 & 2\cos \Theta_- \end{pmatrix}, \quad (2)$$

where $\Theta_\pm = \theta_i \pm \theta_t$, while for total internal reflection it follows that:

$$M_{\mathrm{TIR}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \Delta & -\sin \Delta \\ 0 & 0 & \sin \Delta & \cos \Delta \end{pmatrix}, \quad (3)$$

where

$$\Delta = -2\tan^{-1}\left(\frac{\cos \theta_i \sqrt{n^2 \sin^2 \theta_i - 1}}{n \sin^2 \theta_i}\right).$$

Equations (2) and (3) govern the variations of polarization as a light ray interacts within a dielectric-filled material.

The next challenge is: *How can we design an optical component that is capable of guiding a ray to a special angular direction while simultaneously preserving its polarization?* Technically, the total phase delay (or phase shift) during ray propagation must be kept as 2π, or an integral multiple of 2π, and the deflected angle between the exit and incident beams should match the designed value. Section 3 shows that a wedge plate is a possible candidate to achieve this goal.

3. ISSUE OF POLARIZATION PRESERVATION AND ITS CONDITIONS

In this section, we will show how to resolve the issue indicated in Section 2. Much like the wedge plates considered in [11,12], Fig. 1(a) shows that a ray is incident to the narrow side of the wedge plate as long as the incident angle when the ray first strikes the top wedge surface is greater than the total internal reflection angle θ_c. This ray can continuously propagate between the top and bottom surfaces of the wedge plate until it reaches the wide side (or the exit surface). To avoid unnecessary complexity, the wedge input and exit surfaces were cut such that their directions of the surface normal are the same as the directions of the incident and exit beams, respectively. In Fig. 1(a), α_i is the angle between the ray propagation direction and the horizontal axis after the ith total internal reflection, θ_i is the incident angle before the ith total internal reflection, θ_v is the wedge vertex angle, and n is the refractive index of the plate material. Before leaving the wedge plate, this ray encounters m times of total internal reflection.

Without loss of generality, we assume this ray finally escapes from the top surface of the wedge plate and m is an even number. The geometry in Fig. 1(a) shows that $\theta_i = \theta_1 + (i-1)\theta_v$, $\alpha_i = 90 - \theta_i - \theta_v$ (when i is even), and the ray deflection angle $\Delta\alpha = \alpha_0 - \alpha_m = m\theta_v$. In short,

$$\Delta\alpha = m\theta_v,$$

$$\sum_{i=1}^{m} -2\tan^{-1}\left(\frac{\cos[\theta_1 + (i-1)\theta_v]\sqrt{n^2 \sin^2[\theta_1 + (i-1)\theta_v] - 1}}{n \sin^2[\theta_1 + (i-1)\theta_v]}\right) = -2\pi k, \quad (4)$$

where m is even and k is an integer. After specifying the

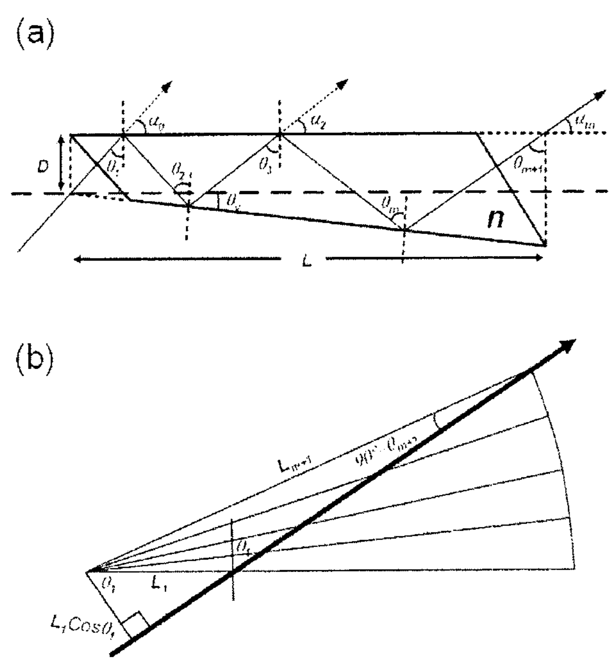

Fig. 1. Schematics of rays propagating in (a) a wedge plate, (b) a virtually folded wedge plate.

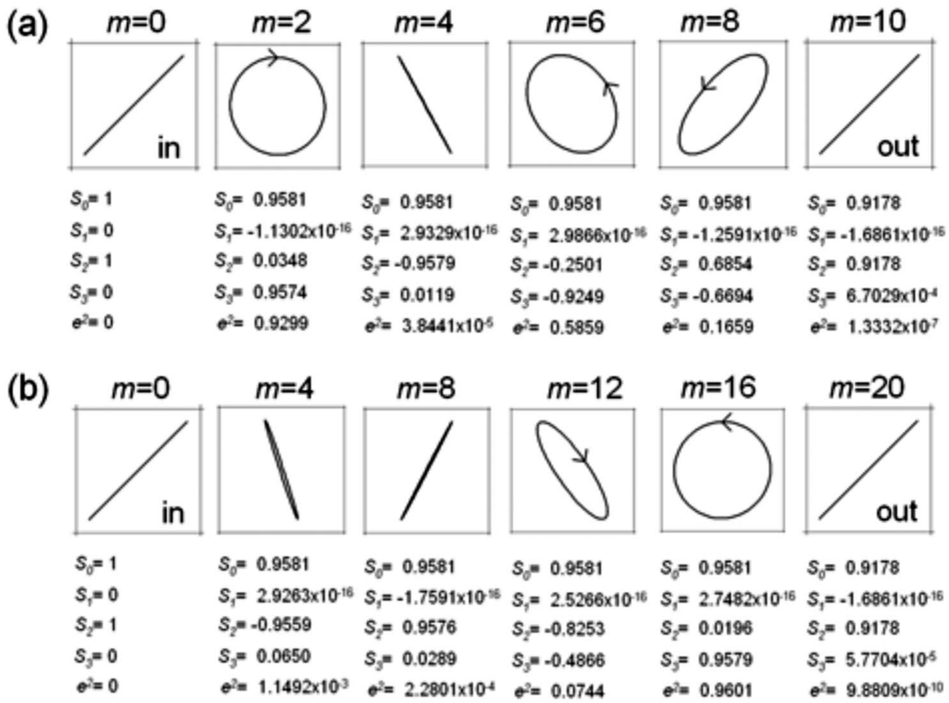

Fig. 2. Polarization variations when total internal reflections happen. m=(a) 10 and (b) 20.

ray deflection angle $\Delta\alpha$, multiple solutions of (θ_1, θ_v) can be derived from Eq. (4) with different values of m and k. Equation (4) is the basic condition of a polarization-preserving angular shifter for a wedge plate. Formally, θ_1 and θ_v are the dominate parameters in Eq. (4), but the real scale of the wedge plate, i.e., the width of input port D and the total length of wedge L, provide the degree of freedom in an angular shifter.

4. NUMERICAL EXPLORATION

For a numerical illustration, consider a typical glass, BK7, where $n=1.51509$ at 632.8 nm and request $\Delta\alpha=30°$. After solving, two solutions were arbitrarily selected for demonstrations: (1) $m=10$, $\theta_1=49.46°$, $\theta_v=3°$, where the total phase delay is $\sim -2\pi$, i.e., $-2\pi-7.3\times10^{-4}$; and (2) $m=20$, $\theta_1=48.69°$, $\theta_v=1.5°$, where the total phase delay is $\sim -4\pi$, i.e., $-4\pi+6.3\times10^{-5}$. Figure 1(b) shows the series of a virtually folded wedge by which the multireflection ray path can be treated as a straight line. In Fig. 1(b), L_1 is the length from the wedge vertex to the first total internal reflection position and L_{m+1} is the length from the wedge vertex to the $(m+1)$th total internal reflection position. Simple mathematics prove that $L_1=D(\cot\theta_v+\tan\theta_1)$ and $L_{m+1}=[L_1\cos\theta_1/\sin(90°-\theta_{m+1})]$, and hence the total length L from the ray incident position to the exit position is

$$L = L_{m+1} - L_1 + D\tan\theta_1,$$

$$= D(\cot\theta_v + \tan\theta_1)\frac{\cos\theta_1}{\cos(\theta_1 + m\theta v)} - D\cot\theta_v. \quad (5)$$

By Eq. (5), one can determine the exact size of the wedge plate by choosing the port width D or the wedge length L.

In the numerical example above, $\theta_v=3°$ and the thickness of the entrance port $D=5$ mm. By Eq. (5), the plate length L is 264.5 mm. For simplicity of presentation (without loss of generality), ignore the absorption for the time being. This study uses TracePro (version 4.0.4), a commercial simulation package, for numerical verification [13]. A collimated beam with +45° linear polarization was propagated into the wedge plate at $\alpha_0=40.54°$ and finally exited the wedge plate at $\alpha_{10}=10.54°$ after ten total internal reflections. Figure 2(a) shows the polarization states (the Stokes parameters) after the ray encounters even times of total internal reflection. Ellipse ratio e of a linearly polarized beam was considered and $e^2=I_{\min}/I_{\max}$ was calculated, where $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities of the optical beam, proportional to the squares of semimajor and semiminor axes of the ellipse, respectively, described by the beam's electric field vector. Results show that the output polarization beam has a good linear property with the square of ellipse ratio $e^2=1.3332\times10^{-7}$ and the azimuth of polarization axis 44.51°. This study also considers an example of 20 total internal reflections, where $D=5$ mm and $\theta_v=1.5°$. In this case, $L=470.9$ mm, the incident angle is $\alpha_0=41.31°$, and hence $\alpha_{20}=11.31°$. Figure 2(b) shows this simulation result, clearly indicating that the output beam also retains very good linear polarization characteristics with the squared ellipse ratio $e^2=9.8809\times10^{-10}$ and the azimuth of polarization axis 44.43°.

5. LIMITATION OF THE POLARIZATION-PRESERVING ANGULAR SHIFTER

Physically, the limit of the ray deflection angle $\Delta\alpha$ should be less than the incident angle α_0, while α_0 should be less than $90°-\theta_c$. For BK7 at a 632.8 nm wavelength, the criti-

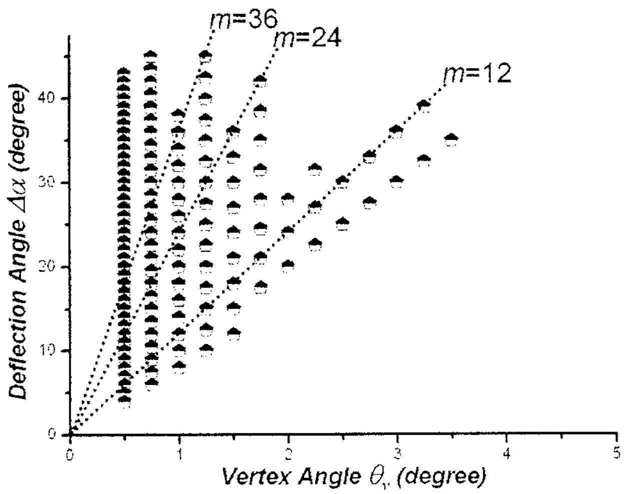

Fig. 3. Possible ray deflection angles versus wedge vertex angle. The vertex angle θ_v is picked up from 0.5° to 5° with a unit of 0.25° to determine the possible ray deflection angle.

cal angle θ_c is 41.3°, hence, the deflection angle $\Delta\alpha$ cannot be greater than 48.7° when the wedge plate has an even number of reflections. Determining the limit of the angular shifter is worthwhile, so this study explores the possible range of deflection angle $\Delta\alpha$ at specific vertex angles θ_v, which range from 0.5° to 5° with a unit of 0.25°. Figure 3 shows the results, indicating that the available wedge vertex angle is limited from 0° to 3.5°, although the range bounded up to 5° has been checked, and 8 total internal reflections are required for the wedge plate to preserve the polarization after ray propagation.

A greater number of reflections increases the system dimensions and enlarges the phase difference caused by the incident angle error. Figure 3 shows different solutions for three reflection numbers—$m=12$, 24, and 36—highlighted by dashed lines. This figure indicates that the maximal deflection angle can reach 39° when the deflection number is 12, while twice as many reflections increase the deflection angle to 42°, and three times as many reflections contributes an additional 3° of deflection angle range.

For simplicity, this study only shows the case of an even number of reflections. Nevertheless, for the case of an odd number of total internal reflections, one can simply multiply the deflection angle $\Delta\alpha$ twice. Furthermore, utilizing multiple polarization-preserving wedges can also increase the deflection angle.

6. TOLERANCE ISSUE

A polarization-preserving wedge can be manufactured using glass molding or plastic injection. The accuracy of the incident plane angle and wedge vertex angle plays an important role in preserving the polarization state. Figure 1(a) shows that the polarized beam is normally incident to the obliquely cut input surface and exits from the output surface after m total reflections. Incident angle or wedge vertex angle errors negatively affect the polarization-preserving shifter performance because the angles of each total internal reflection θ_i are varied while these errors occur.

This paper discusses three variations of ray propagation angles. First, if the ray is normally incident to the input surface that is tilted by a small angle δ_s, i.e., the incident angle $\alpha_0'=\alpha_0+\delta_s$, then the ray angle after the mth reflection is $\alpha_m'=\alpha_0+\delta_s+m\theta_v$ and the ray exit angle becomes $\alpha_{\text{out}}=\alpha_0+\sin^{-1}(n\sin\delta)+m\theta_v=\alpha_m+\sin^{-1}(n\sin\delta_s)$. The deflection angle of the output ray can be altered by another small angle $\sin^{-1}(n\sin\delta_s)-\delta_s$, i.e., $\Delta\alpha'=\Delta\alpha+\sin^{-1}(n\sin\delta_s)-\delta_s=m\theta_v+\sin^{-1}(n\sin\delta_s)-\delta_s$. Second, if a fabrication error occurs in the wedge vertex angle (with an additional small angle δ_v) and the input and output surfaces remain the same, the ray angle, after mth reflection, becomes $\alpha_m'=\alpha_0+m(\theta_v+\delta_v)$. Further, the ray exit angle is $\alpha_{\text{out}}=\alpha_0+\sin^{-1}(n\sin m\delta_v)+m\theta_v=\alpha_m+\sin^{-1}(n\sin m\delta_v)$. The deflection angle's variation is $\sin^{-1}(n\sin m\delta_v)$. The third situation is when the four surfaces of the polarization-preserving wedge are manufactured identically, but the misalignment of the incident beam with the incident plane decreases performance. When the incident beam is tilted by a small angle δ_i, i.e., the incident angle $\alpha_{\text{in}}=\alpha_0+\delta_i$, the ray angle before first reflection should be adjusted to $\alpha_0'=\alpha_0+\sin^{-1}(\sin\delta_i/n)$. The ray angle after the mth reflection is then $\alpha_m'=\alpha_0+\sin^{-1}(\sin\delta_i/n)+m\theta_v$ and the ray exit angle finally becomes $\alpha_{\text{out}}=\alpha_0+\delta_i+m\theta_v=\alpha_m+\delta_i$. Thus, an angular error in the output beam is the same as the incident beam's angular error. In other words, the ray deflection angle $\Delta\alpha$ can remain unchanged when the incident beam is misaligned.

Angular errors on wedge surfaces or incident beam alignment do not seriously affect the deflection angle (or output ray direction) in the three scenarios above, but variation in the output beam's polarization state is unavoidable. The phase shift contributed by input and output surfaces, as deduced in Eq. (2), must be considered as well. Figures 4(a) to 4(f) investigate variations of the square of ellipse ratio e^2 and the azimuth of the polarization axis for the two numerical cases in Section 4, where angular errors occur in the input surface, vertex angle, and incident beam alignment. The simulation results in Figs. 4(a) and 4(b) show that variations of e^2 caused by one degree of the input surface error are less than 1% (for $m=10$) and 3% (for $m=20$), and changes in the azimuth of the polarization axis are less than 0.25° for both cases. However, vertex angle error clearly influences polarization. Figures 4(c) and 4(d) show the variations of e^2 when a manufacturing error occurs in wedge vertex angle. These figures show approximately 6% variation of e^2 with a 30 arc min vertex angular error for the case where reflection number $m=10$. They also show another case where reflection number $m=20$ encountered phase differences exceed 2π and the vertex angular error was less than 30 arc min. Polarization-preserving components must have 5 arc min of angular accuracy and maintain e^2 variations less than 3% for both cases. Finally, Figs. 4(e) and 4(f) show the impact of 1° of misalignment in the incident beam. The impact of the incident beam misalignment is similar to that of input surface tilt.

7. CONCLUSIONS

In conclusion, this study proposes and numerically demonstrates that a wedge plate can angularly shift a collimated beam while maintaining its polarization state. This study provides the conditions for polarization preservation and the method of estimating the system scale to allow others to design and manufacture a polarization-preserving angular shifter based on this approach. For a simulation investigation, two numerical cases of the polarization-preserving shifter were discussed. Simulation results show that this method can indeed transfer a polarization beam into a desired angle while maintaining its polarization state. Tolerance analysis shows that the accuracy of the wedge vertex angle is the most important issue when preserving the polarization state. Any inaccuracy in the vertex angle changes the direction and polarization of the output beam, obviously because the performance degenerations are magnified during multiple total reflections. On the other hand, when the wedge machining is perfect, the incident beam angle's angular error does not affect the deflection angle and the squared ellipse ratio can be kept under 1.5% with 1° accuracy for the incident beam angle.

Polarized beams are widely used in measuring thin film, biological tissue, glass surface stress, and so forth. This study helps designers understand ray polarization behavior within dielectric-filled material and control the phase difference by properly choosing wedge parameters. This method can easily be extended to other applications

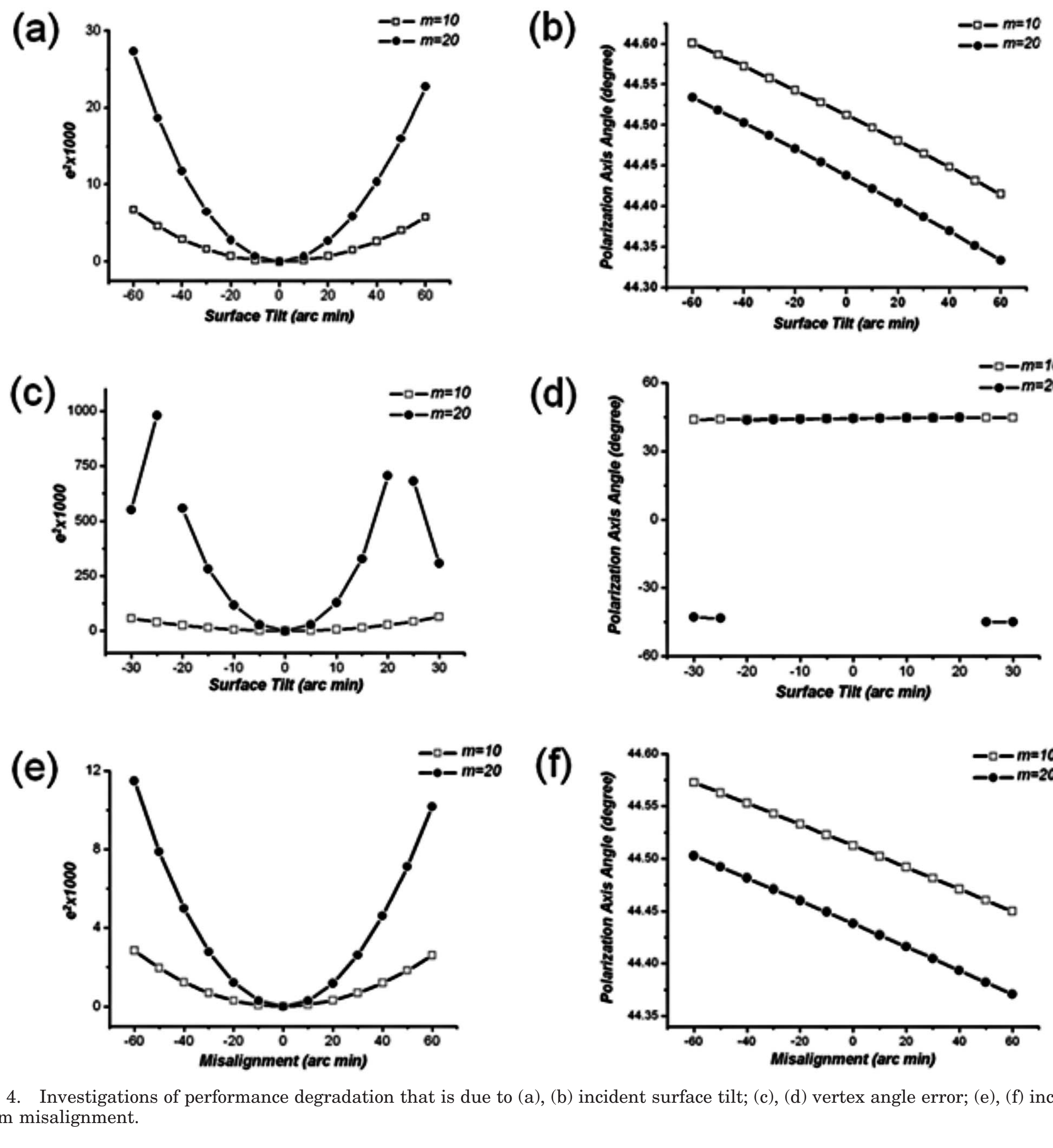

Fig. 4. Investigations of performance degradation that is due to (a), (b) incident surface tilt; (c), (d) vertex angle error; (e), (f) incident-beam misalignment.

of polarization controlling mechanisms. For example, it is possible to replace the requirement of 2π total phase with another value to create a deflecting phase retarder.

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REFERENCES

1. M. L. Faupel, S. B. Bambot, T. Harrell, and A. Agrawal, "Multi-modal optical tissue diagnostic system," U.S. patent 6,975,899 (December 13, 2005).
2. R. M. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, 1989).
3. R. M. Azzam, "Displacement of a monochromatic light beam parallel to itself without change of polarization," Opt. Lett. **7**, 80–82 (1982).
4. R. M. Azzam and M. E. R. Khan, "Polarization-preserving single-layer-coated beam displacers and axicons," Appl. Opt. **21**, 3314–3322 (1982).
5. E. Cojocaru, "Polarization-preserving totally reflecting prisms," Appl. Opt. **31**, 4340–4342 (1992).
6. Z. P. Wang, W. M. Sun, S. L. Ruan, C. Kang, Z. J. Huang, and S. Q. Zhang, "Polarization-preserving totally reflecting prisms with a single medium layer," Appl. Opt. **36**, 2802–2806 (1997).
7. E. J. Galvez, "Achromatic polarization-preserving beam displacer," Opt. Lett. **26**, 971–973 (2001).
8. A.-C. Hsu, C.-F. Ho, and J.-L. Chern, "Tilting tolerance analysis of a broadband polarization-preserving beam displacer," Appl. Opt. **41**, 5956–5962 (2002).
9. M. Born and E. Wolf, *Principles of Optics*, 7th ed. (Cambridge U. Press, 1999), pp. 42–43 and 50–51.
10. E. Collett, *Polarized Light: Fundamentals and Applications* (Marcel Dekker, 1992), pp. 139–161.
11. Y.-K. Cheng, S.-N. Chung, and J.-L. Chern, "Analysis and reduction of dark zone in ultra-thin wedge plate display," J. Soc. Inf. Disp. **14**, 813–818 (2006).
12. Y.-K. Cheng, S.-N. Chung, and J.-L. Chern, "Aberration analysis of a wedge-plate display system," J. Opt. Soc. Am. A **24**, 2357–2362 (2007).
13. See http://lambdares.com for technical information.