行政院國家科學委員會補助專題研究計畫成果報告

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計畫主持人：羅佩禎

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行政院國家科學委員會專題研究計畫成果報告

新快速傅利葉轉換法的研發與頻譜分析

Development of New FFT Algorithms and Their Application to Spectral Analysis

計畫編號：NSC89-2213-E-009-117

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ABSTRACT

In EEG (electroencephalogram) analysis, the time-varying spectra based on the short-time Fourier transform (STFT) provide a long-term monitoring of the signals. This study investigates an efficient method, real-time, shift-pruning FFT algorithm, to deal with the problem. The “shift-pruning FFT” algorithm integrates the recursive procedure with the modified output-pruning scheme to efficiently perform the long-term spectral analysis on EEG. The second part of this research is focused on the exploration and realization of the 2D split-vector-radix FFT (svr-FFT) structure. An important theorem is developed to guide the construction of the 2D svr-FFT structure.

計畫緣由與目的

In our study of the meditation EEG, the rhythmic patterns were used to identify the deepness of meditation [3]. The widely used EEG rhythmic patterns include: delta (f<4Hz), theta (4Hz≤f<8Hz), alpha (8Hz≤f≤13Hz), and beta (13Hz<f) [6]. A number of sophisticated signal processing methods have been introduced in order to extract frequency feature from EEG [2, 9, 14]. Nonetheless, some conventional, yet straightforward, approaches have proved their practicability and usefulness in clinical application and medical research. Among them, the running spectral analysis based on STFT has been widely used in the long-term EEG monitoring. The procedure involves evaluation of the DFTs (discrete Fourier transforms) on successively overlapping segments windowed by the moving frame. We previously introduced a real-time, moving FFT (fast Fourier transform) algorithm [11] which implemented the recursive procedure in the real-time manner.

Since the development of the FFT by Cooley and Tukey [5], efficient algorithms for computing the moving-frame DFTs have been reported [1, 4, 8, 16]. These algorithms were designed to obtain the complete spectrum. In the EEG study, normally only a narrow-band spectrum is of interest. Then the output-pruning scheme may be used to reduce the unnecessary arithmetic operations [13, 15, 17, 18]. In this research work, we present an efficient, integrated algorithm, real-time, shift-pruning FFT, that deals with the above two situations.

The second part of this study is devoted to the realization of 2D svr-FFT computational structure. A brief illustration will be given.

研究方法及成果

Part I. Real-time implementation of shift-pruning FFT

Let $X_i[k]$ denote the N-point DFT of the ith framed segment $x_i[n]=x[iM+n]$, $0 \le n \le (N-1)$, where $N=2^r$. Assume that the moving size is $M=2^{\sim}$ ($1 \le M \le N$), and the first P (=2) spectral samples of $X_i[k]$ are to be computed. As the pruning task must be performed posterior to the recursive procedure, the shift-pruning FFT method requires that $(r-\sim) \le \ \le r$, or $N/M \le P \le N$. Fig. 1 plots the block

Fig. 1 Block diagram of the shift-pruning FFT.

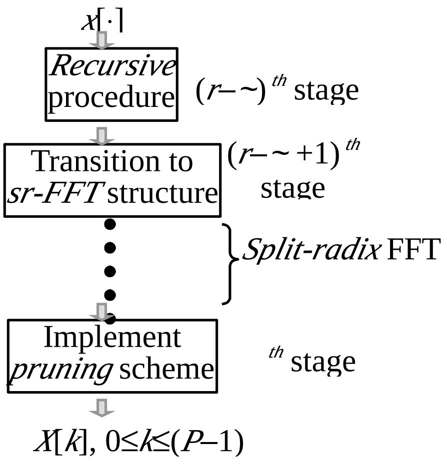


diagram. As shown in Fig. 1, the algorithm performs the recursive procedure at the $(r–\sim)$th stage. To utilize the efficient split-radix FFT (sr-FFT) [7, 10] algorithm, the algorithm transforms the radix-2-FFT order into the sr-FFT structure at the $(r–\sim+1)$th stage [11] and then implements the sr-FFT up to the th stage. At the th stage, output samples are linearly combined with the corresponding twiddle factors to obtain the first P samples of $X_i[k]$. The moving FFT algorithm was presented in detail in [11]. A brief overview of the mathematical basis is given below.

A. Recursive procedure for moving size M=2~, 1á~ár

In [11], we showed that the recursive procedure was implemented in each (N/M)-point local butterfly module at the $(r-\sim)$th stage. The ith N-point DFT $(i\geq 0)$ is

$$X_i[k]=\sum_{n=0}^{N-1}x[iM+n]W_N^{nk}\ ,\ 0\leqq k\leqq N-1. \tag{1}$$

Let n equal $mM+l$, Eq. (1) becomes:

$$X_i[k]=\sum_{l=0}^{M-1}\sum_{m=0}^{(N/M)-1}x[iM+mM+l]W_N^{(mM+l)k}$$
$$=\sum_{l=0}^{M-1}W_N^{lk}G_{i,l}[k]\ , \tag{2}$$

where the $G_{i,l}[k]$ is an (N/M)-point DFT performed at the $(r-\sim)$th stage. The M blocks are arranged according to the index l, from top downwards, in the bit-reversed order. Similarly, the succeeding DFT is:

$$X_{i+1}[k]=\sum_{n=0}^{N-1}x[iM+M+n]W_N^{nk}$$
$$=\sum_{l=0}^{M-1}W_N^{lk}G_{i+1,l}[k]\ . \tag{3}$$

Compared with Eq. (2), the recursive procedure is expressed as

$$G_{i+1,l}[k]=\sum_{m=0}^{(N/M)-1}x[M(i+1+m)+l]W_{N/M}^{mk}$$
$$=W_{N/M}^{-k}\left\{G_{i,l}[k]-x[Mi+l]+x[Mi+N+l]\right\},$$
$$0\leq k\leq\left(N/M-1\right). \tag{4}$$

The above equation relates the DFT of each current frame to that of the preceding frame via a simple recursive formula. Number of arithmetic operations is reduced to $N+M$ complex additions and N complex multiplications for updating the DFT [11].

B. Pruning scheme for output size P=2 , rJ~á ár

The algorithm only constructs the computational branches required to obtain $X[k]$ for $0\leq k\leq(P–1)$. In the following illustration, the subscript i in $x_i[n]$ and $X_i[k]$ is omitted. Here, we propose a scheme to simplify the shift-pruning FFT algorithm without increasing the computational complexity. Consider

$$X[k]=\sum_{n=0}^{N-1}x[n]W_N^{kn}\ ,\ 0\leq k\leq(P–1). \tag{5}$$

Let $n=i(N/P)+j$ where $0\leq i\leq(P–1)$ and $0\leq j\leq(N/P–1)$. The above equation becomes

$$X[k]=\sum_{j=0}^{\frac{N}{P}-1}W_N^{kj}\sum_{i=0}^{P-1}x\left[i\frac{N}{P}+j\right]W_P^{ki}\ , \tag{6}$$

which is the linear combination of (N/P)'s P-point DFTs, at the th stage, with corresponding twiddle factors. Eq. (6) can be expressed as:

$$X[k]=\sum_{j=0}^{\frac{N}{P}-1}W_N^{kj}\cdot X_{j,}\ [k]\ ,\ 0\leq k\leq(P–1). \tag{7}$$

Note that, in consideration of reducing the arithmetic operations, the sr-FFT is implemented after the recursive procedure performed at the $(r-\sim)$th stage. Thus each stage consists of DFT and non-DFT blocks [10]. According to Eq. (7), arrays $X_{j,}$ $[k]$ store all the DFT output values. We developed a strategy for transforming the non-DFT blocks into DFT ones prior to the implementation of Eq. (7) [12].

C. Real-time implementation of the shift-pruning FFT

To implement the shift-pruning FFT in a real-time manner, the algorithm needs to determine amount of work (up to which stage and which block) ready to be accomplished upon receipt of a given data point, $x[n]$. In [10], we first presented a strategy for constructing the sr-FFT butterfly structure in a real-time manner. The strategy can be applied to any modular FFT structure. Accordingly, in the shift-pruning FFT structure, the real-time strategy is applied to those stages after recursive procedure, that is, from the $(r-\sim+1)$th stage to the last stage.

D. Evaluation of efficiency

Evaluation of the computational complexity must consider the complex arithmetic operations required by: 1) the recursive procedure (up to the $(r-\sim)$th stage), 2) a transition to the sr-FFT structure (at the $(r-\sim+1)$th stage), 3) the sr-FFT computation (from the $(r-\sim+2)$th to the   th stage), and 4) the pruning scheme (after the   th stage). Details of the analysis were reported in [11, 12]. An important conclusion is the upper bound of value of P that results in best improvement in computational efficiency when implementing the pruning scheme. Consider the FFT size N=512. We found that, when ≤ 6 (or, $P \leq N/8$), the pruning scheme achieves significant reduction in computational time.

Part II. Development of 2D svr-FFT

In this **part of the research work,** a novel 2-dimensional split-vector-radix fast-Fourier-transform (2D svr-FFT) algorithm **was developed**. The modularizing feature of the 2D svr-FFT structure enables us to explore its characteristics by identifying the local structural property. The block attribute directs the algorithm to construct the local module. We firstly demonstrated that the distribution of DFT blocks can be illustrated by the *Sierpinski triangle*—— a class of fractals generated by IFS (iterated function system). By exploring the structural property of 2D svr-FFT, we come to a conclusion given below

Theorem:

(a,b) *is a DFT block if and only if* $(a,b) \in \mathbf{B}_D$;
(a,b) *is a nonDFT block if and only if* $(a,b) \in \mathbf{B}_N$,

where (a,b) denotes the spatial position of a local block. And two sets $\mathbf{B}_D$ and $\mathbf{B}_N$ are defined below:

$\mathbf{B}_D$: $\{(a,b) \mid \mathrm{LSB}_1(a \vee b)=0$ or $n\mathrm{bit1} \in$ even integers$\}$,

$\mathbf{B}_N$: $\{(a,b) \mid n\mathrm{bit1} \in$ odd integers$\}$.

The operation LSB1($a \vee b$) is used to obtain the least significant bit of logistic operation ($a \vee b$). The variable $n$bit1 denotes the number of consecutive bit 1's counted from the least significant bit position. Following the proposed theorem, the resulting DFT-block distribution exhibits fractal structure.

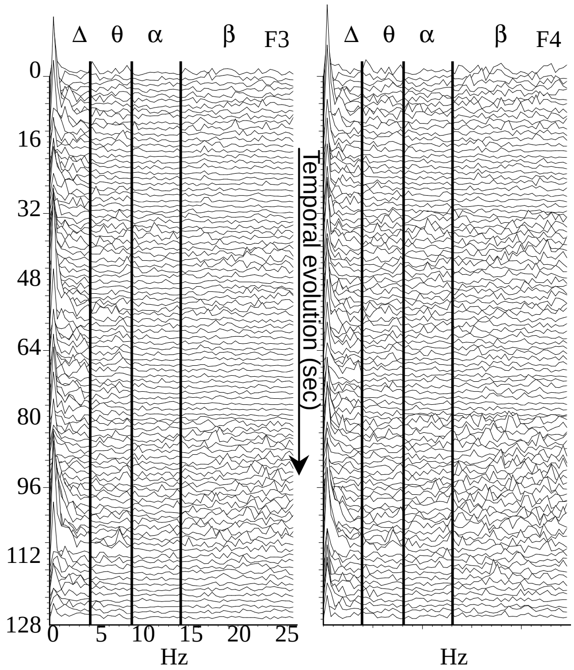
結果與討論

In this section, we present an example of running spectral analysis performed on the EEG under Zen meditation. The EEG signal was collected in a preliminary study developed to investigate the meditation EEG. The subject, a healthy 52-year-old man, has been practicing the orthodox Zen-Buddhism meditation with the Zen master Miao-Tien for more than ten years. The essence of practicing Buddhism via meditation in Zen-Buddhism originated in the affair that Buddha Shakyamuni transmitted his wisdom to the Great Kashiyapa some 2,500 years ago. It was found, in our preliminary study, that the EEG patterns of the Zen-Buddhist disciples, not only in meditation but in consciousness, differed from those of the control subjects (non-meditators). Systematic analysis of meditation EEG using various signal processing methods is under way.

The running spectral analysis based on shift-pruning FFT provides an efficient tool to pre-process the enormous amount of EEG data. In the study, we apply the 8-channel unipolar recording montage with the linked MS1-MS2 (mastoid electrodes) used as the common reference. The 8-channel EEG electrodes are placed at F3, F4, C3, C4, P3, P4, O1, and O2. The sampling rate is 200Hz. According to the naked-eye examination, the β activity (>13Hz) dominates over the entire EEG tracing. Low-frequency θ bursts emerge occasionally. Banquet reported that θ frequencies appeared in the second stage, and β waves were present over the whole scalp in the third stage of deep meditation by advanced subjects [3]. It was also reported [19] that the meditation experts exhibited generalized fast activity when entering the ecstatic or *samadhi* state in meditation.

Fig. 2 displays the running spectra computed by the shift-pruning FFT algorithm using N=512 (2.56s), M=256 (moving size: 1.28s), and P=64 (observing the frequency band 0~25 Hz). The result

Fig. 2 Running spectra of 128-second meditation



EEG signals (channels F3 and F4).

demonstrates the phenomenon of sustained low-frequency activities (θ and Δ bands, 2nd stage meditation) in the entire EEG record. Prominent, low-frequency modulated β-band activities (3rd stage meditation) appear in three time periods: (approximate figures) 0-15, 35-60, and 80-120 seconds. The running spectral analysis reveal the fact that the meditator entered the deep meditation switching between the second- and third-stage meditation, mostly in the *samadhi* state. According to the post-experimental interview, themeditator stated that he had been staying in the state for years, no matter whether he practiced meditation or not.

The running spectral analysis based on shift-pruning FFT algorithm, as a laboratory on-line monitoring tool, provides a simple and effective approach for tracking the meditation stages based on time-varying EEG spectral contents. By integrating the recursive procedure with the output pruning scheme, the shift-pruning FFT algorithm significantly improves the computational efficiency, especially when the pruning size satisfies $P \le N/8$.

To our knowledge none of the published work brought out the algorithm for implementing the svr-FFT directly in 2D. It is the first attempt to explore the structural property of the 2D svr-FFT. The theorem and source code have been developed to realize the 2D svr-FFT computational scheme.

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摘要

在腦電波分析中，時變頻譜可用於監測長時間之腦電波變化；其中短時段傅利葉轉換（short-time Fourier transform, STFT**）最常用來估測腦電波訊號之時變頻譜。本研究之主旨在於提出一高效能之「即時、位移—剪裁快速傅利葉轉換」（**real-time, shift-pruning FFT**）以解決長時間腦電波分析之問題。其中「位移—剪裁快速傅利葉轉換」之運算法則係整合*遞迴程序*和（修正後之）*輸出剪裁*策略，以提高長時間腦電波頻譜分析之效能。本研究之第二部分工作重點在於探究二維「分根基式快速傅利葉轉換」（**split-vector-radix FFT, svr-FFT**）的運算架構，並進一步實現之。在此將提出一重要定理，此定理用於導引二維** svr-FFT **運算架構之實現流程。**