（一）報告封面：

行政院國家科學委員會補助專題研究計畫 ■ 成果報告 □期中進度報告

一般反應值變換不等變異迴歸模型之貝氏推論

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計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：■精簡報告　□完整報告

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中　華　民　國　97　年　10　月　31　日

（二）中、英文摘要及關鍵詞(keywords)。

（1）中文摘要及關鍵詞(keywords)：

國立交通大學統計學研究所

計畫名稱：一般反應值變換不等變異迴歸模型之貝氏推論
研究者：陳志榮
經費來源：行政院國家科學委員會
關鍵詞：貝氏推論；乘冪變換；指數變換；Aranda-Ordaz 變換；不等變異

變換反應值是處理不等變異和非常態誤差的一種常用方法。變換反應值最初是用來當作達成相等變異且常態誤差及產生一個較為簡單的線性迴歸模型(Box 和 Cox，1964)：即經過變換反應值後誤差或標準化誤差為相等變異且常態分布。然而當變換的值域不是所有的實數時，經過變換反應值後誤差或標準化誤差不可能為常態分布。常被使用的變換其值域可能不是所有的實數的例子有乘冪變換(Box 和 Cox，1964)、指數變換(Manly，1976)及 Aranda-Ordaz 變換(Aranda 和 Ordaz，1981)。而且當變換的值域不是所有的實數時，經過變換反應值後誤差或標準化誤差通常有不同的值域，因而有不同的分布。因此，Chen 和 Wang (2003) 提出下列一般反應值變換不等變異 truncated 常態迴歸模型

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \quad i = 1, \ldots, n,$$

的 frequentist 推論，此處 y_i 為第 i 個觀察值；λ 為未知的變換參數向量；$h(\cdot;\lambda)$ 為單調遞增的變換函數；x_i 為已知的第 i 個解釋變數向量；β 為未知的迴歸參數向量；$f(\cdot;\beta)$為迴歸函數；γ 為未知的變異數參數向量；$g(\cdot,\cdot;\gamma)$為正的加權函數；且 ε_i 為獨立標準化 truncated $N(c_i(\lambda,\beta,\gamma), 1)$ 誤差且中位數為 0。然而，此概似函數並沒有公式可以直接計算，因此需要花很多時間用數值疊代方法才可求得。在貝氏架構中，有公式可以直接計算的概似函數是極為重要的。

因此，在這個計畫中，在貝氏架構下，首先我們修改 Chen 和 Wang (2003)的概似函數來提出下列一般反應值變換不等變異 truncated 常態迴歸模型

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \quad i = 1, \ldots, n,$$

的貝氏推論，此處 y_i 為第 i 個觀察值；λ 為未知的隨機變換參數向量且擁有常態(或 truncated 常態或 uniform)先驗分布；$h(\cdot;\lambda)$ 為單調遞增的變換函數；x_i 為已知的第 i 個解釋變數向量；β 為未知的隨機迴歸參數向量且擁有常態(或 truncated 常態或 uniform)先驗分布；$f(\cdot;\beta)$為迴歸函數；γ 為未知的隨機變異數參數向量且擁有 inverse Wishart(或 truncated inverse Wishart 或 vague)先驗分布；$g(\cdot,\cdot;\gamma)$為正的加權函數；且 ε_i 為獨立標準化 truncated $N(0, 1)$ 誤差。其次，我們提出此貝氏迴歸模型的 Markov chain Monte Carlo (MCMC)後驗估計、後驗假設檢定、後驗 credible 區域、後驗預測及相關的有限樣本和大樣本性質。

(2) 英文摘要及關鍵詞(keywords):

Institute of Statistics, National Chiao Tung University

Title:Bayesian Inference under the General Response Transformation Heteroscedastic Regression Model

Principal Investigator:Chih-Rung Chen

Sponsor:National Science Council

Keywords: Bayesian Inference, Power Transformation, Exponential Transformation, Aranda-Ordaz Transformation, Heteroscedasticity

When there exist heteroscedastic errors and/or departures from normality in the data, a popular approach is to transform the response. Originally, transforming the response was proposed both as a means of achieving homoscedasticity and approximate normality and for inducing a simpler linear model for the transformed response (Box and Cox, 1964). In such situations, Box and Cox (1964) proposed the following response transformation normal homoscedastic regression model for modeling independent continuous data:

$$h(y_i;\lambda) = f(x_i;\beta) + \varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i, λ is a finite-dimensional transformation parameter vector, $h(\cdot;\lambda)$ is a strictly increasing and differentiable transformation, x_i is a known covariate vector for subject i, β is a finite-dimensional regression parameter vector, $f(\cdot;\beta)$ is a regression function, and ε_is are i.i.d. $N(0,\sigma^2)$ errors with unknown variance $\sigma^2 > 0$.

When both heteroscedastic errors and departures from normality cannot be removed simultaneously in the data by any single transformation, the Box-Cox model is further generalized to the following response transformation normal heteroscedastic regression model for modeling independent continuous data:

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where γ is a variance parameter vector, $g(\cdot,\cdot;\gamma)$ is a positive weight function, and ε_is are i.i.d. $N(0,1)$ standardized errors.

However, if the range of the response transformation is different from $\boldsymbol{R}$ ($\equiv(-\infty, \infty)$), the corresponding errors cannot be normally distributed. Commonly-used examples are the power transformations (Box and Cox, 1964), exponential transformations (Manly, 1976), and Aranda-Ordaz transformations (Aranda and Ordaz, 1981). Moreover, the corresponding errors don't even have the same distributions, due to the fact that they may have different supports.

Thus, Chen and Wang (2003) proposed the following general response transformation truncated normal heteroscedastic regression model

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where ε_is are independent truncated $N(c_i(\lambda,\beta,\gamma), 1)$ standardized errors with median 0.

However, there does not exist any closed-form formula for the likelihood function proposed by Chen and Wang (2003). Thus, it takes too much time to calculate this likelihood function by any numerical iteration method. A closed-form formula for the likelihood function will be very important in a Bayesian framework; otherwise, it is nearly impossible to do the Bayesian inference in practice.

Thus, in this project, in a Bayesian framework, we first modify the likelihood function proposed in Chen and Wang (2003) and then propose the following general transformation truncated normal heteroscedastic regression model

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i, λ is a finite-dimensional random transformation parameter vector with normal (or truncated normal or uniform) prior distribution, $h(\cdot;\lambda)$ is a strictly increasing and differentiable transformation, x_i is a known covariate vector for subject i, β is a finite-dimensional random regression parameter vector with normal (or truncated normal or uniform) prior distribution, $f(\cdot;\beta)$ is a regression function, γ is a random variance parameter vector with inverse Wishart (or truncated inverse Wishart or vague) prior distribution, $g(\cdot,\cdot;\gamma)$ is a positive weight function, and ε_is are independent truncated $N(0, 1)$ standardized errors. Next, we propose the corresponding Markov chain Monte Carlo (MCMC) posterior estimation, hypothesis testing, credible region, and prediction, and the corresponding finite-sample and large-sample properties for the proposed Bayesian regression model.

(三)報告內容：請包括前言、研究目的、文獻探討、研究方法、結果與討論（含結論與建議）…等。若該計畫已有論文發表者，可以 A4 紙影印，作為成果報告內容或附錄，並請註明發表刊物名稱、卷期及出版日期。若有與執行本計畫相關之著作、專利、技術報告、或學生畢業論文等，請在參考文獻內註明之，俾可供進一步查考。

（1）前言：

For modeling independent continuous data, it is common practice simply to assume the following normal homoscedastic regression model: For $i = 1, \ldots, n$,

(1) $$y_i = f(x_i;\beta) + \varepsilon_i, \quad i = 1, \ldots, n,$$

where y_i is the observation for subject i, x_i is a known covariate vector for subject i, β is a finite-dimensional regression parameter vector, f is a known regression function of both x_i and β, and ε_is are *i.i.d.* $N(0, \sigma^2)$ errors with unknown variance $\sigma^2 > 0$.

When heteroscedastic errors and/or departures from normality exist in the data, a popular approach is to transform the response. Originally, the response transformation was proposed both as a means of achieving homoscedasticity and approximate normality and for inducing a simpler linear model for the transformed response (Box and Cox, 1964). In such situations, we may assume the following response transformation normal homoscedastic regression model to extend the normal homoscedastic regression model (1) for modeling independent continuous data: For $i = 1, \ldots, n$,

(2) $$h(y_i;\lambda) = f(x_i;\beta) + \varepsilon_i, \quad i = 1, \ldots, n,$$

where λ is a finite-dimensional response transformation parameter vector, $h(.;\lambda)$ is a known strictly increasing and differentiable response transformation, and ε_is are *i.i.d.* $N(0, \sigma^2)$ errors with unknown variance $\sigma^2 > 0$.

When both heteroscedastic errors and departures from normality cannot be removed simultaneously in the data by any single response transformation, we may assume the following response transformation normal heteroscedastic regression model to extend the response transformation normal homoscedastic regression model (2) for modeling independent continuous data: For $i = 1, \ldots, n$,

(3) $$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \quad i = 1, \ldots, n,$$

where γ is a finite-dimensional variance parameter vector, g is a known positive weight function of $f(x_i;\beta)$, x_i and γ, and ε_is are *i.i.d.* $N(0, 1)$ standardized errors.

When the range of the response transformation may be different from $\boldsymbol{R}$, Chen and Wang (2003) proposed the following general response transformation truncated normal heteroscedastic regression model to extend the response transformation normal heteroscedastic regression model (3) for modeling independent continuous data: For $i = 1, \ldots, n$,

(4) $$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \quad i = 1, \ldots, n,$$

where ε_is are independent truncated $N(0, 1)$ standardized errors. Three commonly-used families of response transformations with ranges possibly different from $\boldsymbol{R}$ in the literature are presented to illustrate the importance and applicability of the proposed model. The likelihood inference under the proposed model is discussed thoroughly. Finally, when the range of the response transformation is not $\boldsymbol{R}$, the inappropriateness of the likelihood inference under the response transformation normal heteroscedastic regression model (3) is shown to further demonstrate the importance of that work.

Some references are available in the literature for the specification of the prior distributions under the Box-Cox response transformation normal homoscedastic regression model. For example, Box and Cox (1964) developed a Bayesian approach with data-dependent priors. Pericchi (1981) and Sweeting (1984) suggested other non-data-dependent priors.

In the literature, all Box-Cox response transformation Bayesian regression models are misleading because they used incorrect likelihood functions by making the impossible assumption that the Box-Cox transformed response is normally distributed. However, the Box-Cox transformed response cannot be normally distributed because of the support constraint.

（2）研究目的：

In this project, we assume that the transformed response is truncated normally distributed to satisfy the support constraint. So all results in this project are correct and can be applied to response transformation Bayesian regression models.

Since the likelihood function under the general response transformation model with heteroscedastic errors (4) proposed in Chen and Wang (2003) has no closed-form formula, it takes too much time to find this likelihood function by any numerical iteration method. However, a closed-form formula for the likelihood function will be very important in a Bayesian framework; otherwise, it is nearly impossible to do the Bayesian inference in practice.

Thus, in this project, in a Bayesian framework, we first modify the likelihood function proposed in Chen and Wang (2003) and then propose the following general response transformation truncated normal heteroscedastic regression model

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i, λ is a finite-dimensional random transformation parameter vector with normal (or truncated normal or uniform) prior distribution, $h(\cdot;\lambda)$ is a strictly increasing and differentiable transformation, x_i is a known covariate vector for subject i, β is a finite-dimensional random regression parameter vector with normal (or truncated normal or uniform) prior distribution, $f(\cdot;\beta)$ is a regression function, γ is a random variance parameter vector with inverse Wishart (or truncated inverse Wishart or vague) prior distribution, $g(\cdot,\cdot;\gamma)$ is a positive weight function, and ε_is are independent truncated $N(0, 1)$ standardized errors. Next, we propose the corresponding Markov chain Monte Carlo (MCMC) posterior estimation, hypothesis testing, credible region, and prediction, and the corresponding finite-sample and large-sample properties for the proposed Bayesian regression model.

（3）文獻探討：

Originally, the response transformation was proposed both as a means of achieving homoscedasticity and approximate normality and for inducing a simpler linear model for the transformed response (Box and Cox, 1964). In such situations, Box and Cox (1964) proposed the following response transformation model for modeling independent continuous data: For i = 1, …, n,

$$h(y_i;\lambda) = x_i^T\beta + \varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i with support $(0, \infty)$, λ is a real-valued response transformation parameter with $h(y_i;\lambda) = (y_i^{\lambda} - 1)/\lambda$ if $\lambda \neq 0$ and $\log(y_i)$ if $\lambda = 0$, x_i is a known covariate vector for subject i, β is a finite-dimensional regression parameter vector, and ε_is are *i.i.d.* $N(0, \sigma^2)$ errors with unknown variance $\sigma^2 > 0$. However, the Box-Cox transformed response cannot be normally distributed because the support of ε_i is $(-x_i^T\beta - 1/\lambda, \infty)$ if $\lambda > 0$ or $(-\infty, -x_i^T\beta - 1/\lambda)$ if $\lambda < 0$. If ε_is are $N(0, \sigma^2)$ errors with unknown variance $\sigma^2 > 0$, then their supports should be $\boldsymbol{R}$.

When the range of the response transformation may be different from $\boldsymbol{R}$, Chen and Wang (2003) proposed the following general response transformation model with heteroscedastic errors to extend the response transformation model with heteroscedastic errors (3) for modeling independent continuous data: For $i = 1, \ldots, n$,

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta), x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where ε_is are independent truncated $N(0, 1)$ standardized errors. Three commonly-used families of response transformations with ranges possibly different from $\boldsymbol{R}$ in the literature are presented to illustrate the importance and applicability of the proposed model. The likelihood inference under the proposed model is discussed thoroughly. Finally, when the range of the response transformation is not $\boldsymbol{R}$, the inappropriateness of the likelihood inference under the response transformation model with heteroscedastic errors (3) is shown to further demonstrate the importance of that work.

Some references are available in the literature for the specification of the prior distributions under the Box-Cox response transformation model with homoscedastic errors. For example, Box and Cox (1964) developed a Bayesian approach with data-dependent priors. Pericchi (1981) and Sweeting (1984) suggested other non-data-dependent priors.

In the literature, all Box-Cox response transformation Bayesian regression models are misleading because they used incorrect likelihood functions by making the impossible assumption that the Box-Cox transformed response is normally distributed. However, the Box-Cox transformed response cannot be normally distributed because of the support constraint.

(4) 研究方法：

In this project, in a Bayesian framework, we first modify the likelihood function proposed in Chen and Wang (2003) and then propose the following general transformation truncated normal heteroscedastic regression model

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i, λ is a finite-dimensional random transformation parameter vector with normal (or truncated normal or uniform) prior distribution, $h(\cdot;\lambda)$ is a strictly increasing and differentiable transformation, x_i is a known covariate vector for subject i, β is a finite-dimensional random regression parameter vector with normal (or truncated normal or uniform) prior distribution, $f(\cdot;\beta)$ is a regression function, γ is a random variance parameter vector with inverse Wishart (or truncated inverse Wishart or vague) prior distribution, $g(\cdot,\cdot;\gamma)$ is a positive weight function, and ε_is are independent truncated $N(0, 1)$ standardized errors.

First of all, let $\theta \equiv (\lambda^T, \beta^T, \gamma^T)^T$ be the d-dimensional parameter vector and let Θ be the corresponding parameter space, where θ is chosen to have a subjective proper or non-informative improper prior density function with a closed-form formula $\pi(\theta)$. For example, for $i = 1, \ldots, n$, the support of y_i is $(0, \infty)$, $h(y_i;\lambda) = (y_i^{\lambda} - 1)/\lambda$ if $\lambda \neq 0$ and $\log(y_i)$ if $\lambda = 0$, $f(x_i;\beta) = x_i^T\beta$, and $g(f(x_i;\beta),z_i;\gamma) = \sigma \exp[\gamma_1 f(x_i;\beta) + z_i^T\gamma_2]$ for $\gamma = (\sigma, \gamma_1, \gamma_2^T)^T$. Moreover, $\theta \equiv (\lambda, \beta^T, \sigma, \gamma_1, \gamma_2^T)^T$ is chosen to have the prior density function $\pi(\theta) = \pi_1(\lambda)\pi_2(\beta,\sigma)\pi_3(\gamma_1,\gamma_2)$, where $\pi_1(\lambda)$ is a known subjective normal probability density function (p.d.f.) or uniform, $\pi_2(\beta,\sigma)$ is a known subjective normal-inverse-gamma p.d.f. or $\propto 1/\sigma$, and $\pi_3(\gamma_1,\gamma_2)$ is a known subjective normal p.d.f. or uniform.

Next, derive the closed-form formula of the proposed likelihood function of θ, i.e., the conditional p.d.f. $p(y|\theta)$ of y given θ, by a similar method in Chen and Wang (2003), where $y \equiv (y_1, \ldots, y_n)^T$. Then the posterior likelihood function of θ, i.e., the conditional p.d.f. $p(\theta|y)$ of θ given y, is $\propto \pi(\theta)p(y|\theta)$. A Markov chain Monte Carlo (MCMC) approach is applied to generate an MCMC sample $\{\theta^t, \ldots, \theta^{t+m}\}$ of size m from $p(\theta|y)$ ($\propto \pi(\theta)p(y|\theta)$) for some large t and m.

Finally, the corresponding MCMC posterior estimation, hypothesis testing, credible region, and prediction, and the corresponding finite-sample and large-sample properties for the proposed Bayesian regression model can be easily derived via the techniques of the traditional Bayesian inference.

（5）結果與討論（含結論與建議）：

Since the likelihood function in Box and Cox (1964) is incorrect due to the support constraint, we need to find a correct likelihood function to use in response transformation regression model.

Since the likelihood function under the general response transformation truncated normal heteroscedastic regression model (4) proposed in Chen and Wang (2003) has no closed-form formula, it is nearly impossible to calculate its likelihood function by any numerical iteration method.

Thus, in this project, in a Bayesian framework, we first modify the likelihood function proposed in Chen and Wang (2003) and then propose the following general response transformation truncated normal heteroscedastic regression model

$$h(y_i;\lambda) = f(x_i;\beta) + g(f(x_i;\beta),x_i;\gamma)\varepsilon_i, \qquad i = 1, \ldots, n,$$

where y_i is the observation for subject i, λ is a finite-dimensional random transformation parameter vector with normal (or truncated normal or uniform) prior distribution, $h(\cdot;\lambda)$ is a strictly increasing and differentiable transformation, x_i is a known covariate vector for subject i, β is a finite-dimensional random regression parameter vector with normal (or truncated normal or uniform) prior distribution, $f(\cdot;\beta)$ is a regression function, γ is a random variance parameter vector with inverse Wishart (or truncated inverse Wishart or vague) prior distribution, $g(\cdot,\cdot;\gamma)$ is a positive weight function, and ε_is are independent truncated $N(0, 1)$ standardized errors. Next, we propose the corresponding MCMC posterior estimation, hypothesis testing, credible region, and prediction, and the corresponding finite-sample and large-sample properties for the proposed Bayesian regression model by utilizing the techniques of the traditional Bayesian inference.

(四)參考文獻。

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(五)計畫成果自評。

In the proposal of this project, I would like to propose the Bayesian inference under the general response transformation truncated normal heteroscedastic regression model (4) proposed in Chen and Wang (2003). However, I found that it is nearly impossible to do the posterior inference utilizing the likelihood function proposed in Chen and Wang (2003). The main reason is that there is no closed-form formula for its likelihood function. So I need to use a numerical iteration method to calculate the likelihood function at many possible θ values. Even though I have calculated the likelihood function at several possible θ values, it only gives me a very rough picture of the likelihood function rather than the whole likelihood function. Afterwards, I found a simple way to modify the general response transformation truncated normal heteroscedastic regression model (4) proposed in Chen and Wang (2003) in order to have a closed-form formula for its likelihood function. Finally, the corresponding MCMC posterior estimation, hypothesis testing, credible region, and prediction, and the corresponding finite-sample and large-sample properties for the proposed Bayesian regression model can be easily derived via the techniques of the traditional Bayesian inference.

In the literature, all Box-Cox response transformation Bayesian regression models are misleading because they used wrong likelihood functions. They assumed that the Box-Cox transformed response is normally distributed. However, it cannot be normally distributed because of the support constraint. In this project, we assume that the transformed response is truncated normally distributed to satisfy the support constraint. So all results in this project are correct and can be applied to response transformation Bayesian regression models.

(六)可供推廣之研發成果資料表。

(七)附錄