行政院國家科學委員會補助專題研究計畫成果報告

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※ **模糊環境下之能力集合擴展** ※
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計畫主持人：會國雄　教授

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行政院國家科學委員會專題研究計畫成果報告

模糊環境下之能力集合擴展

Expanding Competence Sets in Fuzzy Environments

計畫編號：NCS 89-2416-H-009-003

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中文摘要

所謂能力集合擴展問題是指，在已知能力間的擴展成本下，找出一種有效的方法，如最低成本，將已經獲得的能力集擴展到解決問題真正所需的能力集。然而，在實際問題中，能力間的擴展成本經常是一模糊量。本研究即探討在以可能性分佈來表徵模糊擴展成本下，如何分析能力集擴展的問題。首先，本研究提出模糊最小成本擴展樹的概念，以及如何構建最小成本擴展樹之可能性分佈的演算法。接著，應用此演算法及MST**方法來求解模糊環境下，不含復合能力之最佳能力集擴展。最後，本研究以運輸規劃專業人員之能力集擴展為例說明。**

關鍵字：能力集分析、最佳擴展、模糊最小擴展樹、可能性分佈、習慣領域

Abstract. The problem of competence set expansion refers to finding an effective way, say with minimum cost, for expanding the already acquired skills set to the truly needed competence set with the given expansion cost among skills. However, in many practical problems, the expansion costs are characterized by fuzzy quantities. This paper aims at how to analyz the problem of competence set expansion with the fuzzy expansion cost represtented by possibility distribution. First, the concept of fuzzy minimum spanning tree is introduced, and the algorithm to construct the possibility distribution of minimum spanning tree is proposed. Next, we apply the algorithm and MST method to solve the problem of optimal competence set expansion without compound skills in fuzzy environments. Finally, theoretical support is accompanied by an illustrative example.

Keywords: competence set analysis, optimal expansion, fuzzy minimum spanning tree, possibility distribution, habitual domains

1. Introduction

Because of rapid change of our technology and environment, many problems we are facing are new, complex, and nontrivial. The solutions to these problems are usually outside our day-to-day experience, competence, or our habitual domains. Thus, they are fuzzy and challenging. In order to effectively solve this kind of fuzzy or challenging problems, we need to continually expand our competence or habitual domains so that we can make a good decision with confidence.

Competence set analysis was first introduced by Yu (1988) as an application part of habitual domains (Yu 1988, 1990). Its analytical applications and mathematical foundation was reported in Yu and Zhang (1989, 1990). Mathematical methods to attain more efficient ways to acquire the needed competence sets under various assumptions have been reported in many research (Yu and Zhang 1990, 1992; Li and Yu 1994; Shi and Yu 1996; Li et al. 2000). These previous research have been focus on finding the optimal expansion process of competence sets in non-fuzzy environment, that is the expansion cost among skills are crisp values.

However, in many practical problems, a decision maker or planner only partly knows the information about the expansion costs. For example, a veteran transportation planner may not give the exactly time for learning transportation demand forcasting for a green planner, instead he may only estimate the learnning time will be "long" according to his experience. In such case, we must use fuzzy theory to deal with the fuzziness of lingustic terms.

Wang and Wang (1998) reported the model of optimal expansion of a fuzzy competence set, which aimed at the fuzzy *background strength* that can be represented by the product of fuzzy proficiency of background competence and fuzzy background relation. For a given skill, *g*, if someone's background strength, $_g$, is larger than or equal to the critical level, θ_g, then the skill is called someone's *skill competence* with learnning cost being zero. Otherwise, the skill is called someone's *non-skill competence* with a certain learnning cost, which is a crisp value. Though Wang and Wang (1998) first suggested the fuzzy set concept to the problem of competence set expansion, they did not to deal with the problem of expansion cost with fuzzy quantities.

Since a competence set expansion process is similar to the construction of minimum spanning tree (Shi and Yu 1996, Feng and Yu 1998), thus the concept of constructing fuzzy minimum spanning tree can be applied to solve the expansion problem with fuzzy cost. Delgado et al. (1985) gave the definition and some properties of fuzzy tree based on the Rosenfeld's study. On the basis of the initial Rosenfeld's definition, Delgado et al. tried to specify the concept of connectedness by introducing the representation by α-cut. Moreover, the connectedness was represented by the relation taking value in [0,1]. Liu and Wang (1991) analyzed the shortest path with fuzzy quantity, represented by fuzzy number or possibility distribution, on edges, and proposed the method to construct the possibility distribution of shortest path.

Therefore, in this study, we shall analyze the optimal expansion process with fuzzy expansion cost represented by possibility distribution. The rest of this paper is organized as follows. Section 2 presents the basic concept of possibility theory, and then we proposed an algorithm to establish the possibility distribution of minimum spanning tree. Section 3 presents the process of solving competence set expansion with fuzzy cost. In section 4, a numerical example will be given to illustrat how to apply the proposed process. Finally, we give a conclusion.

2. Fuzzy Minimum Spanning Tree

In this section, we will first introduce some basic concepts of possibility theory. With the possibility theory, the fuzzy minimum spanning tree can be established from a graph with fuzzy quantities on edges. Then we propose an algorithm to construct the fuzzy minimum spanning tree.

2.1 Possibility Theory

Definition 1. [Zimmermann, 1991] Let $\tilde{F}$ be a fuzzy set of the universe U characterized by a membership function $\mu_{\tilde{F}}(u)$. $\tilde{F}$ is a *fuzzy restriction* on the variable *X*, denoted by $R(X)$, if $\tilde{F}$ acts as an elastic constraint on the values that may be assigned to *X*.

Definition 2. [Zadeh, 1978] Let X be a variable taking values in U and $\tilde{F}$ act as a fuzzy restriction, $R(X)$, associated with *X*. Then the proposition "X is $\tilde{F}$," which translates into $R(X)=\tilde{F}$ associates a *possibility distribution*, Π_X, with X which is postulated to be equal to $R(X)$.

According to the definition 2, the *possibility distribution function*, π_X, characterizing the possibility distribution Π_X, is defined to be numerically equal to the membership function $\mu_{\tilde{F}}(u)$ of $\tilde{F}$, that is $\pi_X \equiv \mu_{\tilde{F}}(u)$.

Definition 3. Let $\tilde{A}$ be a fuzzy set in the universe U and Π_X a possibility distribution associated with a variable X which takes values in *U*. The *possibility measure*, $\pi(A)$, of $\tilde{A}$ is then defined by

$$Poss(X \text{ is } \tilde{A}) \equiv \pi(\tilde{A})$$

$$\equiv \sup_{u\in U} \min\{\mu_{\tilde{A}}(u), \pi_x(u)\} \qquad (1)$$

With the definition 2, we give the multi-dimensional possibility distribution as follows:

Definition 4. Let $X=(X_1,\ldots,X_n)$ be a vector variable taking values in cartesian product $U=U_1\times\cdots\times U_n$, namely X_i takes value in U_i, i=1, …, *n*. Let $\tilde{F}$ act as a fuzzy restriction associated with X in *U*. Then the proposition "X is $\tilde{F}$," which translates into $R(X)=\tilde{F}$ associates an *n dimensional possibility distribution*, $\Pi_{(X_1,\ldots,X_n)}$, with X which is postulated to be equal to $R(X)$. Therefore, an *n dimensional possibility distribution function* is denoted by

$$\pi_{(X_1,\ldots,X_n)}(u_1,\ldots,u_n) \equiv \mu_{\tilde{F}}(u_1,\ldots,u_n) \qquad (2)$$

If $\tilde{F}$ is cartesian product of *n* one-dimension fuzzy restriction, $F_1,\ldots,F_n$ then the above equation can be write as follows:

$$\pi_{(X_1,\ldots,X_n)}(u_1,\ldots,u_n) \equiv \pi_{X_1}(u_1)\wedge\cdots\wedge\pi_{X_n}(u_n), \quad (3)$$

where $\pi_{X_i}(u_i)=\mu_{\tilde{F}_i}(u_i)$, $u_i\in U_i$, $i=1,\ldots,n$, and $\wedge$ represents the min operator.

2.2 Fuzzy Minimum Spanning Tree

Definition 5. [Katagiri and Ishii, 2000] Let $G=(N, E)$ denote undirected graph consisting of vertex set $N=\{v_1, v_2, \ldots, v_n\}$ and edge set $E=\{e_1, e_2, \ldots, e_m\} \subset N\times N$. Moreover cost c_j is attached to edge e_j. A *spanning tree* $T=(N, S)$ of G is a partial graph satisfying the following conditions: (i) T has same vertex set as G; (ii) $|S|=n-1$, where $|S|$ denotes the cardinality of set S; (iii) T is connected.

According to Cayley's theorem, the number of spanning trees for n distinct vertices is n^{n-2}. Assume each cost of edge of graph $G(N, S)$ is positive. The graph $T' = G'(N, S')$ whose total cost, $\sum_{j\in S'} c_j$, is minimum is called a *minimum spanning tree*.

In the fuzzy minimal spanning tree (FMST) problem, the cost, c_j, between each pair of vertexes are fuzzy quantities. The fuzzy quantity of cost can be represented in two ways: possibility distribution or fuzzy number.

For the way of possibility distribution, the cost of edges is regarded as a linguistic variable X taking value in a universe U. The value can be expressed in terms of either "high", "very high", etc, or "close to r", where r is a crisp number. Therefore, the cost of edges can be expressed as the proposition: "c_j is $\tilde{r}_j$," and the corresponding possibility distribution function is:

$\pi_{c_j}(x_j) \equiv \mu_{\tilde{r}_j}(u_j)$.

The other way to represent the fuzzy quantity of cost is using fuzzy number. The key point of this way is to give a membership function not only satisfying the definition of fuzzy number, but also suitably reflecting the features of the problem. In this paper we will use possibility distribution function to express the fuzzy quantity.

Now we can give the definition of fuzzy graph.

Definition 6. Let $\tilde{G}=(N, E)$ denote undirected graph consisting of vertex set $N=\{v_1, v_2, \ldots, v_n\}$ and edge set $E=\{e_1, e_2, \ldots, e_m\} \subset N\times N$, where each edge, e_j, is attached by fuzzy cost, $\tilde{c}_j$, associated with possibility distribution $\Pi_{\tilde{c}_j}$.

Definition 7. For a spanning tree $T(N, S)$, let the possibility distribution of cost of edge j be Π_{c_j}. Let the linguistic variable of edge's cost be denoted by X_j, $j\in S$. When they take values as $x_1, \ldots, x_j$, $j\in S$, the total cost of tree T is $\sum_{j\in S} x_j$. By the definition of multi-dimensional possibility distribution, it is follows that the possibility distribution of the tree T is

$$\Pi_{C^T}(y) = \min\{\Pi_{c_j}(x_j) \mid j\in S\} . \quad (4)$$

According to Cayley's theorem, there exist n^{n-2} spanning trees in a graph G. Let the set of spanning trees and minimum spanning trees be denoted by $K=\{T_1, \ldots, T_k\}$, $k= n^{n-2}$, and $Q=\{T_q | C^{T_q} = \min(C^{T_i}), T_i \in K\}$, respectively. Now we will give the definition of possibility distribution of cost of minimum spanning tree.

Definition 8. Let T' be a minimum spanning tree associated with total cost $C^{T'}$. Then the total cost of T' is constrained by the possibility distribution $\Pi_{C^{T'}}$. This possibility distribution can be determined by

$$\begin{aligned}\Pi_{C^{T'}} &= \sup_{T_i\in Q}\{\Pi_{C^{T_i}}\} \\ &= \sup_{T_i\in Q}\{\min(\Pi_{c_j}(x_j))\}.\end{aligned} \quad (5)$$

Next we propose the algorithm for constructing the possibility distribution of minimum spanning tree. The following are steps of the algorithm.

Step 1. Give the membership function, shown as Fig. 1, of each edge's cost as follows:

$$\mu_{c_j}(x_j) = \begin{cases} \dfrac{x_j - a_j}{m_j^1 - a_j}, & a_j \le x_j \le m_j^1 \\ 1, & m_j^1 < x_j < m_j^2 \\ \dfrac{x_j - b_j}{m_j^2 - b_j}, & m_j^2 \le x_j \le b_j \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

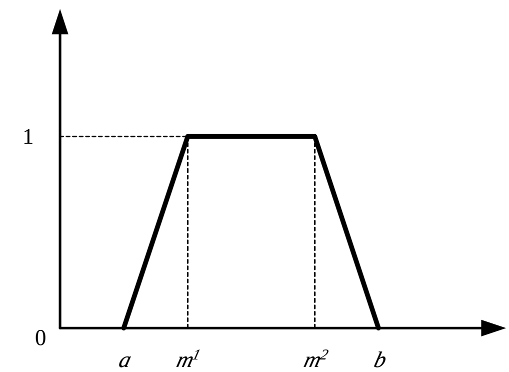

Fig. 1. Membership function

Step 2. Take $\alpha \in [0, 1]$, and calculate

$$L_j^*(\alpha) = (m_j^1 - a_j)\alpha + a_j , \quad (7)$$

$$U_j^*(\alpha) = (m_j^2 - b_j)\alpha + b_j . \quad (8)$$

Step 3. Construct the minimum spanning tree and calculate the total cost with edge's cost $L_j(\alpha)$ and $U_j(\alpha)$, respectively. The results are denoted by $C^{T'_L}(\alpha)$ and $C^{T'_U}(\alpha)$.

Step 4. Define the membership function as follows:

$$\mu_{C^{T'}}(x) = \begin{cases} \alpha, \ x = C^{T'_L}(\alpha), 0 \le \alpha < 1 \\ 1, \quad C^{T'_L}(1) \le x \le C^{T'_U}(1) \\ \alpha, \ x = C^{T'_U}(\alpha), 0 \le \alpha < 1 \\ 0, \quad \text{otherwise} \end{cases} \tag{9}$$

According to the definition 2, the possibility distribution of total cost of minimum spanning tree $\Pi_{C^{T'}}$ is equivalent to $\mu_{C^{T'}}(x)$.

Theorem 1. The membership function established by the algorithm is the possibility distribution of minimum spanning tree.

Proof: Since the membership function of fuzzy edge is shown as Fig. 1, there are five cases for edges to take value for a given $\alpha \in [0, 1]$

(i) $C^{T'} < C^{T'_L}(0)$,
(ii) $C^{T'_L}(0) \le C^{T'} < C^{T'_L}(1)$
(iii) $C^{T'_L}(1) \le C^{T'} \le C^{T'_U}(1)$
(iv) $C^{T'_U}(1) < C^{T'} \le C^{T'_U}(0)$
(v) $C^{T'_L}(0) < C^{T'}$.

Since case (i) and (v) are similar, and (ii) and (iv) are similar, thus we need only to discuss the cases (i), (ii), and (iii). Suppose there exists a minimum spanning tree $T'(N, S')$ whose total cost is $\sum_{j \in S'} x_j$.

Case (*i*) Suppose $\Pi_{C^{T'}}(y) = \mu(C^{T'}) > 0$. By the definition 7 and 8, $\Pi_{C^{T'}}(y) = \min\{\Pi_{c'_j}(x_j) | j \in S'\} > 0$, thus $\Pi_{c'_j}(x_j) > 0$ for all *j*. It follows that $\mu_{c'_j}(x_j) > 0$ for all *j*. That is the value of μ is somewhere between [a, b]. But the value of $C^{T'_L}(0)$ is calculated by condition $\alpha = 0$, in which $L_j(\alpha)$ locates on the left side of a_j, i.e. $\Pi_{c'_j}(x_j) = 0$. Contradiction. Therefore, $\Pi_{C^{T'}}(y) = 0$, if $C^{T'} < C^{T'_L}(0)$.

Case (*ii*) Given a $\alpha^* \in [0, 1]$, then there exists the corresponding $C^{T'} = e^{-1}(\alpha^*)$. Let the left-end point of α^*-cut of each edge be denoted by $L_j(\alpha^*)$. Moreover, calculate the minimum spanning tree by $L_j(\alpha^*)$. Thus according to definition 7, it follows that $\Pi_{C^{T'}}(y) = \alpha^*$.

Suppose the above equation is not hold, say $\Pi_{C^{T'}}(y) > \alpha^*$, then there must exist an edge *k*, whose length is larger than $L_j(\alpha^*)$, such that the other edges are all larger than edge *k*. If so, then the $C^{T'} = \sum x_j$ must be larger than $\sum L_j(\alpha^*)$, which leads contradiction.

Suppose $\Pi_{C^{T'}}(y) < \alpha^*$. This is impossible because of the definition 8.

Case (*iii*) If $C^{T'} \in [C^{T'_L}(1), C^{T'_U}(1)]$, then for each edge, there must exist $c_j \in [m_j^{+}, m_j^{-}]$, $j \in S'$. If there is a minimum spanning tree $T'_q \in Q$ such that $c_{j_q} \notin [m_{j_q}^{+}, m_{j_k}^{-}]$, then by the definition 8, $\Pi_{C^{T'_q}}(y)$ will be ignored. Thus there is only one situation will hold, i.e. $\Pi_{C^{T'_q}}(y) = 1$. ∎

3. Competence Set Expansion with Fuzzy Cost

The problem of competence set expansion is referred to finding an effective way, say with minimum cost, for expanding the already acquired skills set, *Sk*, to the truly needed competence set, *Tr*, with the given expansion cost among skills (Yu and Zhang, 1990). If the expansion costs among skills are fuzzy quantities, then this kind of expansion problem is called a competence set expansion with fuzzy cost.

A competence set expansion can be regarded as a tree construction process if there are not compound skills (Shi and Yu, 1996). In other words, to find the effective way to expanding competence set from *Sk* to *Tr* is similar to find the minimum spanning tree for given nodes and edges in a digraph. Thus, we can use the techniques for solving minimum spanning tree, such as integer mathematical programming or minimum spanning table method, MST, (Feng and Yu, 1998), to deal with the problem of competence set expansion.

Since MST method enjoies many advantges compared to integer mathematical programming, we will use the MST method to construct the minimum spanning tree, i.e. optimal expansion process for competence set. Let us first briefly describe some terminology in the MST method. An *expansion table*, as shown in table 1, is a matrix representation of a digraph, in which the component of row *i* and column *j* stands for the cost function denoted by $c_{ij}=c(i, j)$ of acquiring from skill x_i to skill x_j.

Table 1 Expansion table

	x_1	…	x_j	…	x_m
x_1					
:					
x_i			$c(i, j)$		
:					
x_m					

Except for complete graphs, an expansion table may have empty cells indicating that the corresponding edge, say < x_i, x_j >, does not exist in the digraph. When the expansion cost c_{ij} is well defined, it is called a *connecting element*, or simply *conn-element*. Note, a conn-element c_{ij} is connecting x_j from x_i. Thus in the problem of competence set expansion with fuzzy cost, the conn-element will be represented by fuzzy number or possibility distriburion, denoted by c_{ij}.

The MST method has seven procedures consisting of initializing condition, selecting and marking procedure, cycle detecting procedure, crossing out procedure, stopping rule, compressing procedure, and unfolding procedure. For a detailed description of the MST method and the correspounding computer program, MINST, refer to Feng and Yu (1998) and Chiang et al. (2000).

As mentioned before, the expansion process of competence set is a kind of construction of minimum spanning tree, so we can use the results of section 2 to solve the problem of competence set expansion with fuzzy cost. Next we introduce the algorithm for constructing the possibility distribution of expansion process. The following are steps of the algorithm.

Algorithm.

Step 1. Give the membership function of expansion cost, shown as Fig. 1.

Step 2. Take $\sim \in [0, 1]$, and calculate the lower bound and upper bound of conn-element c_{ij} by equations (7) and (8), respectively. Denote by $L_{ij}(\sim)$ and $U_{ij}(\sim)$.

Step 3. Construct the minimum spanning tree by MST method and calculate the total cost with $L_{ij}(\sim)$ and $U_{ij}(\sim)$, respectively. The results are denoted by $C^{t_L}(\sim)$ and $C^{t_u}(\sim)$.

Step 4. Construct the membership function by equation (9).

4. Numeric Example

There are a number of all major issues facing transportation professionals and decision-makers today including increasing traffic congestion, declining mobility, air quality and environment concerns, deterioration of the transportation infrastructure, and limited resources. Thus, to ensure that transportation planners from urban and regional planning graduate schools possess the necessary training and skills is of great importance to public-sector agencies and private firms responsible for all aspects of transportation planning, operation, and management.

To respond to the change of transportation planning market (E), transportation planners will need a wide range of skills and knowledge. Turnbull (1991) identified 12 knowledge and 9 skill areas as important for future transportation professionals. Table 2 summarizied the future demands of the transportation marketplace by knowledge areas and technical skill areas.

Table 2 Summary of knowledge and skill areas for future transportation planners

Knowledge Areas	**Skill Areas**
Intermodal /Multimodal Focus	Travel Demand Modeling
Individual Mode Characteristics	Air Quality and Environmental Analysis Techniques
Transportation/ Land Use Interrelationships	Financial Analysis Techniques
Traffic Engineering	GIS
Air Quality and Environmental Impact of Modes	Database Management
TSM, TDM, and TCM	Mode Specific
Travel Demand Forecasting Process	Evaluation Techniques
ITS and Advanced Technologies	Problem Solving Techniques
Federal and State Requirements	Communication Skills
Transportation Planning and Decision-Making Process	
Public Participation Process	
Management	

Suppose a green planner has already acquired the competence of communication skills (c), problem solving techniques (p), and evaluation techniques (e). To meet with the future need for transportation planner, he or she still need to learn other skills consisting of travel demand modeling (t), air quality and environmental analysis techniques (a), GIS (g), mode specific (m), database management (d), and financial analysis techniques (f). By the terminology of competence set analysis, the already acquired skill set and the truly needed competence set can be denoted by $Sk=\{c, p, e\}$, and $Tr=\{c, p, e, t, a, g, m, d, f\}$, respectively. Thus the set of skills to be acquired can be represented as $Tr\backslash Sk=\{t, a, g, m, d, f\}$.

Suppose the fuzzy expansion cost among skills can be identified as table 3. Note that each membership function of cost is represented by four parameters (a, m^1, m^2, b). Moreover, we define the expansion cost from the acquired skills set, Sk, to other skills as follows:

$c(Sk, y) = \min\{c(x, y), x \in Sk, y \in Tr \setminus Sk\}$.

Table 3. Fuzzy expansion cost among skills

	Sk	t	a	g	m	d	f
Sk	×	3.6, 4.3, 5.7, 6.4	8.6, 8.8, 9.2, 9.4	7.4, 7.7, 8.3, 8.6	5.4, 6.72, 9.28, 10.6	2.9, 3.4, 4.6, 5.1	3.8, 3.9, 4.1, 4.2
t	×	×	6.1, 7.6,	5.7, 6.9,	2.7, 3.4,	6.5, 7.7,	3.4, 3.7,

			10.4, 11.9	9.1, 10.2	4.6, 5.3	10.3, 11.5	4.3, 4.6
a	×	6.5, 7.7, 10.3, 11.5	×	2.7, 3.4, 4.6, 5.3	5, 5.5, 6.5, 7	9, 9, 9, 9	8.6, 8.8, 9.2, 9.4
g	×	2, 2.5, 3.5, 4	2.7, 3.4, 4.6, 5.3	×	3, 3, 3, 3	3.4, 4.2, 5.8, 6.6	2.7, 3.4, 4.6, 5.3
m	×	3, 3.5, 4.5, 5	5.8, 6.9, 9.1, 10.2	4.2, 4.6, 5.4, 5.8	×	6.2, 6.6, 7.4, 7.8	6.7, 7.4, 8.6, 9.3
d	×	2.6, 2.8, 3.2, 3.4	3.8, 3.9, 4.1, 4.2	2.8, 2.9, 3.1, 3.2	6.7, 6.9, 7.1, 7.3	×	5.5, 5.8, 6.2, 6.5
f	×	3, 3, 3, 3	2.7, 3.4, 4.6, 5.3	7.4, 7.7, 8.3, 8.6	3.4, 4.2, 5.8, 6.6	4, 4, 4, 4	×

According to the algorithm, the possibility distribution of cost of minimum spanning tree can be constructed step by step shown as fig. 2.

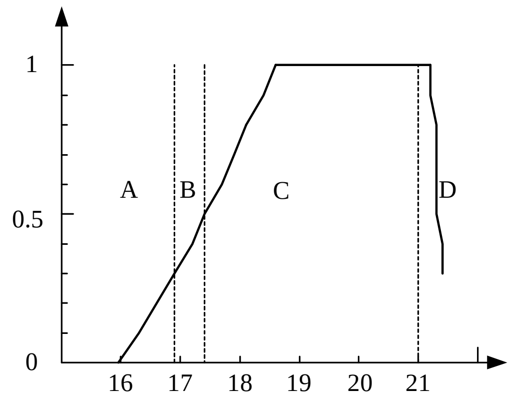

Fig. 2 Possibility Distribution of expansion cost

The results of constructing process with ~-cut level ranging from 0 to 1 are shown as table 4. The second column and fourth column represent the lower bound and upper bound of expansion cost under a certain ~-cut level. The third and fifth columns represent the expansion process. For example, when ~-cut level is 0, the lower bound of expansion cost, $C^{\alpha_L}(0)$, is 15.96, and the corresponding expansion process is type A, shown as Fig. 3. Similarly, the upper bound of expansion cost, $C^{\alpha_U}(0)$, is 21.44, and corresponding expansion process is type D, shown as Fig. 6. The expansion processes of type B and C are shown as Fig. 4 and Fig. 5, respectively.

Table 4. The lower and upper bound of expansion cost and the corresponding expansion process under ~-cut level

α	$C^{\alpha_L}(\sim)$	Expansion Process	$C^{\alpha_U}(\sim)$	Expansion Process
0	15.96	A	21.44	D
0.1	16.26	A	21.42	D
0.2	16.56	A	21.40	D
0.3	16.87	B	21.37	D
0.4	17.16	B	21.35	D
0.5	17.43	C	21.33	D
0.6	17.67	C	21.31	D
0.7	17.91	C	21.29	D
0.8	18.14	C	21.26	D
0.9	18.38	C	21.24	D
1.0	18.62	C	21.22	D

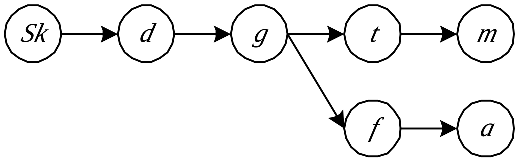

Fig. 3 Type A Expansion Process

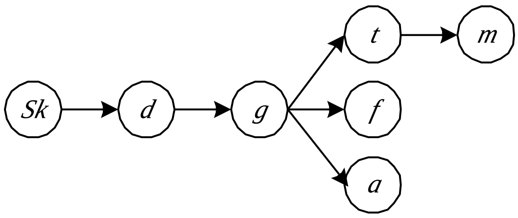

Fig. 4 Type B Expansion Process

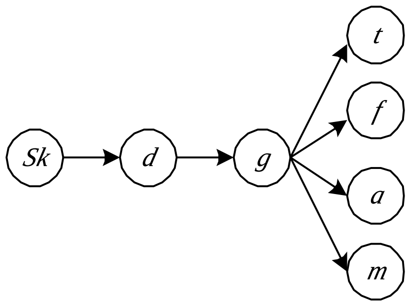

Fig. 5 Type C Expansion Process

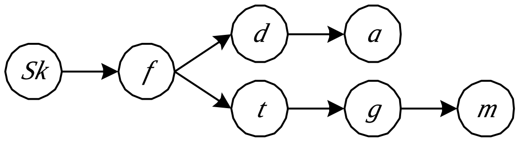

Fig. 6 Type D Expansion Process

Table 4 shows that the expansion process, which constructed by lower bound of conn-element will vary from type A to type C when ~-cut level increase from 0 to 1. However, the expansion process constructing by upper bound of conn-element will keep no matter how ~-cut level varies. On the other hand, as depicted by Fig. 2, the expansion process gradually varies from type A to type D when the cost of expansion increases.

5. Conclusions

We described an algorithm to establish the possibility distribution of minimum spanning tree. More information can be provided by this method compared to Chang and Lee's (1999) method that constructed the minimum spanning tree by using overall existence ranking index (OERI) method to defuzzify the edges' cost. Then with the algorithm we applied to find the possibility distribution of the optimal expansion process of competence set.

Because of the fuzziness of expansion cost, the expansion process may vary with different ~ level. According to the results, the fuzzy cost of optimal expansion process is characterized by possibility distribution, rather than a crisp value. It is a certain result of fuzziness of expansion process when the fuzzy expansion costs are considered. However, it contains more information. We may give explanation as follows. Because of the fuzziness of expansion costs, the so-called optimal expansion process also is a fuzzy concept. Therefore, the optimal expansion process may be process A or another process.

In the case containing many expansion processes, how can a decision maker make decision to choose expansion process? We suggest two ways for decision maker to choose expansion processes. First, given a ~, which represents the decision level, and then choose the expansion processes with possibility larger than ~. Second, rank the expansion process according to the rule that "up is better than down, and left is better than right." For instance, by the numerical example given in section 4, we can rank the optimal expansion process as C B A D.

Finally, in this study we did not consider the case that compound skills are included in *Tr**Sk*, therefore, it remains still a further research.

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