行政院國家科學委員會專題研究計畫成果報告

多率訊號處理於轉化多工器上的應用
Multirate Signal Processing for Transmultiplexing Systems

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1 中文摘要

近來離散多調調變系統(DMT)或轉化多工器被廣泛的應用到各類的系統,最佳化的DMT系統成爲非常有趣的研究課題.在這篇報告中,我們證明最佳化的DMT系統的設計可被轉換成一個最佳化的系統的設計我們可以系統化的方式來設計最佳化的DMT系統.

關鍵詞：轉化多工器,離散多調調變系統(DMT),最佳化DMT

Abstract. Recently discrete multitone modulation (DMT) systems (or transmultiplexers) have been widely applied to various applications. There has been considerable interest in the design of optimal DMT systems. The M-band DMT system can be viewed as a dual of an M-band subband coder by interchanging the analysis and synthesis bank. In this report, we will show that the design of optimal perfect DMT systems can be converted to the design problem of a hypothetical subband coder. Optimal perfect DMT systems can be designed in a systematic approach.

Keywords: transmultiplexer, discrete multitone modulation (DMT), optimal DMT

2 緣由與目的

The discrete multitone modulation (DMT) systems have been shown to be a very useful for transmission over frequency selective channels [1][2]. Recently there has been considerable interest in the design of optimal DMT systems [3]. Fig. 2 shows an M-band DMT system over a frequency selective channel $C(z)$ with additive channel noise $e(n)$. The transmitting and receiving filters are respectively $T_k(z)$ and $R_k(z)$, and the DMT system $\mathcal{D}$ is denoted by $\mathcal{D} = \{T_k(z), R_k(z)\}$. The inputs $x_k(n)$ of the transmitter are modulation symbols, e.g., PAM or QAM symbols. Each symbol of the k-th band contains b_k bits. The average bit rate is $b = 1/M \sum_{k=0}^{M-1} b_k$. We say the DMT system is perfect if the outputs $\widehat{x}_k(n) = x_k(n)$, for $k = 0, 1, \cdots, M-1$ in the absence of channel noise $e(n)$. In this case, there is no inter- and intra-band ISI. When there is channel noise, $\widehat{x}_k(n) = x_k(n) + e_k(n)$, where the noise $e_k(n)$ of the k-th band comes entirely from the channel noise $e(n)$. For a given average bit rate, the optimal DMT system minimizes the transmitted power $\mathcal{P}$, i.e., the variance of the transmitted signal $y(n)$ as indicated in Fig. 2.

The M-band DMT system can be viewed as

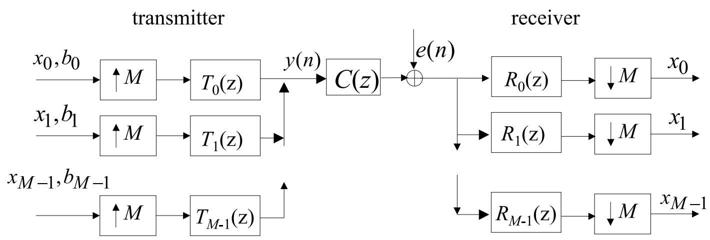

圖 1: An M-band DMT system over a frequency selective channel $C(z)$ with additive channel noise $e(n)$.

a dual of an M-band subband coder (Fig. 2) by interchanging the analysis and synthesis bank. The filter bank with analysis filters $H_k(z)$ and synthesis filters $F_k(z)$, denoted by $\mathcal{F} = \{H_k(z), F_k(z)\}$, is said to be biorthogonal or have perfect reconstruction (PR) property if

$$\left(F_k(e^{j\omega})H_m(e^{j\omega})\right)_{\downarrow M} = \delta(k-m),$$

where $\downarrow M$ denotes M-fold decimation. When there is quantization noise, the output $\widehat{x}(n) = x(n)+q(n)$, where $q(n)$ comes entirely from the quantization noise $q_k(n)$. A PR filter bank is called orthonormal if $F(e^{j\omega}) = H^*(e^{j\omega})$. For a given class of filter banks, the optimal solution is one that minimizes the output noise variance σ_q^2.

In the context of optimal subband coder design, great advance has been made recently [4][5]. It has been shown that, for the class of orthonormal filter banks, the Principle Component Filter Bank (PCFB) minimizes the output noise variance σ_q^2. For the design of biorthogonal filter banks, the structure of cascading orthonormal (ParaUnitary) filter banks with pre- and post-filters (PPU structure) is proposed in [6] to minimize the output noise. Recently, Moulin et. al. show that [5] there is no loss of generality in assuming the PPU structure in the design of optimal biorthogonal filter banks. More recently, it is shown that PCFB is also optimal for designing DMT with orthonormal transmitter.

In this report, we will formulate the design problem of optimal DMT systems and point out the duality in the design of optimal DMT systems and optimal biorthogonal filter banks. We will show that the design of optimal perfect DMT systems can be converted to the design problem of a hypothetical subband coder and hence can be solved using existing techniques for designing optimal biorthogonal filter banks in most cases.

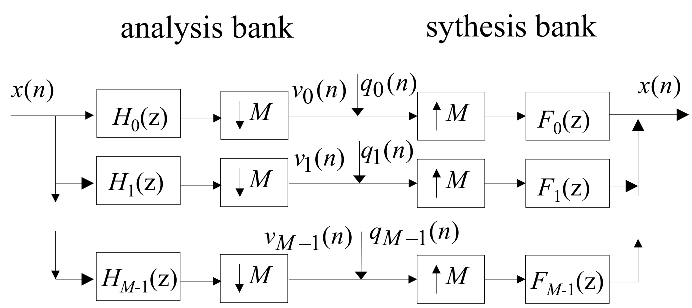

圖 2: An M-band subband coder.

3 結果與討論:

DMT Systems

The problem of designing optimal DMT system for a given channel $C(z)$ and channel noise $e(n)$ can be formulated as follows. We assume $x_k(n)$ are PAM symbols for simplicity. Each of the symbols $x_k(n)$ of the k-th band carries b_k bits. The average bit rate is

$b = 1/M \sum_{k=0}^{M-1} b_k$. Assuming the input modulation symbols $x_k(n)$ are white and uncorrelated, which can always be done with proper interleaving. The transmitted power $\mathcal{P}$ is given by

$$\mathcal{P} = \sum_{k=0}^{M-1} g(P_e, b_k) \|t_k\|_2^2 \int_0^{2\pi} S_{ee}(e^{j\omega}) |R_k(e^{j\omega})|^2 \frac{d\omega}{2\pi} \quad (1)$$

where $g(P_e, b_k) = \left[Q^{-1} \left(\frac{P_e}{2(1-2^{-b_k})} \right) \right]^2 \frac{2^{2b_k} - 1}{3}$

Subband Coders An M-band filter bank $\mathcal{F} = \{H_k(z), F_k(z)\}$ is as shown in Fig. 2. The quantization noises $q_k(n)$ are usually assumed to be wide sense stationary random processes that are white, zero mean and uncorrelated. The variance of the k-th quantization noise $q_k(n)$ is related to the variance of the k-th subband signal $v_k(n)$ by a distortion function,

$$\sigma_{q_k}^2 = D(b_k) \sigma_{v_k}^2,$$

where b_k is the number of bits allocated to the k-th subband. The variance of the output quantization noise is

$$\sigma_q^2 = \sum_{k=0}^{M-1} D(b_k) \|f_k\|_2^2 \int_0^{2\pi} S_{xx}(e^{j\omega}) |H_k(e^{j\omega})|^2 \frac{d\omega}{2\pi} \quad (2)$$

Principle Component Filter Banks (PCFB). In recent years, great advance has been made in the study of optimal orthonormal filter banks or the so-called Principle Component Filter Banks (PCFB) [4], . The development is based on the majorization theorem. Given 2 ordered sequences $\{a_n\}_{n=0}^{M-1}$ and $\{b_n\}_{n=0}^{M-1}$ with $a_n \geq a_{n+1}$ and $b_n \geq b_{n+1}$, we say $\{a_n\}_{n=0}^{M-1}$ majorizes $\{b_n\}_{n=0}^{M-1}$ if

$$\sum_{n=0}^{N} a_n \geq \sum_{n=0}^{N} b_n, \ 0 \leq N \leq M-1,$$

with equality when $N = M - 1$.

Consider a class of filter banks $\mathcal{C}$. The class can be the collection of FIR filter banks or the set of ideal filter banks. A filter bank $\mathcal{F}$ in the class $\mathcal{C}$ is a PCFB for the given input $S_{xx}(e^{j\omega})$ if the set $\{\sigma_{v_k}^2\}_{k=0}^{M-1}$ formed by its subband variances majorizes the set $\{\sigma_{v_k}'^2\}_{k=0}^{M-1}$ formed by the subband variances of any other filter bank $\mathcal{F}'$ in the class $\mathcal{C}$. The PCFB, when it exists, minimizes the output quantization noise in (2). This result does not require that $q_k(n)$ be white and uncorrelated. Also the PCFB is optimal for any given bit allocation. In particular, it is optimal under optimal bit allocation.

Prefilters for Orthonormal Filter Banks. To minimize the quantization noise or to maximize the coding gain, [6] considers a class of biorthogonal filter banks by cascading orthonormal or paraunitary (PU) filter banks with pre- and post filters (Fig. 3). This will be called PPU

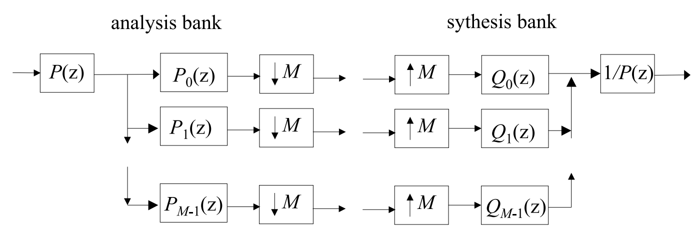

圖 3: An M-band filter bank with pre-filter $P(z)$ and post filter $1/P(z)$.

structure. The analysis and synthesis filters of the biorthogonal filter banks are of the form

$$H_k(e^{j\omega}) = P(e^{j\omega}) P_k(e^{j\omega}), \quad F_k(z) = \frac{P_k^*(e^{j\omega})}{P(e^{j\omega})}, \quad (3)$$

where $\{P_k(e^{j\omega}), P_k^*(e^{j\omega})\}$ form an orthonormal filter bank. Under high bit rate assumption $D(b_k) = c2^{-2b_k}$ and optimal bit allocation, it is shown that [6] the optimal prefilter $P(e^{j\omega})$ should be the half whitening filter for the input power spectrum $S_{xx}(e^{j\omega})$, i.e.,

$$P(e^{j\omega}) = 1/S_{xx}^{1/4}(e^{j\omega}).$$

Furthermore, $\{P_k(e^{j\omega}), P_k^*(e^{j\omega})\}$ should be the PCFB for the input power spectrum $\sqrt{S_{xx}(e^{j\omega})}$. That is, the design problem decouples as 2 problems: the problem of designing a half whitening filter for $S_{xx}(e^{j\omega})$ and the problem of designing the PCFB for $\sqrt{S_{xx}(e^{j\omega})}$.

Optimal Biorthogonal Filter Banks. More recently, Moulin et. al. [5] shows that it is not a loss of generality assuming the PPU structure in designing optimal biorthogonal subband coders. The problem of designing optimal biorthogonal filter banks can be decoupled as the problem of designing a half whitening filter and a PCFB. Without assuming optimal bit allocation, this is true in most cases [5].

Optimal DMT. For the design of the optimal DMT systems for the most general class, let us consider the DMT system

$$\mathcal{D} = \{F_k(z), H_k(z)/C(z)\},$$

where $H_k(z)$ and $F_k(z)$ are the analysis and synthesis filters of a biorthogonal filter bank $\mathcal{F} = \{H_k(z), F_k(z)\}$. The transmitted power in (1) can be rewritten as

$$\mathcal{P} = \sum_{k=0}^{M-1} g(P_e, b_k)\|f_k\|^2 \int_0^{2\pi} \frac{S_{ee}(e^{j\omega})}{|C(e^{j\omega})|^2}|H_k(e^{j\omega})|^2 \frac{d\omega}{2\pi} \quad (4)$$

For the above objective function, we can convert it to the following hypothetical filter bank design problem: consider the M-band filter bank in Fig. 2 with input power spectrum $S_{ee}(e^{j\omega})/|C(e^{j\omega})|^2$. Suppose the distortion function $D(b_k)$ is replaced by $g(P_e, b_k)$; the variance of quantization noise $q_k(n)$ is

$$\sigma_{q_k}^2 = g(P_e, b_k)\sigma_{v_k}^2.$$

Then the output quantization noise σ_q^2 is given exactly by (4)! Note that in the design of optimal biorthogonal filter banks, the problem decouples as prefilter design and PCFB design in most cases without making assumptions on $D(b_k)$. This means that, except in pathological cases, we can solve the design problem of optimal perfect DMT systems in the same manner using the design method for optimal biorthogonal filter banks .

4 計畫成果自評:

In this report we show how to design optimal DMT systems. Without complexity constraint, the optimal systems are ideal filters. The performance will serve as a bound for the all the DMT systems. The bound will be very useful for future design of DMT systems.

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