新定期盤存補貨政策
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I. Introduction

One of the most widely used stochastic inventory models is an order-up-to periodic review policy. Unlike a continuous review model where the order quantity is fixed, an order-up-to periodic policy places an order in every period that will raise inventory to a target level. Since demand fluctuates period by period, the order quantity varies too. This may result in costs of adjustment in practice due to, for example, changes in capacity or production plans. These costs are incurred by the supplier but may be charged to the buyer (e.g., Urban, 2000). However, an ordinary order-up-to policy implicitly assumes that these costs do not exist (or simply ignores these costs).

It is possible that the order quantity is made (almost) fixed in periodic review systems. Such a scenario is attractive and desirable to both the supplier and the buyer. It simplifies the production, order picking, delivery, unloading (for the supplier), inspection process, and inventory record updating procedure (for the buyer). It avoids any extra costs that might be incurred due to variations in the order quantity. It also agrees with the JIT management philosophy. However, since demand is stochastic in the real world, a certain mechanism is needed to absorb variations of demand. In general, there are several ways to achieve this. One is through a standing order inventory system where provision is made for procuring extra units in the case of an emergency and selling off excess inventory. Standing order systems are first considered by Rosenshine and Obee (1976) and recently studied in Chiang (2007). Another is through a two-supplier inventory system (Janssen and de Kok, 1999) in which while one supplier delivers a fixed quantity in each period, the other ships any quantity needed to raise inventory to a target level.

In this research, we propose a third approach to facilitating the scenario of a (almost) fixed order quantity in periodic review systems. We suppose that every n periods the buyer, who employs an order-up-to policy, plans for the quantity delivered for each upcoming period. Assuming that demand not filled immediately is backlogged, we suggest that a fixed quantity Q be delivered by the supplier for each of the upcoming n periods *except* perhaps the first several periods. To be more specific, the immediately upcoming period's shipment size is adjusted first, and if not sufficient, the upcoming second period's shipment size is adjusted next, and so on, such that the quantity shipped in each of the subsequent periods is exactly Q. Thus, the quantity to be delivered in the immediately upcoming period is of any size and the quantity delivered in each of the subsequent (n – 1) periods is equal to or smaller than Q. For example, suppose that $Q = 5$ and $n = 5$. If demand of the previous 5 periods is 28 (resp. 22), the quantity delivered in each of the upcoming 5 periods is 8 (resp. 2), 5, 5, 5, 5, respectively. On the other hand, if demand of the previous 5 periods is 17 (resp. 13), the quantity delivered in each of the upcoming 5 periods is 0, 2 (resp. 0), 5 (resp. 3), 5, 5, respectively. For a certain n, the optimal order-up-to level for the delivery scenario described is computed by minimizing the average n-period's cost. Next, assuming that there is a fixed cost incurred every n periods for auditing the inventory level as well as planning and adjusting order quantities, the optimal n can be obtained by minimizing the average cost per period through a simple procedure. Finally, the optimal Q can also be determined. Hence, the proposed replenishment policy is easy to implement.

Note that Flynn and Garstka (1990) consider a related but more complex problem where one schedules delivery quantities for the next n periods that are generally not equal to one another. Flynn and Garstka (1997) further extend the analysis to determine the optimal review period. Chiang (2001) also studies a delivery splitting periodic model where n shipments are scheduled in future time points that are evenly separated. However, Three major shortcomings limit the applicability of Chiang's model. First, these n shipments are of fixed but different sizes and the costs of adjustment due to changes in the shipment size are not included. Second, only the immediately upcoming period's shipment size is used to absorb variations in demand. Third, the interval between delivery epochs may not be an integral multiple of a basic time unit (e.g., a day). Thus, the suggested model may not be easily implemented in practice. Other related periodic review models are investigated in Ehrhardt (1997) and Urban (2000). The former considers the problem of selecting a fixed replenishment quantity to be delivered in each of n consecutive periods in the future, while the latter describes a multi-period "recurrent" newsvendor problem where changes in the order quantity result in an additional cost to the buyer.

II. A Periodic Review Replenishment Model

Consider the following replenishment problem: every n periods (one period is one day, for example) the buyer reviews an item and plans its shipment size for each upcoming period to raise the inventory position (i.e., inventory on hand minus backorders plus inventory on order) to a target level (i.e., an order-up-to policy is used). Assume that demand of each period is independently and identically distributed. As explained in section 1, it benefits the supplier as well as the buyer if a fixed quantity Q is shipped in each period. Such a situation occurs only when demand in the previous n periods is exactly nQ. If demand of the previous n periods is not nQ, the buyer would like to adjust first the immediately upcoming period's shipment size so that the quantity shipped in each of the subsequent (n – 1) periods is exactly Q; if adjustment to the immediately upcoming period's shipment size is not sufficient, the buyer would adjust next the upcoming second period's shipment size so that the quantity shipped in each of the subsequent (n – 2) periods is Q; and so on. Notice that we have implicitly assumed in the above scenario that the immediately upcoming period's shipment size can be adjusted if requested by the buyer. If it takes a positive lead time (an integral multiple of the period length) to adjust the shipment size, our replenishment model can be modified by appropriately defining $G_i(Y|n, Q)$ introduced below, as in an ordinary periodic review model (e.g., Porteus, 1990).

For the proposed replenishment policy to be clearer, let $f_k(\cdot)$ be the probability density function of k-period's demand, $k = 1,\ldots, n$, and D the demand during the previous n periods (its probability density function is thus $f_n(\cdot)$). If $D \geq (n - 1)Q$, the quantity delivered for each of the upcoming n periods is $D - (n - 1)Q, Q,\ldots, Q$, respectively; if $(n - 1)Q > D \geq (n - 2)Q$, the quantity delivered for each upcoming period is 0, $D - (n - 2)Q, Q,\ldots, Q$, respectively; if $(n - 2)Q > D \geq (n - 3)Q$, the quantity delivered for each upcoming period is 0, 0, $D - (n - 3)Q, Q,\ldots, Q$, respectively; and so on. See Chiang (2008) for details of the proposed model.

III. A Simplified Policy

In the above delivery scenario, the quantity shipped in the immediately upcoming period is of any size and the quantity shipped in each of the subsequent (n – 1) periods is equal to or smaller than Q. Suppose that excess inventory in the immediately upcoming period can be salvaged or retuned to the supplier at c per unit. Then, only the quantity shipped in this period is variable and the quantity shipped in each of the subsequent (n – 1) periods is exactly Q. This greatly simplifies the replenishment policy. Referring to the second paragraph of section 2, if $D < (n - 1)Q$, the quantity shipped for each upcoming period is 0, $Q, Q,\ldots, Q$, respectively, and the excess units $(n - 1)Q - D$ in the immediately upcoming period are salvaged or returned to the supplier. See Chiang (2008) for details of the simplified policy.

IV. Computational Results

To illustrate, consider the base case: μ = 4/period, h = \$1, and p = \$100. Demand is assumed to follow a Poisson process. If Q = 4 and n = 5, then Y^* = 29 and $G(Y^*|n, Q)/n$ = \$11.06. We vary μ and Q as well as n and p in the base case (specifically, μ = 2, 4, and 6, Q = μ – 1, μ, μ + 1, μ + 2, and μ + 3, n = 1, 2,…, 20, and p = \$10, \$100, and \$1000) and solve 900 problems. It is found that $G(Y^*|n, Q)/n$ is increasing in n. This implies that if K = 0, n^* = 1, i.e., the proposed model reduces to an ordinary periodic policy where an order is placed in every period. See Chiang (2008) for the rest of the computational results.

V. Conclusion

This research considers a single-item replenishment problem where every n periods the buyer plans for the quantity delivered for each upcoming period. Depending on the demand of the previous n periods, the quantity delivered in the immediately upcoming period may be of any size and the quantity delivered in each of the subsequent (n – 1) periods is equal to or smaller than a fixed quantity Q. If excess inventory in the immediately upcoming period can be salvaged or retuned to the supplier at the original purchase cost, the quantity shipped in each of the subsequent (n – 1) periods is exactly Q.

Computation shows that the optimal Q is greater than or equal to the mean period demand. However, as the ratio p/h decreases, the optimal n increases and the optimal Q may approach the mean period demand. In addition, as K increases, the optimal n increases, and if K = 0, the optimal n is equal to 1. In this sense, the ordinary order-up-to policy where an order is placed in every period could be regarded as a special case of the proposed replenishment model. More importantly, as K is larger, the proposed model becomes more attractive relative to the ordinary order-up-to policy.

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In this research, we propose a periodic review replenishment model where every n periods the buyer plans for the quantity delivered for each upcoming period. We suggest that the quantity delivered in the immediately upcoming period be of any size and the quantity delivered in each of the subsequent (n – 1) periods be equal to or smaller than a fixed quantity Q. The ordinary order-up-to policy where an order is placed in every period could be regarded as a special case of the proposed replenishment model. This makes a good contribution to the inventory literature.