Asset Substitution, Convertible Bonds, and Asymmetric Information

Abstract

This article provides an analytical pricing formula for a callable convertible bond with consideration of tax benefits, bankruptcy costs, bond maturities, and the capital structure of the bond issuer. Our structural model allows optimal strategies for call, voluntary conversion, and bankruptcy to be endogenously determined. The numerical results predict when the call redemption, the forced conversion, the voluntary conversion, and the bankruptcy of a callable convertible bond may occur. The literature findings of late calls associated with dividend payments and tax benefits are confirmed, and the hypothesis that using convertible bonds can reduce the asset substitution problem is also validated.

JEL Classification: G13, G32, G33

Keywords: Convertible Bond, Call Policy, Conversion Strategy, Endogenous Bankruptcy, Asset Substitution

中文摘要

本文建構一個可轉換公司債結構式模型，其中考慮稅盾，破產成本，債券到期日以及發行者之資本結構。模型中發行者之贖回與違約策略以及持有者之轉換策略皆由模型內生最適決定。由數值模擬分析發現下列結果：由於發行者可節省股利支出，或因舉債融資所產生之稅盾，所導致之延遲贖回行為與文獻上之實證結果一致。此外，發行可轉換公司債可有效減少資產替換之代理問題，亦符合既有之文獻實證結果。

關鍵詞：可轉換公司債，贖回政策，轉換策略，內生違約，資產替換

Introduction

Convertible bonds, spanning the dimensions from common stocks on the one hand to straight bonds on the other, are one of the most popular hybrid financing instruments. Most convertible bonds have call provisions, making the valuation and the determination of the optimal strategies for call and conversion more complicated. Similar to ordinary bondholders, investors of convertible bonds are entitled to receive coupon payments and principal payments, and thus the default risk of the bond issuer is also essential to the valuation of convertible bonds.

The pioneered work of Merton (1974) provides a structural model and explains how the risky debt can be viewed as a European contingent claim on the value of firm's assets. He further derives the closed-form valuation by using the Black-Scholes option pricing formula. Subsequently, Black and Cox (1976) first utilize the first-passage-time approach to extend Merton's model and consider the possibility that the bond issuer may default prior to the maturity. Leland (1994) further takes the tax benefits and the bankruptcy costs into account, which are viewed as the perpetual contingent claims on the unlevered asset value of a firm. By the pricing method of a perpetual American option, he provides the closed-form pricing formulas of these contingent claims, and furthermore, he uses the smooth-pasting condition to endogenize the bankruptcy strategy of the equityholders. Leland and Toft (1996), based on Leland (1994), use a (single) barrier option approach and construct a stationary debt structure[1] to price a finite-maturity coupon debt with consideration of endogenous bankruptcy.

As for the valuation of convertible bonds by a structural model, Ingersoll (1977a) first uses the Black-Scholes methodology and derives the closed-form pricing formula with some simplifying assumptions. In addition, he obtains the optimal call trigger which is equal to the call price multiplied by the conversion ratio, and shows that the conversion will occur only at the time of call or at the maturity of the bond in a perfect market. Meanwhile, Brenann and Schwartz (1977) price a more general convertible bond by the finite difference method where they solve a partial differential equation with more realistic boundary conditions. Subsequently, Brenann and Schwartz (1980) allow for stochastic interest rates and take consideration of the senior debt in the issuer's capital structure. Their numerical results suggest, in a striking manner, that for a reasonable range of interest rates, the errors from the certain interest rate model are likely to be small. For practical purposes, therefore, it may be preferable to use a simple model with the constant interest rate for valuing convertible bonds. Nyborg (1996) provides an excellent survey on the valuation of convertible bonds and reviews the reasons why firms issue convertible bonds. All of the works above focus on the case with a positive net-worth covenant in which bankruptcy is triggered when the firm's asset value falls to the total outstanding debt's principal value. Recently, Sarkar (2001) and Sarkar and Hong (2004), based on the endogenous bankruptcy framework of Leland (1994), price a callable corporate bond and analyze the call probability as well as the effective duration with consideration of tax benefits and bankruptcy costs. In addition, Sarkar (2003) explores early and late calls of convertible bonds still under the perpetual maturity setting of Leland (1994), which seems unreasonable. Moreover, Sarkar (2003) only considers the possibility of forced conversion when the call is triggered but neglects the possibility of voluntary conversion by bondholders.

Agency problems are a central concern in corporate finance. The pioneering works include Jensen and Meckling (1976), investigating optimal capital structure with agency costs, Myers (1977), exploring the underinvestment problem under risky debt financing, and Myers (1984), proposing the static tradeoff and financing pecking order hypotheses of optimal capital structure.

The asset substitution/risk shifting problem states that shareholders wish to increase the riskiness of firm's activities so as to transfer value from bondholders to themselves. For example, shareholders may adopt a riskier investment project with negative net present value (NPV). Some structural models, such as Merton (1974), explicitly regard the equity value as a call option on the firm's asset value due to the limited liability property. The asset substitution problem thus appears in such models due to the Vega of call options, that is, increasing the return volatility of the firm's asset will result in higher equity values. Barnea et al. (1980) explore this analogy and suggest that issuing shorter-term debts may reduce the incentives of shareholders to increase risk. In addition, the monitoring role of convertible debts in resolving risk shifting problems is also studied in the literature. For instance, Green (1984) shows that convertible bonds could be used to restore the positive NPV maximization rule of shareholders. Staring with Merton (1974), there are a lot of structural models developed to value prices of contingent claims or to analyze issues of corporate finance. Some of them are concerning about asset substitution problem of corporate bonds, but most of them only investigate such problem of straight bonds. For example, Ericsson (2000) provides quantitative illustrations of how the capital structure decision is affected by asset substitution problem. Recently, Ju and Ou-Yang (2006) examine the straight-bond-induced asset substitution problem within a dynamic structural model. Chesney and Gibson-Asner (2001) is the first work of quantifying the effect of convertible bonds on reconciling asset substitution problem. Similar to Merton (1974), they regard the equity as down-and-out options on the firm value, and in turn, show that the optimal volatility selection of shareholders is lower in the case where the firm is financed by convertible bonds. However, the call and conversion strategies of convertible bonds and bankruptcy decision of the firm does not endogenously determined by the model. On the other hand, Francois et al. (2006), by the Nash equilibrium of a non-cooperative sequential game between shareholders and convertible bondholders, show that convertible debt appears to be a poor risk mitigating tool. Nevertheless, the tax benefits and bankruptcy costs are not taken into account, which are of importance to convertible bonds.

This paper provides a simple but complete structural model to price a callable convertible bond with finite maturity using the pricing technique of double-barrier options, where the optimal strategies for call, voluntary conversion, and bankruptcy are endogenously determined by shareholders and bondholders. Our model not only takes tax benefits, bankruptcy costs, and bond maturities into account, but also considers the possibilities that the call, the voluntary conversion, and the bankruptcy may occur prior to the maturity of the bond. In addition, our numerical results predict that when the call redemption, the forced conversion, the voluntary conversion, and the bankruptcy of a callable convertible bond may happen. The empirical literature findings of late calls associated with dividend payments and tax benefits are confirmed in our numerical analyses, and furthermore, the hypothesis that shorter-term debts and convertible debts can be used to reduce the asset substitution agency problem is also numerically validated by our model.

The remainder of this paper is organized as follows. In Section 1, we set up the modeling framework. Section 2 is devoted to present the analytical valuation of a callable convertible bond. Next, we show the numerical results of the optimal strategies for call, voluntary conversion and bankruptcy, and analyze the prices of the callable convertible bond in Section 3. In Section 4, the asset substitution problem associated with convertible bonds is also examined. Finally, Section 5 summarizes the article and makes concluding remarks.

1. Valuation Framework

Consider a bond issuer (or an objective firm) where the callable convertible bond is the only senior issue, which continuously pays a constant coupon flow, C, with the finite time to maturity, T, and the par value, P. The other claim of the firm is the common share. Let $V(t)$ designate the unlevered asset value of the bond issuer at time t. The dynamics of $V(t)$ on the risk-neutral filtered probability space are given by

$$dV(t) = V(t)\Big((r-q)dt + \sigma dW^{Q}(t)\Big), \tag{1}$$

where r denotes the constant risk-free interest rate,[2] q is the constant payout ratio of the issuer, σ is the constant return volatility, and W^Q is a Wiener process.

As usual, if bondholders convert convertible bonds into common shares, then they will receive a fraction γ of the unlevered asset value of the issuer. Here we implicitly assume the conversion in our model is “block conversion”, that is, all the bondholders will convert the convertible bonds into the common shares at the same time. If the issuer of the callable convertible bonds calls back all outstanding callable convertible bonds at the same time, then all the bondholders have to immediately choose either to convert callable convertible bonds into common shares, or to receive the pre-specified call price (the redemption value), $(1+\beta)P$, where βP is the call premium of the callable convertible bonds.

At the initial time, assumed to be time zero for simplicity, we suppose that the upper constant call barrier, V_{Call}^0, and the upper constant conversion barrier, V_{Con}^0, are both greater than the initial unlevered asset value of the bond issuer, $V(0)$. As soon as the unlevered asset value of the bond issuer goes up and touches either V_{Call}^0 or V_{Con}^0, then either the call of the bond issuer or the voluntary conversion of the bondholders is triggered. Therefore, two first passage times can be further defined as $\tau_{Call}^0 \equiv \inf\left(t>0:V(t)\geq V_{Con}^0\right)$ and $\tau_{Con}^0 \equiv \inf\left(t>0:V(t)\geq V_{Con}^0\right)$, where τ_{Call}^0 and τ_{Con}^0 are the time that the bond issuer decides to call back the bonds and the time that the bondholders determine to voluntarily convert the bonds into common shares, respectively.

In addition to the results of being called or being voluntarily converted, there are still two other possible outcomes for callable convertible bonds. One is that the bond issuer declares bankruptcy prior to the time of the call, the time of the voluntary conversion, and the maturity of the bond; the other one is that callable convertible bonds mature and none of the call, the voluntary conversion and the bankruptcy occurs. Subsequently, another lower constant bankruptcy barrier is defined as V_B^0, which is less than $V(0)$. As soon as the unlevered asset value of the bond issuer goes down and touches V_B^0, the bankruptcy of the bond issuer is triggered. Once the bond issuer declares bankruptcy, the bondholders receive the recovery value, $(1-\alpha)V_B^0$, at the time of default, where α, between 0 and 1, is the ratio of bankruptcy costs or restructuring costs. Again, another first passage time can be denoted as $\tau_B^0 \equiv \inf\left(t>0:V(t)\leq V_B^0\right)$, where τ_B^0 is the time that the bond issuer announces bankruptcy. In the next section, we will endogenously determine the optimal strategies for call, voluntary conversion, and bankruptcy by taking the desired objectives of the bond issuer and the bondholders into consideration.

2. Pricing Callable Convertible Bonds with Default Risk

2.1. Pricing a non-callable convertible bond with default risk

For a non-callable convertible bond, the bond issuer can decide when to go bankrupt and the bondholders can determine when to voluntarily convert the bonds into common shares. Leland and Toft (1996) use a (single) barrier option approach to valuing a risky corporate coupon bond, motivating this article to use a double-barrier option approach to pricing a risky non-callable convertible bond. Similar to Leland and Toft (1996), the initial lower barrier, V_{B1}^0, represents the bankruptcy trigger of the issuer. We further denote the initial upper barrier, V_{Con}^0, as the voluntary conversion trigger of the bondholders. Initially, these two barriers are treated as exogenously given constants and will be endogenously determined later through the Nash-equilibrium argument.

Under our risk-neutral framework, the initial value of a non-callable convertible bond, $NCCB(0)$, can be written as

$$NCCB(0) = \mathrm{E}^Q\left[e^{-r\tau_{B1}^0} 1_{\left\{\tau_{B1}^0 < \tau_{Con}^0, \tau_{B1}^0 \le T\right\}} (1-\alpha) V_{B1}^0\right] + \mathrm{E}^Q\left[e^{-r\tau_{Con}^0} 1_{\left\{\tau_{Con}^0 < \tau_{B1}^0, \tau_{Con}^0 \le T\right\}} \gamma V_{Con}^0\right]$$

$$+\mathrm{E}^Q\left[e^{-rT} 1_{\left\{\min(\tau_{Con}^0, \tau_{B1}^0) > T\right\}} P\right] + \mathrm{E}^Q\left[\int_0^{\tau_{B1}^0 \wedge \tau_{Con}^0 \wedge T} Ce^{-rt} dt\right], \quad (2)$$

where $1_{\{A\}}$ denotes the indicator function with value 1 if event A occurs and with value zero otherwise, $t \wedge s \equiv \min(t,s)$, and $\tau_{B1}^0 \equiv \inf\left(t > 0 : V(t) \le V_{B1}^0\right)$ and $\tau_{Con}^0 \equiv \inf\left(t > 0 : V(t) \ge V_{Con}^0\right)$ stand for the time of the bankruptcy of the bond issuer and the time of the voluntary conversion of the bondholders, respectively.

We can then rewrite Equation (2) as

$$NCCB(0) = \frac{C}{r} + \mathrm{E}^Q\left[e^{-r\tau_{B1}^0} 1_{\left\{\tau_{B1}^0 < \tau_{Con}^0, \tau_{B1}^0 \le T\right\}} \left((1-\alpha) V_{B1}^0 - \frac{C}{r}\right)\right]$$

$$+\mathrm{E}^Q\left[e^{-rT} 1_{\left\{\min\left(\tau_{Con}^0, \tau_{B1}^0\right) > T\right\}} \left(P - \frac{C}{r}\right)\right] + \mathrm{E}^Q\left[e^{-r\tau_{Con}^0} 1_{\left\{\tau_{Con}^0 < \tau_{B1}^0, \tau_{Con}^0 \le T\right\}} \left(\gamma V_{Con}^0 - \frac{C}{r}\right)\right]. \quad (3)$$

Since V_{B1}^0 and V_{Con}^0 are two constants at the initial time, Equation (3) can be further simplified as follows:

$$NCCB(0)=\frac{C}{r}+\left((1-\alpha)V_{B1}^{0}-\frac{C}{r}\right)G_{\tau_{B1}^{0}}+\left(P-\frac{C}{r}\right)G_{T}+\left(\gamma V_{Con}^{0}-\frac{C}{r}\right)G_{\tau_{Con}^{0}}, \tag{4}$$

where

$$G_{\tau_{B1}^{0}}=\left(\frac{V_{B1}^{0}}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Con}^{0})^{2n}V(0)}{(V_{B1}^{0})^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+1}}{V(0)(V_{Con}^{0})^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B1}^{0})^{2n+1}}{(V_{Con}^{0})^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+1}}{V(0)(V_{Con}^{0})^{2n}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Con}^{0})^{2n+2}}{(V_{B1}^{0})^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+1}V(0)}{(V_{Con}^{0})^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$\left.\left.+\left(\frac{(V_{B1}^{0})^{2n+1}V(0)}{(V_{Con}^{0})^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+1}V(0)}{(V_{Con}^{0})^{2n+2}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]\right\},$$

$$G_{T}=e^{-rT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{V_{Con}^{0}}{V_{B1}^{0}}\right)^{\frac{2n\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n}}{V(0)(V_{Con}^{0})^{2n-1}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+1}}{V(0)(V_{Con}^{0})^{2n}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\right.$$

$$\left.-\left(\frac{(V_{B1}^{0})^{n+1}}{(V_{Con}^{0})^{n}V(0)}\right)^{\frac{2\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{Con}^{0})^{2n+1}V(0)}{(V_{B1}^{0})^{2n+2}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{Con}^{0})^{2n}V(0)}{(V_{B1}^{0})^{2n+1}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\right\},$$

$$G_{\tau_{Con}^{0}}=\left(\frac{V_{Con}^{0}}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Con}^{0})^{2n+1}}{(V_{B1}^{0})^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n}V(0)}{(V_{Con}^{0})^{2n+1}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B1}^{0})^{2n}V(0)}{(V_{Con}^{0})^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n}V(0)}{(V_{Con}^{0})^{2n+1}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Con}^{0})^{2n+1}V(0)}{(V_{B1}^{0})^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+2}}{(V_{Con}^{0})^{2n+1}V(0)}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$+\left(\frac{(V_{B1}^{0})^{2n+2}}{(V_{Con}^{0})^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B1}^{0})^{2n+2}}{(V_{Con}^{0})^{2n+1}V(0)}+\lambda^* T}{\sigma\sqrt{T}}\right)\Bigg]\Bigg]\Bigg\}.$$

In what follows, we take the initial tax benefits of future coupon payments, $TB(0)$, and the initial value of potential bankruptcy costs, $BC(0)$, as two contingent claims upon the unlevered asset value of the firm. By risk-neutral valuation method, the cumulative discounted tax benefits at the initial time can be represented by

$$TB(0)=\mathrm{E}^{Q}\left[\int_{0}^{\tau_{B1}^{0}\wedge\tau_{Con}^{0}\wedge T}\tau C e^{-rt}dt\right]=\mathrm{E}^{Q}\left[\frac{\tau C}{r}\left(1-e^{-r(\tau_{B1}^{0}\wedge\tau_{Con}^{0}\wedge T)}\right)\right]=\frac{\tau C}{r}\left[1-G_{\tau_{B1}^{0}}-G_{\tau_{Con}^{0}}-G_{T}\right],$$

(5)

where τ is the constant corporate tax rate. Similarly, the discounted bankruptcy costs at the initial time can be written as

$$BC(0)=\mathrm{E}^{Q}\left[e^{-r\tau_{B1}^{0}}\alpha V\left(\tau_{B1}^{0}\right)1_{\{0<\tau_{B1}^{0}\le T\}}\right]=\alpha V_{B1}^{0}\mathrm{E}^{Q}\left[e^{-r\tau_{B1}^{0}}1_{\{0<\tau_{B1}^{0}\le T\}}\right]=\alpha V_{B1}^{0}F_{\tau_{B1}^{0}}, \quad (6)$$

Where $F_{\tau_{B1}^{0}}=\left(\frac{V(0)}{V_{B1}^{0}}\right)^{\frac{\lambda^*-\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{V_{B1}^{0}}{V(0)}-\lambda^* T}{\sigma\sqrt{T}}\right)+\left(\frac{V(0)}{V_{B1}^{0}}\right)^{\frac{-2\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{V_{B1}^{0}}{V(0)}+\lambda^* T}{\sigma\sqrt{T}}\right)\right].$

The initial total firm value, $F_{NCCB}(0)$, is therefore equal to the initial unlevered asset value plus the initial tax benefits and less the initial value of the potential bankruptcy costs, i.e., $F_{NCCB}(0)=V(0)+TB(0)-BC(0)$. Since the accounting identity of the balance sheet states that the total firm value must equal to the sum of the equity value and the liability value, the initial equity value of the bond issuer, $E_{NCCB}(0)$, must equal to the initial total firm value minus the initial value of the non-callable convertible bond, i.e., $E_{NCCB}(0)=F_{NCCB}(0)-NCCB(0)$.

To endogenize the optimal voluntary conversion policy, V_{Con}^{*}, and the optimal bankruptcy strategy, V_{B1}^{*}, we first apply the following smooth-pasting conditions to determine the initial constant voluntary conversion trigger, $V_{Con}^{*,0}$, and the initial constant bankruptcy trigger, $V_{B1}^{*,0}$.

$$\left.\frac{\partial E_{NCCB}(0)}{\partial V(0)}\right|_{V(0)=V_{B1}^{*,0}}=\frac{\partial E_{NCCB}(0)\,|_{V(0)=V_{B1}^{*,0}}}{\partial V_{B1}^{*,0}}=0, \quad (7)$$

$$\left.\frac{\partial NCCB(0)}{\partial V(0)}\right|_{V(0)=V_{Con}^{*,0}} = \frac{\partial NCCB(0)\,|_{V(0)=V_{Con}^{*,0}}}{\partial V_{Con}^{*,0}} = \gamma \,. \tag{8}$$

These two conditions represent that at the initial time, the shareholders choose $V_{B1}^{*,0}$ to maximize the equity value, and the bondholders determine $V_{Con}^{*,0}$ to maximize the value of the non-callable convertible bond, respectively.[3] Furthermore, the Nash-equilibrium argument is employed to endogenously determine the optimal strategies for the voluntary conversion and the bankruptcy. Given any $V_{Con}^{*,0}$, the shareholders determine the optimal bankruptcy strategy as a function of $V_{Con}^{*,0}$, denoted as $V_{B1}^{*,0}\left(V_{Con}^{*,0}\right)$; on the other hand, given any $V_{B1}^{*,0}$, the bondholders also decide the optimal conversion strategy as a function of $V_{B1}^{*,0}$, denoted as $V_{Con}^{*,0}\left(V_{B1}^{*,0}\right)$. Under the assumption that both the shareholders and the bondholders are fully informed, the optimal (Nash-equilibrium) strategies for the voluntary conversion and the bankruptcy can be obtained by jointly solving Equations (7) and (8) numerically.

2.2. Pricing a call-forcing convertible bond with default risk

Consider a call-forcing convertible bond, where the bond issuer can decide when to go bankrupt and when to call the bonds back, and the bondholders, however, can not convert voluntarily. Once the bond issuer announces to call the bonds, the bondholders can, at the same time, choose either to accept and then receive the redemption price, or to be forced to convert the bond into the common shares. The risk-neutral pricing method implies that the initial value of a call-forcing convertible bond, $CFCB(0)$, can be written as

$$CFCB(0) = \mathrm{E}^Q\left[e^{-r\tau_{B2}^0} 1_{\left\{\tau_{B2}^0<\tau_{Call}^0,\,\tau_{B2}^0\le T\right\}}(1-\alpha)V_{B2}^0\right] + \mathrm{E}^Q\left[e^{-rT} 1_{\left\{\min\left(\tau_{Call}^0,\,\tau_{B2}^0\right)>T\right\}} P\right]$$
$$+\mathrm{E}^Q\left[e^{-r\tau_{Call}^0} 1_{\left\{\tau_{Call}^0<\tau_{B2}^0,\,\tau_{Call}^0\le T\right\}} \max\left(\gamma V_{Call}^0,(1+\beta)P\right)\right] + \mathrm{E}^Q\left[\int_0^{\tau_{B2}^0\wedge\tau_{Call}^0\wedge T} Ce^{-rt}dt\right], \tag{9}$$

where $\tau_{B2}^0 \equiv \inf\left(t>0 : V(t)\le V_{B2}^0\right)$ and $\tau_{Call}^0 \equiv \inf\left(t>0 : V(t)\ge V_{Call}^0\right)$ stand for the time of bankruptcy and the time of call, respectively. Since V_{B2}^0 and V_{Call}^0 are assumed to be two constants initially, we can also simplify Equation (9) as follows:

$$CFCB(0)=\frac{C}{r}+\left((1-\alpha)V_{B2}^{0}-\frac{C}{r}\right)H_{\tau_{B2}^{0}}+\left(P-\frac{C}{r}\right)H_{T}$$

$$+\left(\max\left(\gamma V_{Call}^{0},(1+\beta)P\right)-\frac{C}{r}\right)H_{\tau_{Call}^{0}}, \qquad (10)$$

where

$$H_{\tau_{B2}^{0}}=\left(\frac{V_{B2}^{0}}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Call}^{0})^{2n}V(0)}{(V_{B2}^{0})^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+1}}{V(0)(V_{Call}^{0})^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B2}^{0})^{2n+1}}{(V_{Call}^{0})^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+1}}{V(0)(V_{Call}^{0})^{2n}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Call}^{0})^{2n+2}}{(V_{B2}^{0})^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+1}V(0)}{(V_{Call}^{0})^{2n}}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$\left.\left.+\left(\frac{(V_{B2}^{0})^{2n+1}V(0)}{(V_{Call}^{0})^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+1}V(0)}{(V_{Call}^{0})^{2n+2}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]\right\},$$

$$H_{T}=e^{-rT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{V_{Call}^{0}}{V_{B2}^{0}}\right)^{\frac{2n\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n}}{V(0)(V_{Call}^{0})^{2n-1}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+1}}{V(0)(V_{Call}^{0})^{2n}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\right.$$

$$\left.-\left(\frac{(V_{B2}^{0})^{n+1}}{(V_{Call}^{0})^{n}V(0)}\right)^{\frac{2\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{(V_{Call}^{0})^{2n+1}V(0)}{(V_{B2}^{0})^{2n+2}}-\lambda T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{\ln\frac{(V_{Call}^{0})^{2n}V(0)}{(V_{B2}^{0})^{2n+1}}-\lambda T}{\sigma\sqrt{T}}\right)\right]\right\},\text{ and}$$

$$H_{\tau_{Call}^{0}}=\left(\frac{V_{Call}^{0}}{V(0)}\right)^{\frac{\lambda}{\sigma^2}}\sum_{n=0}^{\infty}\left\{\left[\left(\frac{(V_{Call}^{0})^{2n+1}}{(V_{B2}^{0})^{2n}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n}V(0)}{(V_{Call}^{0})^{2n+1}}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{(V_{B2}^{0})^{2n}V(0)}{(V_{Call}^{0})^{2n+1}}\right)^{\frac{\lambda^*}{\sigma^2}}\right.\right.$$

$$\left.\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n}V(0)}{(V_{Call}^{0})^{2n+1}}+\lambda^*T}{\sigma\sqrt{T}}\right)\right]-\left[\left(\frac{(V_{Call}^{0})^{2n+1}V(0)}{(V_{B2}^{0})^{2n+2}}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^{0})^{2n+2}}{(V_{Call}^{0})^{2n+1}V(0)}-\lambda^*T}{\sigma\sqrt{T}}\right)\right.$$

$$+\left(\frac{(V_{B2}^0)^{2n+2}}{(V_{Call}^0)^{2n+1}V(0)}\right)^{\frac{\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{(V_{B2}^0)^{2n+2}}{(V_{Call}^0)^{2n+1}V(0)}+\lambda^*T}{\sigma\sqrt{T}}\right)\Bigg]\Bigg]\Bigg\}.$$

Similar to the case of the non-callable convertible bond, the various risk-neutral probabilities can also be calculated in this case. The total firm value in the call-forcing convertible bond case is expressed as

$$F_{CFCB}(0)=V(0)+\frac{\tau C}{r}\left[1-H_{\tau_{B2}^0}-H_{\tau_{Call}^0}-H_T\right]+\tau\beta P 1_{\{\gamma V_{Call}^0<(1+\beta)P\}}F_{\tau_{Call}^0}-\alpha V_{B2}^0 F_{\tau_{B2}^0}\,, \tag{11}$$

where $F_{\tau_{Call}^0}=\left(\frac{V_{Call}^0}{V(0)}\right)^{\frac{\lambda-\lambda^*}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{V(0)}{V_{Call}^0}+\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{V_{Call}^0}{V}\right)^{\frac{2\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{V(0)}{V_{Call}^0}-\lambda^*T}{\sigma\sqrt{T}}\right)\right]$, and

$$F_{\tau_{B2}^0}=\left(\frac{V(0)}{V_{B2}^0}\right)^{\frac{\lambda^*-\lambda}{\sigma^2}}\left[\Phi\left(\frac{\ln\frac{V_{B2}^0}{V(0)}-\lambda^*T}{\sigma\sqrt{T}}\right)+\left(\frac{V(0)}{V_{B2}^0}\right)^{\frac{-2\lambda^*}{\sigma^2}}\Phi\left(\frac{\ln\frac{V_{B2}^0}{V(0)}+\lambda^*T}{\sigma\sqrt{T}}\right)\right].$$

We are now going to determine the optimal call and bankruptcy strategies for the bond issuer who initially chooses these optimal policies by the corresponding smooth-pasting conditions as given by

$$\left.\frac{\partial E_{CFCB}(0)}{\partial V(0)}\right|_{V(0)=V_{B2}^{*,0}}=\frac{\partial E_{CFCB}(0)\,|_{V(0)=V_{B2}^{*,0}}}{\partial V_{B2}^{*,0}}=0\,, \tag{12}$$

$$\left.\frac{\partial E_{CFCB}(0)}{\partial V(0)}\right|_{V(0)=V_{Call}^{*,0}}=\frac{\partial E_{CFCB}(0)\,|_{V(0)=V_{Call}^{*,0}}}{\partial V_{Call}^{*,0}}=\begin{cases}1-\gamma, & \text{if } \gamma V_{Call}^{*,0}\geq(1+\beta)P.\\ 1, & \text{if } \gamma V_{Call}^{*,0}<(1+\beta)P.\end{cases} \tag{13}$$

Based on the same arguments of the previous section, jointly solving $V_{Call}^*(0)$ and $V_{B2}^*(0)$ from Equations (12) and (13) and then substituting them back into Equation (10), we complete the analytical valuation of a call-forcing convertible bond with consideration of the issuer's default risk.

2.3. Pricing a callable convertible bond with default risk

To price a callable convertible bond, we have to determine its optimal strategies for the call, the voluntary conversion, and the bankruptcy. For this purpose, there are two essential assumptions which will be summarized below and then interpreted in the next paragraph. We first assume that the possibility of a voluntary conversion (a call) does not affect the optimal call policy (the optimal voluntary conversion strategy).

This assumption, which ensures the uncorrelation between the optimal voluntary conversion strategy and the optimal call policy of the callable convertible bond, is used to keep the model tractable. The optimal voluntary conversion trigger of the non-callable convertible bond, and the optimal call trigger of the call-forcing convertible bond are therefore applied to the callable convertible bond, where all three bonds are otherwise identical. In addition, we further suppose that the optimal bankruptcy trigger of the call-forcing convertible bond is the same as that of the callable convertible bond which is otherwise the same. It can be concluded that the optimal voluntary conversion trigger of the non-callable convertible bond, and the optimal call and bankruptcy triggers of the call-forcing convertible bond are employed to the callable convertible bond, where all three bonds are otherwise identical. The analytical valuation of a callable convertible bond subject to the default risk of the bond issuer can therefore be expressed as follows:

$$CCB\left(0;V_{B2}^{*}(0),V_{Con}^{*}(0),V_{Call}^{*}(0)\right)=\begin{cases} NCCB\left(0;V_{B2}^{*}(0),V_{Con}^{*}(0)\right), \text{ if } V_{Con}^{*}(0)\le V_{Call}^{*}(0). \\ CFCB\left(0;V_{B2}^{*}(0),V_{Call}^{*}(0)\right), \text{ if } V_{Con}^{*}(0)> V_{Call}^{*}(0). \end{cases} \tag{14}$$

3. Numerical Examples

3.1. Optimal strategies for call and voluntary conversion

The parameters in the base case, taken from Sarkar (2003), are as follows: $P=100$, $C=7$, $\tau=0.35$, $\alpha=0.5$, $r=0.07$, $q=0.04$, $\sigma=0.2$, $\beta=0.05$, $\gamma=0.2$, and $T=5$.[5] All parameters in this article are the same as the base case unless otherwise stated. Also notice that in the numerical analyses of this article, the desired pricing formula of the callable convertible bond, involving some infinite sums (from zero to infinity and from negative infinity to infinity), has been replaced with the finite sums (assumed from zero to ten and from minus ten to ten, respectively).[6]

Figure 1 illustrates the optimal call triggers as a function of the time to maturity for various return volatilities of the unlevered asset. The optimal call trigger is an increasing function of the time to maturity when the time to maturity becomes shorter, and is a decreasing function of the time to maturity otherwise, that is, the optimal call trigger is concave to the time to maturity. The concavity is more obvious as the return volatility becomes larger, and therefore, the riskier bond issuer will call back the bond at higher unlevered asset value, which results in late calls.

The optimal voluntary conversion triggers, plotted in Figures 2, behave much similar to the optimal call triggers in Figures 1. Some implications of our model are discussed as follows. The optimal voluntary conversion triggers are usually greater than the optimal call triggers for the most part, that is, the voluntary conversion will not occur in most of the cases. Nevertheless, our model predicts that for a callable convertible bond with very low coupon payments or with shorter time to maturity, smaller return volatility, and higher risk-free interest rate, the voluntary conversion may happen. In addition, the numerical results also show that when the call is triggered, the forced conversion occurs in most of the cases, especially for the case of lower call premium. However, for a callable convertible bond with higher call premium, higher coupon payment, shorter time to maturity, smaller return volatility and intermediate risk-free interest rate, the call redemption may take place.

3.2. Optimal bankruptcy strategy

Figure 3 plots the optimal bankruptcy trigger as a function of the time to maturity for various return volatilities of the unlevered asset. Observe that the optimal bankruptcy trigger is a decreasing function of the time to maturity and is concave to the time to maturity. In addition, similar to Leland and Toft (1996), the greater the return volatility, the lower the optimal bankruptcy trigger due to the limited liability of the equityholders.

3.3. Values of the callable convertible bond

Figure 4 shows the values of a callable convertible bond as a joint function of the unlevered asset value and the time to maturity. The value of a callable convertible bond is a non-decreasing function of both the unlevered asset value and the time to maturity. For lower unlevered asset value, the callable convertible bond value, concave to the unlevered asset value, is analogous to the price of a risky coupon bond because the possibilities of the call and the voluntary conversion are extremely small. On the other hand, for higher unlevered asset value, the callable convertible bond value, convex to the unlevered asset value, is similar to the equity value due to increases in the possibilities of the call and the voluntary conversion. Moreover, when the time to maturity is short and the unlevered asset value is in the middle range, the callable convertible bond value, similar to the risk-free coupon bond value, is very close to the par value, which equals to 100 in our base case. This is because the events of call, voluntary conversion, and bankruptcy rarely happen in this case.

Figure 5 illustrates the prices of a callable convertible bond as a function of the unlevered asset value for various return volatilities. Not only will greater return volatilities increase the probability of the bankruptcy but also will raise the probabilities of the call and the voluntary conversion. For lower unlevered asset value, the former effect is dominant and thus the callable convertible bond value decreases as the return volatility goes up. On the other hand, for higher unlevered asset value, the latter effect dominates and the callable convertible bond value, therefore, increases with rising return volatilities. In addition, related to our earlier discussion in Figure 7, we observe that higher and lower unlevered asset values make the callable convertible bond behave like the equity and the risky coupon bond, respectively. As a result, under higher unlevered asset value, an increase in the volatility can raise the price of a callable convertible bond due to the property of the equity value. On the other hand, the callable convertible bond acts as the risky coupon bond under lower unlevered asset value and thus the higher the return volatility, the lower the price of a callable convertible bond.

Table 1 exhibits the values of the callable convertible bond for varying coupon payments, risk-free interest rates, and unlevered asset values. An increase in the coupon payment can raise the value of the callable convertible bond in most of the cases, which accords well with the intuition. However, there are some significant exceptions when the unlevered asset is 700, the coupon payment equals to 0, 1, and 3 among all various risk-free interest rates (excluding the case of $V = 700$, $C = 3$ and $r = 0.01$). In view of Figure 2, we can observe that in these exceptional cases, the optimal call triggers are less than 700, and thus the callable convertible bond has been called back and will be forced to convert into common shares. As a result, it is similar to the equity whose value falls as the coupon payment increases.

4. Asset Substitution

To clearly illustrate whether issuing convertible bonds instead of coupon bonds can reduce the risk shifting problem, we first consider a risky coupon bond as a sole debt obligation in our framework. Following the same methodology in Section 2, the risky coupon bond price, the total firm value, and the equity value can then be obtained, and the optimal bankruptcy strategy can also be endogenously determined by the corresponding smooth-pasting condition, which is similar to Equation (7) or Equation (12). Moreover, we can define the risk shifting intensity as the partial derivative of the equity value with respect to the return volatility of the unlevered asset. As a consequence, the positive risk shifting intensity represents that shareholders have incentives to increase the riskiness of firm’s activities. Using the parameters of the base case in the previous section, we provide Figure 9 to compare the risk shifting intensities between the coupon-bond-based model (where the coupon bond is the only debt obligation) and the callable-convertible-bond-based model (where the callable convertible bond is the only debt of the firm, i.e., the same model as in Section 2.3).

Figure 6 plots the risk shifting intensities as a function of the unlevered asset value. Panels 1-1 and 1-2 plot the risk shifting intensities of the coupon-bond-based model with the time to maturities of 6 months and 5 years, respectively. Panels 2-1 and 2-2 plot the risk shifting intensities of the callable-convertible-bond-based model with the time to maturities of 6 months and 5 years, respectively. Observe that (i) Panels 1-1 and 1-2 display that the risk shifting intensities approach to zero as the unlevered asset value goes up, that is, there is almost no asset substitution problem when the default risk is rather small in the coupon-bond-based model; (ii) shorter time to maturities will reduce the asset substitution problem both in the coupon-bond-based model and the callable-convertible-bond- based model, which is generally consistent with Barnea et al. (1980); (iii) the hypothesis that callable convertible bonds can be used to resolve the risk shifting problem is numerically validated by comparing Panels 1-1 and 1-2 with the corresponding Panels 2-1 and 2-2; (iv) Panels 2-1 and 2-2 show that positive risk shifting intensities appear again as the unlevered asset value becomes higher since these callable convertible bonds have been called back and forced to convert into common shares.

5. Concluding Remarks

In this article, we construct a structural model to derive the analytical valuation of a callable convertible bond by the pricing method of double-barrier options with consideration of the possibilities that the call, the voluntary conversion, and the bankruptcy can occur prior to the maturity of the bond. Our model also takes the bankruptcy costs, the tax benefits, and the time to maturity of the bond into account. Not only are the optimal call and bankruptcy strategies endogenously determined by the shareholders as the equity value is maximized, but also the optimal voluntary conversion strategy is obtained by the bondholders while the value of the convertible bond is maximized.

In summary, our numerical results predict that (i) late calls are in most of the cases, and higher coupon, lower risk-free interest rate, greater return volatility, and medium time to maturity will lead to late calls where the optimal call triggers become extraordinarily high, which is generally consistent with Ederington et al. (1997); (ii) the voluntary conversion may occur in the cases of the callable convertible bond with very low coupon payment, or with shorter time to maturity, smaller return volatility, and higher risk-free interest rate; (iii) when the call is triggered, the forced conversion usually happens whereas the call redemption may take place in the case of the callable convertible bond with greater call premium, higher coupon payment, shorter time to maturity, smaller return volatility and intermediate risk-free interest rate. In addition, our model confirms that shorter-term bonds are useful to reconcile the asset substitution problem, which is consistent with Barnea et al. (1980). Furthermore, the hypothesis that callable convertible bonds can be used to reduce the risk shifting problem is also validated.

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Table 1

Values of the callable convertible bond for various coupon payments *C* risk-free interest rates *r*, and unlevered asset values *V* [*]

		$C=0$	$C=1$	$C=3$	$C=5$	$C=7$	$C=9$
$V=100$	$r=0.01$	70.1304	79.8774	99.9895	118.831	129.262	139.016
	$r=0.03$	76.0017	78.064	81.3014	83.4202	84.5099	84.6794
	$r=0.05$	73.8349	76.6368	81.1219	84.0518	85.3629	84.9772
	$r=0.07$	69.1157	72.5535	78.6639	83.6004	87.1546	89.0153
	$r=0.09$	63.3819	67.0840	74.0637	80.3516	85.7869	90.0955
$V=400$	$r=0.01$	85.2181	104.093	115.312	121.729	130.018	139.270
	$r=0.03$	85.6840	92.7388	105.112	114.616	122.886	130.972
	$r=0.05$	85.1181	89.3716	97.9861	106.110	113.915	121.681
	$r=0.07$	84.4142	87.4606	93.8555	100.333	106.946	113.800
	$r=0.09$	83.5488	85.9600	91.0112	96.2024	101.657	107.494
$V=700$	$r=0.01$	172.525	136.598	155.878	155.601	150.698	150.474
	$r=0.03$	170.140	146.043	138.438	146.410	155.064	161.722
	$r=0.05$	157.296	145.580	138.539	141.096	146.529	152.405
	$r=0.07$	151.048	143.882	139.065	140.444	144.093	148.402
	$r=0.09$	148.521	142.986	139.323	140.526	143.488	146.918

[*] All parameters in this table are the same as the base case in the text unless otherwise noted, and the optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously.

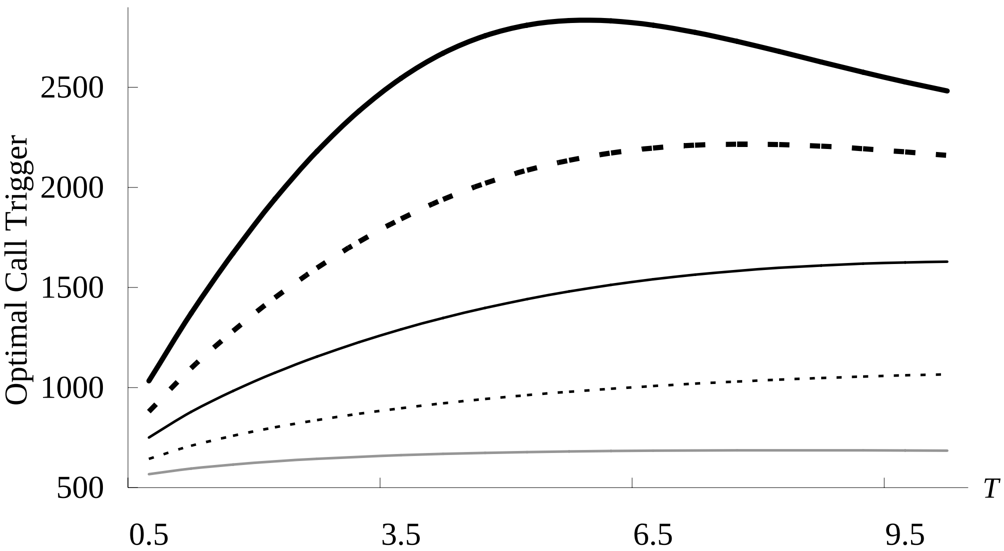

Figure 1

Optimal call triggers as a function of the time to maturity T for various return volatilities σ

The lines plot the optimal call triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

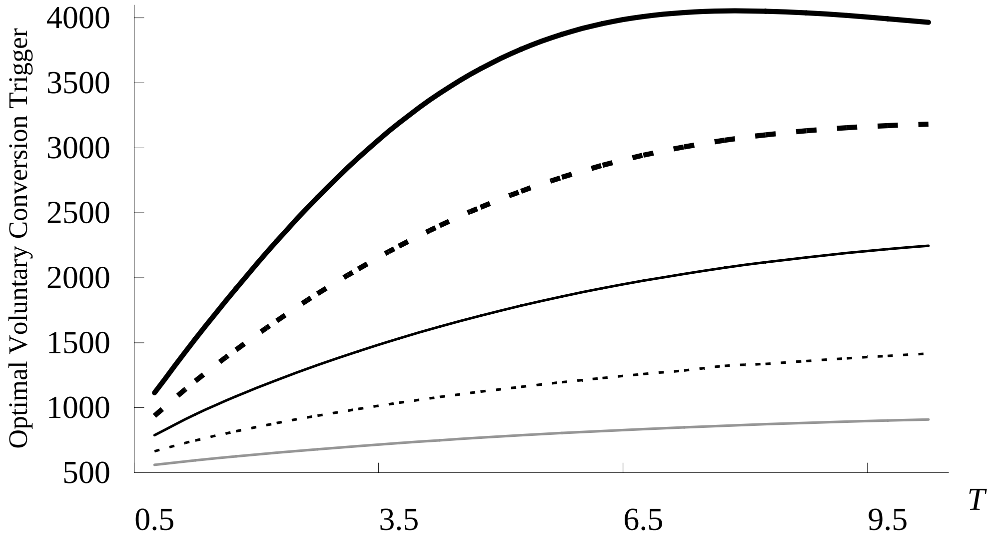

Figure 2

Optimal voluntary conversion triggers as a function of the time to maturity T for various return volatilities σ

The lines plot the optimal voluntary conversion triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

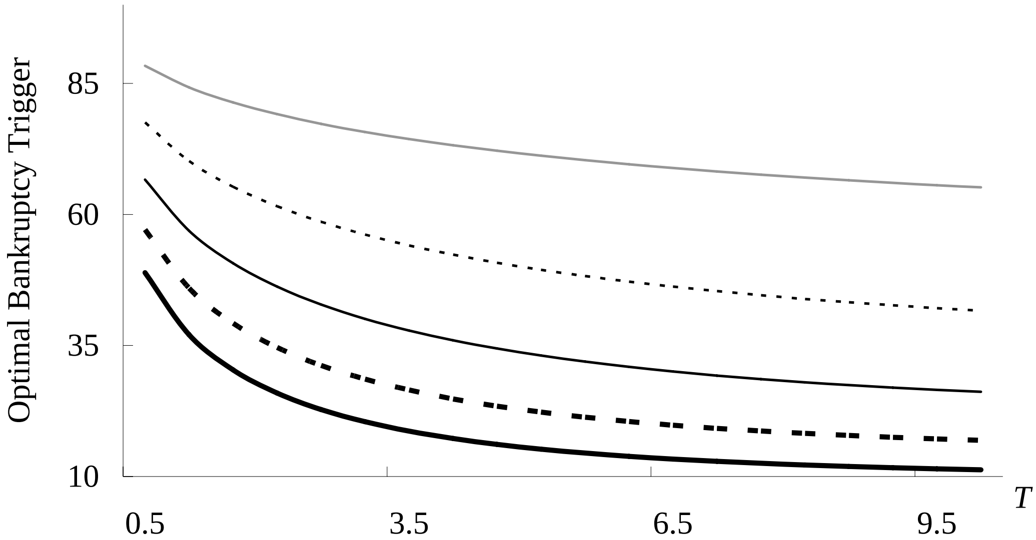

Figure 3

Optimal bankruptcy triggers as a function of the time to maturity *T* for various return volatilities σ

The lines plot the optimal bankruptcy triggers as a function of the time to maturity with return volatilities of 0.1 (gray line), 0.3 (dashed line), 0.5 (solid line), 0.7 (bold dashed line), and 0.9 (bold solid line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, and $\gamma = 0.2$.

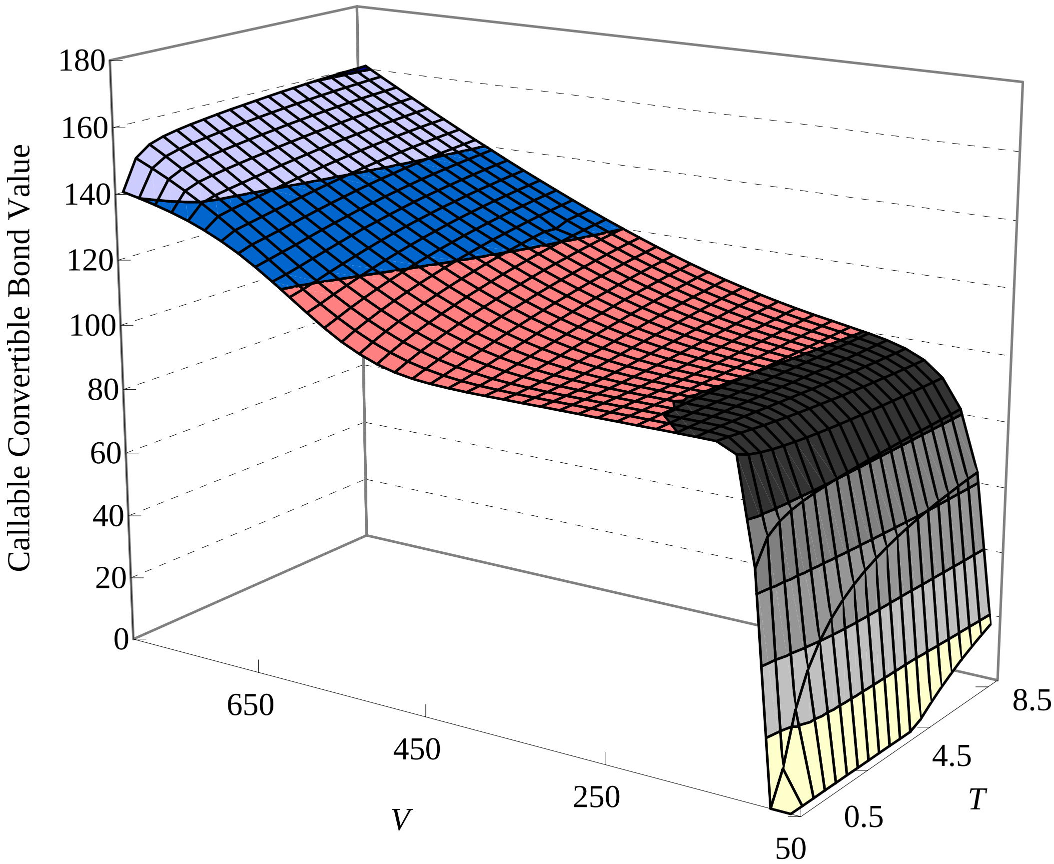

Figure 4

Values of the callable convertible bond as a joint function of the time to maturity *T* and the unlevered asset value *V*

The surface plots the value of the callable convertible bonds for varying levels of the unlevered asset values and the time to maturities. The optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously. The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\sigma = 0.2$, $\beta = 0.05$, and $\gamma = 0.2$.

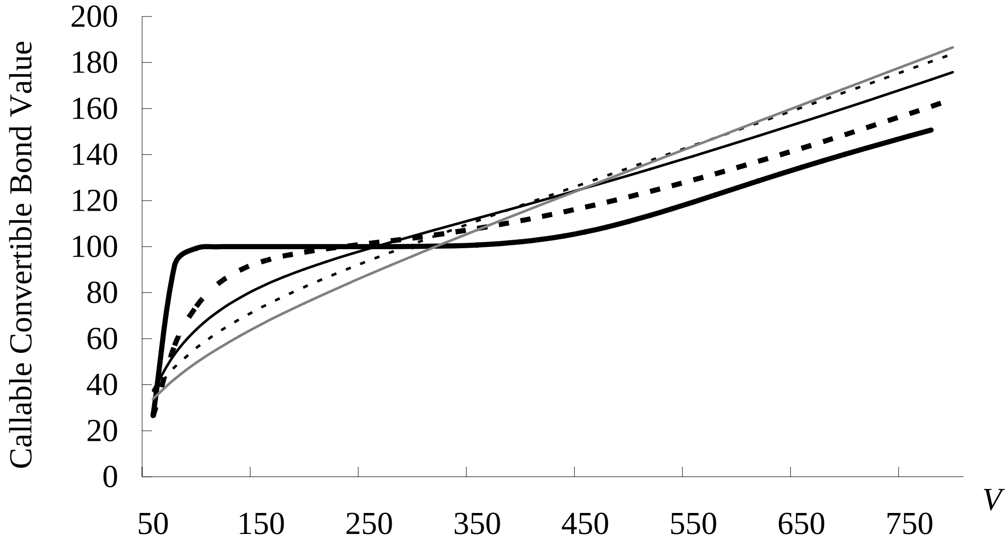

Figure 5

Values of the callable convertible bond as a function of the unlevered asset value V for various return volatilities σ

The lines plot the prices of the callable convertible bond as a function of the unlevered asset value with return volatilities of 0.1 (bold solid line), 0.3 (bold dashed line), 0.5 (solid line), 0.7(dashed line), and 0.9 (gray line). The parameters are given as follows: $P = 100$, $C = 7$, $\tau = 0.35$, $\alpha = 0.5$, $r = 0.07$, $q = 0.04$, $\beta = 0.05$, $\gamma = 0.2$, and $T = 5$.

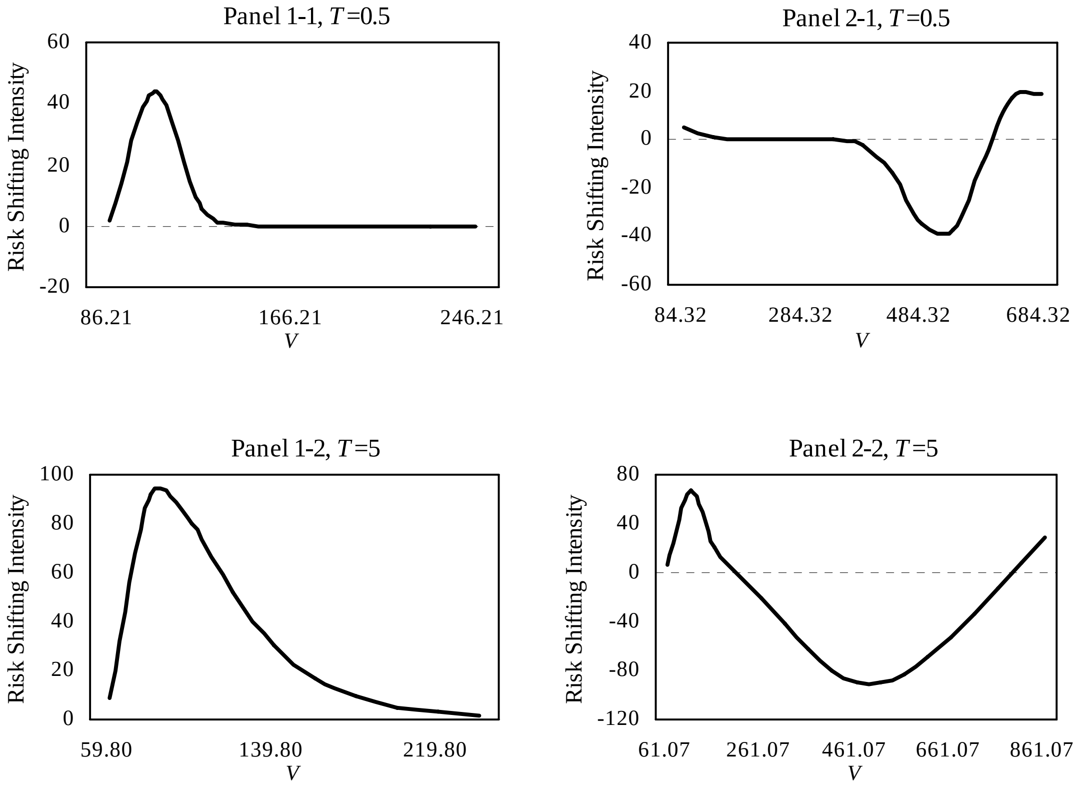

Figure 6

Risk shifting intensities as a function of the unlevered asset value *V* for the coupon-bond-based model and the callable-convertible-bond-based model

The panels plot risk shifting intensities, which stand for the partial derivatives of the equity value with respect to the return volatility, as a function of the unlevered asset value. Panels 1-1 and 1-2 show risk shifting intensities for the coupon-bond-based model (where the coupon bond is the only debt obligation) with the time to maturities 0.5 and 5, respectively. Panels 2-1 and 2-2 show risk shifting intensities for the callable-convertible-bond-based model (where the callable convertible bond is the only debt obligation) with the time to maturities 0.5 and 5, respectively. The optimal strategies for call, voluntary conversion, and bankruptcy are determined endogenously. The parameters are given as follows: $P=100$, $C=7$, $\tau=0.35$, $\alpha=0.5$, $r=0.07$, $q=0.04$, $\sigma=0.2$, $\beta=0.05$, $\gamma=0.2$, and $T=5$.

Evaluating the Results of the Project

1. The present results are consistent with most of the objectives of the proposed project, except for the asymmetric information between the convertible bond issuer and holders due to some technique difficulties.
2. The report here could provide some characteristics for call policy and the conversion strategy of a callable convertible bond, and guide the bond issuer and the bondholders to exercise their call and conversion options optimally.
3. After improving the modeling and writing contents presenting in the paper, I'm planning to submit the final result to an international academic journal in the next year.