Analytical Solutions of Multilevel Space-Vector PWM for Multiphase Voltage Source Inverters

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***Abstract*—This study systematically analyzes multilevel space-vector pulse width modulation for a multiphase voltage source inverter (VSI). The instantaneous output voltages of the VSI, which are called space vectors, can be classified according to the switching states of the VSI. By applying the eigenspace decomposition of the system matrix, the *n*-phase VSI control problem can be solved analytically. This analysis leads to a switching strategy that uses the fewest space vectors and minimum total conduction time. This yields a minimum switching number and efficient dc supply utilization. Further, the switching strategy of a multilevel multiphase VSI system can be solved based on the switching strategy of a two-level multiphase VSI. Simulations and experimental results confirm the effectiveness of the proposed algorithm.**

***Index Terms*—Maximum modulation index, multiphase, pulse width modulation (PWM), space vector, voltage source inverters (VSI).**

I. Introduction

THE preliminary idea of using multiphase inverters for variable-speed drives was first proposed in the 1960s [1]. Using multiphasemachines instead of three-phase machines has several advantages such as reduction in copper loss and attenuation of phase-belt harmonics [2], improvements in efficiency, and reduction in torque pulsation. Further, an improved fault tolerance without additional hardware [3], [4] and reduced power handling requirement for each phase [1], [5] are achieved using multiphase machines. In recent years, multiphase drives have been applied to control multiphase brushless machines or multiphase permanent-magnet motors for electrical vehicles and ships and for low-torque propulsion [5]. The additional degrees of freedom in multiphase inverters are adopted to control other multiphase machines independently [6]–[8], i.e., a multiphase multimotor system driven by a single voltage source inverter (VSI). Notably, the allowable number of motors that can be serially connected in a system depends on the phase number of the motors.

Advantages such as low-harmonic current waveforms, low-voltage drop cross switches for the same current ratings, and relatively low switching frequencies compared with two-level inverters are found in the multilevel inverters [9], [10]. In recent years, multilevel inverters driven by the hysteresis current regulation method were proposed to improve output voltage [11]. Further, they were also applied to the direct torque control to reduce switching losses [12].

The strategy of generating pulse width modulation (PWM) switching signals for a multilevel multiphase inverter used in multiphase machine is an important means of controlling multiphase machines. The study in [13] presents a control method of a six-phase inverter for dual three-phase machines via vector space decomposition. This strategy requires the analysis of three two-dimensional orthogonal subspaces. In [14], the analysis of a nine-phase inverter for sinusoidal phase voltage is conducted according to different load circuits induced by gating patterns. The concept of multiple *d–q* spaces is presented in [15] and [16] for five-phase nonsinusoidal voltages and seven-phase sinusoidal voltages, respectively. As mentioned in [15], for nonsinusoidal phase voltages, torque per ampere is maximized when multiphase motors have concentrated winding and a nonsinusoidal air-gap flux density distribution. For multilevel three-phase systems, a harmonically optimal strategy to allow the full usage of voltage levels is proposed in [17] and a general method for over-modulation operation is suggested in [10]. The method to reduce the common mode voltage of the multilevel inverter is advised in [18] and [19]. Further, a simple control strategy for multilevel multiphase inverters is recommended in [20]. This strategy achieves the objective of simplicity by solving a two-level multiphase inverter control problem. A similar approach to [20] that further considers state redundancy is proposed in [5], i.e., increasing the modulation index.

While several factors can be considered for multilevel multiphase switching signal generation, the fundamental aim is still to match the reference signal waveform given limited switching states. Despite recent research progress, a complete theoretical analysis from the perspective of reference signal matching is lacking. The potential benefits for theoretical development is a relatively simpler implementation scheme as well as extensions to different applications. For instance, the methods in [13] and [15] require calculation of the matrix inverse and trigonometric functions; otherwise, a memory space is needed to store data. The approach in [14] focused on sinusoidal phase voltages [15], i.e., this method has limited applications. The techniques for multilevel inverters [10]–[12], [17]–[19] are for three-phase systems only. The extension to the *n*-phase system is needed especially when the system is analyzed using the space-vector PWM concept. Although the implementation in [20] is relatively simple, it does not allow full use of supplied voltage. By considering the redundant switching state, the system in [5] achieves an extended modulation index compared to that in [20]

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with additional computations. Under the reasonable modulation index, the only difference between strategies of [20] and [5] is the distribution of zero switching states when two-level VSI is considered. For multilevel VSI systems, there exists a selection of switching states in [5] such that only the equivalent integer part of reference input is different from that in [20]. Therefore, no additional operations should be needed to achieve the extended modulation index.

This study proposes a novel and rigorous mathematical analysis of a multiphase inverter under a multilevel switching topology for a multiphase machine. The treatment of this problem differs from the vector analysis adopted in many published studies [5], [13]–[16], [20]. This study is based on eigenspace decomposition of the matrix composed of inverter output voltages. This algebraic formulation allows one to systematically derive general solutions of a multilevel space-vector PWM that satisfies the signal matching condition. Among existing solutions, designing a particular scheme that uses the fewest space vectors per duty cycle is possible. An immediate benefit is that the switching number of the power switches (e.g., MOSFET) can be minimal. Further, if one defines total conduction time of the power stage as the duty ratio containing all nonzero space vectors, this scheme also results in minimum total conduction time. Given these two properties, maximum modulation indices for all phase numbers can be well explained. The proposed scheme requires fewer arithmetic computations compared with those for the method proposed in [5]. The analysis of a two-level switching scheme can be extended to multilevels using integer–fraction decomposition with sum-of-zero compensation. This treatment is similar to that in [20] with a modification that ensures the sum-of-zero condition for reference signals is matched by two-level space vectors. Computing the maximum modulation indices given the number of levels is then straightforward.

The rest of this paper is organized as follows. Section II gives the mathematical foundation, proposed switching schemes, and the analysis of the two-level multiphase switching scheme for a VSI. The extension of the scheme to a multilevel system is explained in Section III. Section IV presents simulation results for the proposed method. Experimental results on an *R–L* circuit are given in Section V. Conclusion is finally drawn in Section VI, along with recommendations for future research.

II. Two-Level Switching Strategy for Multiphase Systems

The analysis of space vectors, generic switching strategy, minimum conduction time strategy, and modulation index analysis are discussed in this section. To have a compact presentation, all formulations and expressions are shown based on the general *N*-phase VSI system. Nevertheless, the explanation for the three-phase system is provided in Appendix A. For readers familiar with the three-phase setting, please refer to Appendix A for a better understanding of the underlying theory.

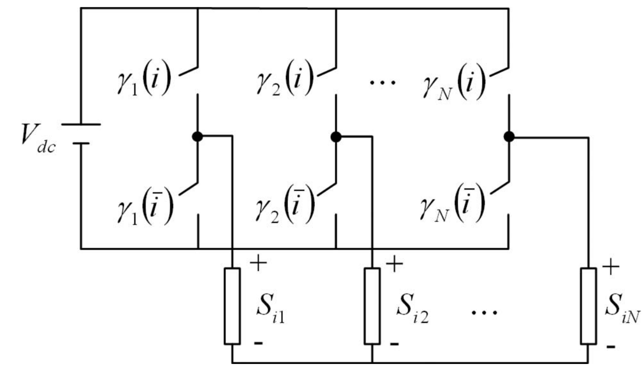


Fig. 1 *N*-phase system with a two-level switching topology.

A. Space Vector Analysis

For an *N*-phase system under a two-level switching topology (see Fig. 1), let $\mathbf{s}_i \in \mathbb{R}^{N\times 1}$, $i = 1$–$M - 1$, be the resulting phase (line-to-neutral) voltage vectors (or so-called space vectors). One can easily show that $M = 2^N$. Among the switching states, two zero vectors result in zero voltage for all phases, i.e., all lower switches or all upper switches are closed. Without loss of generality, the supply voltage is normalized to 1 in the following.

The switching condition can also be described by number *i* in the *N*-bit binary form. Each bit in *i* represents the status of switches in each leg. The value of 1 means the upper switch is closed and 0 means the lower switch is closed. Let $\sigma(i)$ be the number of bits in *i*, whose values are 1, and $\gamma_j(i)$ is the value of bit *j* in *i*, $j = 1 - N$. As the vector $\mathbf{s}_i$ is denoted as $\mathbf{s}_i = [\, s_{i1} \quad s_{i2} \quad \cdots \quad s_{iN} \,]^T$, the *j*th element, s_{ij}, can be written as follows:

$$s_{ij} = \gamma_j(i)\frac{N-\sigma(i)}{N} + (\gamma_j(i)-1)\,\frac{\sigma(i)}{N} = \gamma_j(i) - \frac{\sigma(i)}{N}. \tag{1}$$

Consequently, the space vectors can be organized into groups according to their values of $\sigma(i)$. For *N* phases, one can organize them into *N* groups (group 0 $\sim$ *N*–1). Group 0 contains two zero space vectors. The remaining groups, group *n*, contain $\mathbf{C}_n^N$ vectors. As an example, 128 space vectors exist in a seven-phase system. These vectors are categorized into seven groups. Group 1 contains seven vectors whose binary number representations have only one bit that is one, and the rest of the bits are 0. These vectors are $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_4, \ldots, \mathbf{s}_{64}$ and $\mathbf{s}_1 = [(6/7) \;\; -(1/7) \;\; \cdots \;\; -(1/7)]^T$, according to (1). Groups 2 (21 vectors) to 6 (seven vectors) can be found similarly. The vectors in group 1 ($\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_4, \ldots, \mathbf{s}_{2^{N-2}}$ and $\mathbf{s}_{2^{N-1}}$) are defined as fundamental space vectors (*FSVs*). Putting all *FSVs* together, one can form an *N*-dimensional square matrix as follows:

$$\mathbf{S}_c = \begin{bmatrix} \frac{N-1}{N} & -\frac{1}{N} & \cdots & -\frac{1}{N} \\ -\frac{1}{N} & \frac{N-1}{N} & \cdots & -\frac{1}{N} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{N} & -\frac{1}{N} & \cdots & \frac{N-1}{N} \end{bmatrix}. \tag{2}$$

Further, define the vector $\mathbf{\Gamma}_i$ as the bitwise expansion of positive integer *i*, i.e.,

$$\mathbf{\Gamma}_i = [\,\gamma_1(i) \quad \gamma_2(i) \quad \cdots \quad \gamma_N(i)\,]^T. \tag{3}$$

Then from (1), the space vectors in group *n*, $n = 2$–$N-1$, can be represented as the addition of *n FSVs* (see Appendix B). Specifically, one can rewrite the space vectors according to (2) and (3) as follows:

$$\mathbf{s}_i = \mathbf{S}_c\mathbf{\Gamma}_i. \tag{4}$$

Additionally, an alternative representation of (1) by considering the bitwise complement of integer *i* can be derived as follows:

$$s_{ij} = -\left(\gamma_j(\bar{i}) - \frac{\sigma(\bar{i})}{N}\right) \tag{5}$$

where $\bar{i}$ is the bitwise complement of *i*. Therefore, a second space vector relationship to *FSVs* is

$$\mathbf{s}_i = -\mathbf{S}_c\mathbf{\Gamma}_{\bar{i}} \tag{6}$$

where $\mathbf{\Gamma}_{\bar{i}} = [\,\gamma_1(\bar{i}) \quad \gamma_2(\bar{i}) \quad \cdots \quad \gamma_N(\bar{i})\,]^T$. By combining (4) and (6), a general representation of space vectors in terms of *FSVs* can be derived as follows:

$$\mathbf{s}_i = \mathbf{S}_c\left(\beta_i\mathbf{\Gamma}_i - (1-\beta_i)\,\mathbf{\Gamma}_{\bar{i}}\right). \tag{7}$$

where all β_i values are numbers between 0 and 1.

B. Generic Switching Strategy for Averaged Response

Let $u_i(k)$, $i = 1$–M, be the duty ratio of space vector $\mathbf{s}_i$ at the PWM sequence *kT*, where *T* is the period and $\sum_{i=1}^{M} u_i(k) = 1$. Consider that the averaged response of PWM signals for *M* line-to-neutral voltages is equal to the sampled reference signals

$$T\mathbf{r}(k) = T\sum_{i=1}^{M} u_i(k)\mathbf{s}_i \tag{8}$$

where $\mathbf{r}(k) = [\,r_1(k) \quad r_2(k) \quad \cdots \quad r_N(k)\,]^T$ is the vector of the reference *N*-phase voltage sampled at *kT*. Let $\mathbf{u}(k) = [\,u_1(k) \quad \cdots \quad u_M(k)\,]^T \in \mathbb{R}^{M\times 1}$ be the duty ratio vector and $\mathbf{S} = [\,\mathbf{s}_1 \quad \cdots \quad \mathbf{s}_M\,] \in \mathbb{R}^{N\times M}$ be the quantized phase voltage matrix; thus, one can derive the equation in that resulting PWM signals match the reference signal vector **r**(*k*) as follows:

$$\mathbf{S}\mathbf{u}(k) = \mathbf{r}(k) \tag{9}$$

From (2) to (7)

$$\mathbf{S} = \mathbf{S}_c\mathbf{H} \tag{10}$$

where

$$\mathbf{H} = [\,\mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_M\,] \in \mathbb{R}^{N\times M} \text{ and}$$
$$\mathbf{h}_i = \beta_i\mathbf{\Gamma}_i - (1-\beta_i)\,\mathbf{\Gamma}_{\bar{i}}. \tag{11}$$

Putting (11) in matrix form yields

$$\mathbf{H} = \mathbf{H}_1\mathbf{B} - \mathbf{H}_2\,(\mathbf{I} - \mathbf{B}) \tag{12}$$

where $\mathbf{H}_1 = [\,\mathbf{\Gamma}_1 \quad \mathbf{\Gamma}_2 \quad \cdots \quad \mathbf{\Gamma}_M\,]$, $\mathbf{H}_2 = [\,\mathbf{\Gamma}_{\bar{1}} \quad \mathbf{\Gamma}_{\bar{2}} \quad \cdots \quad \mathbf{\Gamma}_{\bar{M}}\,]$, and $\mathbf{B} = \mathrm{diag}\{\beta_i\} \in \mathbb{R}^{M\times M}$. As a result, (9) becomes

$$\mathbf{S}_c\mathbf{u}_c(k) = \mathbf{r}(k) \tag{13}$$

where $\mathbf{u}_c(k) = \mathbf{H}\mathbf{u}(k)$. Thus, $\mathbf{S}_c$ is a circulant matrix and can be represented as follows:

$$\begin{aligned}\mathbf{S}_c &= \frac{N-1}{N}\mathbf{I} - \frac{1}{N}\mathbf{P} - \frac{1}{N}\mathbf{P}^2 - \cdots - \frac{1}{N}\mathbf{P}^{N-1}\\ &= \mathbf{I} - \frac{1}{N}\left(\mathbf{I} + \mathbf{P} + \mathbf{P}^2 + \cdots + \mathbf{P}^{N-1}\right)\end{aligned} \tag{14}$$

where **P** is an *N*-dimensional circulant permutation matrix as

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \ddots & 0\\ 0 & 0 & 0 & \ddots & 0\\ \vdots & \vdots & \vdots & \ddots & 1\\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

As a result, the eigenvalues of $\mathbf{S}_c$, ξ_n, $n = 0$–$N-1$, are [21]

$$\xi_n = 1 - \frac{1}{N}\sum_{m=0}^{N-1}\phi_n^m,$$
$$\text{where } \phi_n = e^{-(2\pi n/N)j} \text{ (the eigenvalues of } \mathbf{P}). \tag{15}$$

One can then easily verify that the matrix $\mathbf{S}_c$ has an eigenvalue of zero and all other eigenvalues are 1. The associated eigenvectors of $\mathbf{S}_c$ are

$$\mathbf{v}_n = \frac{1}{\sqrt{N}}[\,1 \quad \phi_n \quad \phi_n^2 \quad \cdots \quad \phi_n^{N-1}\,]^T$$

Therefore, the eigenvector corresponding to the zero eigenvalue is $\mathbf{v}_0 = \left[\,1/\sqrt{N} \quad 1/\sqrt{N} \quad \cdots \quad 1/\sqrt{N}\,\right]^T$. Hence, the eigenvalue decomposition of the matrix $\mathbf{S}_c$ becomes

$$\mathbf{S}_c = \begin{bmatrix}\mathbf{V}_c\\ \mathbf{v}_0^T\end{bmatrix}^T \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & 0\end{bmatrix} \begin{bmatrix}\mathbf{V}_c\\ \mathbf{v}_0^T\end{bmatrix} \tag{16}$$

where $\mathbf{V}_c \in \mathbb{R}^{(N-1)\times N}$, whose row vectors are eigenvectors corresponding to the eigenvalue of 1. From (16), (13) becomes

$$\begin{bmatrix}\mathbf{V}_c\\ \mathbf{0}\end{bmatrix}\mathbf{u}_c(k) = \begin{bmatrix}\mathbf{V}_c\\ \mathbf{v}_0^T\end{bmatrix}\mathbf{r}(k). \tag{17}$$

This leads to

$$\mathbf{V}_c\left(\mathbf{u}_c(k) - \mathbf{r}(k)\right) = 0 \tag{18}$$

and

$$\mathbf{v}_0^T\mathbf{r}(k) = 0. \tag{19}$$

Equation (19) depicts the condition in which $\sum_{i=1}^{N}\mathbf{r}_i(k) = 0$. The solution of (17) must lie in the right null space of the matrix $\mathbf{V}_c$ [see ([18])]. One can easily see that the right null space is the vector $\mathbf{v}_0$ as all row vectors of $\mathbf{V}_c$ are orthogonal to $\mathbf{v}_0$. Therefore, the general solution of (17) can be represented by

$$\mathbf{u}_c(k) = \mathbf{r}(k) + \lambda\mathbf{d} \tag{20}$$

where $\mathbf{d} = [1 \quad 1 \quad \cdots \quad 1]^T$ and λ is an arbitrary real value. From (12) to (13) after some algebraic manipulations, the following equation is derived (see Appendix C)

$$\mathbf{H}_1 \mathbf{u} = \mathbf{r}(k) + (\lambda + b)\mathbf{d} \tag{21}$$

where $b = \sum_{i=1}^{M} (1 - \beta_i)\alpha_i$. Since λ can be any number, one can always choose $\lambda = \lambda_1 - b$. Therefore, the aforementioned equation becomes

$$\mathbf{H}_1 \mathbf{u} = \mathbf{r}(k) + \lambda_1 \mathbf{d} \tag{22}$$

Equation (22) is called the generic switching strategy equation under the averaged response formulation of (8). The value λ_1 now becomes a design parameter. The duty ratios satisfying (22) must also be positive and the sum must equal 1.

C. Minimal Total Conduction Time

Total conduction time is defined as the ratio in each PWM duty cycle containing nonzero space vectors. Since all components of resulting vectors on the left side of (22) are non-negative, parameter λ_1 must be selected, such that $\mathbf{r}(k) + \lambda_1 \mathbf{d} \geq 0$. Since those components are a partial sum of the duty ratio, a necessary condition for minimal total conduction time is $\sum_{i=1}^{N} |r_i(k) + \lambda_1|$ reaches the minimum. Given these two constraints, one can easily verify that λ_1 should be the negative value of the minimum of $r_i(k)s$. Let $\mathbf{P}_M$ be the permutation matrix for arranging the components of the vector $\mathbf{r}(k)$ in a descending order, i.e., $\mathbf{P}_M \mathbf{r}(k) = \boldsymbol{\rho}(k)$ such that elements of the vector $\boldsymbol{\rho}(k)$ satisfy $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_{N-1} \geq \rho_N$. This means $\lambda_1 = -\rho_N$. As a result, (22) can be rewritten as follows:

$$\begin{aligned}\mathbf{P}_M \mathbf{H}_1 \mathbf{u} &= \boldsymbol{\rho}(k) - \rho_N \mathbf{d} = \boldsymbol{\rho}_d(k) \\ &= [\rho_1 - \rho_N \quad \rho_2 - \rho_N \quad \cdots \quad 0]^T.\end{aligned} \tag{23}$$

Among the solutions for (23), this study is particularly interested in the solutions with the fewest space vectors. Since the last element of $\boldsymbol{\rho}_d(k)$ is zero, choosing $N-1$ space vectors to satisfy (23) is sufficient. Clearly, the last element of those vectors must also be zero because all duty ratios are nonnegative. Let $\mathbf{u}_1 = [\bar{u}_1 \quad \cdots \quad \bar{u}_{N-1}]^T \in \mathbb{R}^{(N-1)\times 1}$ contain the corresponding duty ratios. By extracting the columns from the matrix $\mathbf{P}_M \mathbf{H}_1$, (23) becomes

$$\begin{bmatrix} \mathbf{H}_d \\ \mathbf{o} \end{bmatrix} \mathbf{u}_1 = [\rho_1 - \rho_N \quad \rho_2 - \rho_N \quad \cdots \quad 0]^T$$

or

$$\mathbf{H}_d \mathbf{u}_1 = [\rho_1 - \rho_N \quad \rho_2 - \rho_N \quad \cdots \quad \rho_{N-1} - \rho_N]^T \tag{24}$$

where $\mathbf{H}_d \in \mathbb{R}^{(N-1)\times(N-1)}$ and $\mathbf{o} \in \mathbb{R}^{1\times(N-1)}$ is a zero vector. Notably, one has $C_{(N-1)}^{(2^{N-1}-1)}$ choices. This study seeks some special structures of the matrix $\mathbf{H}_d$ as follows:

Case 1: Identity matrix

$\mathbf{H}_d = \mathbf{I}$.

In this case, only the *FSVs* are considered; thus, the duty ratios are $\bar{u}_i = \rho_i - \rho_N$. Total conduction time per duty is $\sum_{i=1}^{N-1} \rho_i - (N-1)\rho_N$.

Case 2: Band matrix

$$\mathbf{H}_d = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & 1 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix}.$$

The duty ratios are $\bar{u}_i = \sum_{n=0}^{\lfloor (N-i-1)/2 \rfloor} \rho_{i+2n} - \rho_{i+2n+1}$. The total conduction time per duty is $\sum_{i=1}^{(N-1)/2} \rho_{2i-1} - ((N-1)/2)\rho_N$ for odd *N* and $\sum_{i=1}^{N/2} \rho_{2i-1} - (N/2)\rho_N$ for even *N*.

Case 3: Upper triangular matrix

$$\mathbf{H}_d = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}.$$

This is exactly the same as the study in [20]. The duty ratios are $\bar{u}_i = \rho_i - \rho_{i+1}$ and total conduction time is $\rho_1 - \rho_N$.

Remark 1: Duties obtained in case 3 are the minimum conduction time solution (see Appendix D).

Remark 2: In case 3, only one phase leg switches its state at each state transition. Therefore, the proposed strategy that uses the least number of states has the minimum switching number.

Remark 3: The corresponding space vectors for the solution to (24) are obtained by applying the inverse of permutation, i.e.,

$$\hat{\mathbf{H}}_s = \mathbf{P}_M^T \begin{bmatrix} \mathbf{H}_d \\ \mathbf{o} \end{bmatrix}$$

where $\hat{\mathbf{H}}_s \in \mathbb{R}^{N\times(N-1)}$ contains (*N*–1) space vectors $\boldsymbol{\Gamma}_i$ of (3). Alternatively, the aforementioned equation can be replaced by the space vectors of (1). For example, in considering case 3, the resulting reference vector can be reached as

$$\mathbf{P}_M^T \begin{bmatrix} \frac{N-1}{N} & \frac{N-2}{N} & \cdots & \frac{1}{N} \\ \frac{-1}{N} & \frac{N-2}{N} & \ddots & \vdots \\ & \ddots & \ddots & \frac{1}{N} \\ \frac{-1}{N} & \cdots & -\left(\frac{N-2}{N}\right) & -\left(\frac{N-1}{N}\right) \end{bmatrix} \times \begin{bmatrix} \rho_1 - \rho_2 \\ \rho_2 - \rho_3 \\ \vdots \\ \rho_{N-1} - \rho_N \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}. \tag{25}$$

Notably, (25) is valid only when $\sum_{i=1}^{N} r_i(k) = 0$. Multiplying the two matrices on the left side of (25) yields

$$\mathbf{P}_M^T \left(\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_N \end{bmatrix} - \frac{1}{N} \sum_{i=1}^{N} \rho_i \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \right) = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}. \tag{26}$$

The summation of elements on the left vector is zero (note that the summation remains the same after permutation). However, if $\sum_{i=1}^{N} \rho_i = \sum_{i=1}^{N} r_i(k) \neq 0$, (26) does not hold. That is, if the reference vector does not satisfy $\sum_{i=1}^{N} r_i(k) = 0$, the phase voltages synthesized by the space vector using the case 3 strategy become $r_i(k) - (1/N)\sum_{j=1}^{N} r_j(k) = 0$ for $i = 1$–N; i.e., the voltage at each leg has an offset to the reference voltage.

D. Modulation Index Analysis

For an N-phase system, minimum total conduction time is $\rho_1 - \rho_N$, where ρ_1 (ρ_N) is the maximum (minimum) value in the reference vector, $r(k)$. To avoid overmodulation, total duty ratios should be ≤ 1. This leads to the following constraints on reference signals:

$$\rho_1 - \rho_N \leq 1. \tag{27}$$

For N-phase sinusoidal reference signals with amplitude R, (27) becomes (Appendix E)

$$R \leq \frac{1}{2\cos(\pi/2N)} \quad \text{for odd } N \text{ and } R \leq \frac{1}{2} \text{ for even } N. \tag{28}$$

This computational result provides a complete explanation of modulation indices for sinusoidal reference signals in [5] and the derivation is similar to that of [22]. The indices for 3, 5, 7, 9, and 11 phases are 0.577, 0.526, 0.513, 0.508, and 0.505, respectively. The index remains the same for even-numbered phases. For odd-numbered phases, the modulation index approaches 0.5 as phase number increases.

III. Multilevel Switching Strategy for Multiphase Systems

For a multilevel VSI system, Fig. 2 shows the schematic diagrams for 2, 3, and L levels of each phase leg [23]. The voltage levels produced in each phase leg with respect to ground (or capacitor negative terminal) are $0, V_{\rm dc}, 2V_{\rm dc}, 3V_{\rm dc}, \cdots (L-1)V_{\rm dc}$ for an L-level VSI system. Notably, the number of line-to-line voltage levels that can be produced is $2L-1$, i.e., five different line-to-line voltages $(-2V_{\rm dc}, -V_{\rm dc}, 0, V_{\rm dc}, 2V_{\rm dc})$ can be produced by a three-level system [see Fig. 2(b)]. Further, $4L-3$ phase voltage levels (with respect to the neutral point) are produced by an L-level system for three-phase wye-connected load [23].

Using normalized voltage, output voltage (with reference to the ground) of each phase leg for the L-level VSI system belongs to the set of $\mathbf{L} = \{0, 1, 2, \ldots, (L-1)\}$. The switching strategy can be easily derived following the method described in [20], i.e., using the decomposition of input reference, $\mathbf{r}(k) = \mathbf{r}_i(k) + \mathbf{r}_f(k)$, where the elements of $\mathbf{r}_i(k)$ are all integers and the absolute values of elements of $\mathbf{r}_f(k)$ are <1. The multilevel multiphase control problem can be transformed into a two-level multiphase problem. Notably, the decomposition aforementioned is not unique. As explained in Remark 2 in Section III-C, the two-level switching strategy cannot produce $\mathbf{r}_f(k)$ when the sum of its elements is not zero. Therefore, one must derive a decomposition scheme for the vectors $\mathbf{r}_i(k)$ and $\mathbf{r}_f(k)$, such that the sum of their elements is zero. This

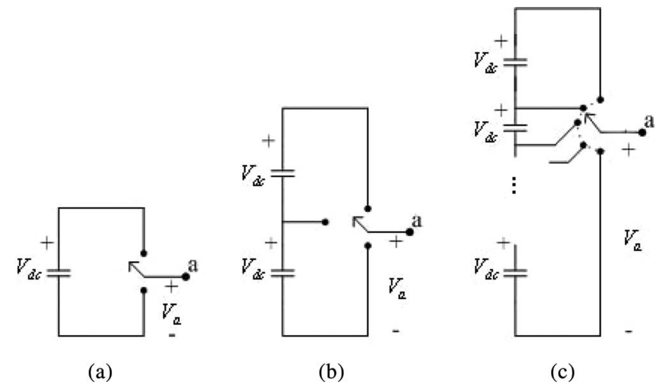


Fig. 2 Each phase leg of the VSI for (a) 2 levels, (b) 3 levels, and (c) L levels.

issue, which is not handled in [20], is solved in the following section.

A. Switching Strategy

To express decomposition, one must first define the function $floor(y)$ as the nearest integer of y toward negative infinity. Thus, $\mathbf{r}_i(k)$ and $\mathbf{r}_f(k)$ can then be written as follows:

$$\mathbf{r}_i(k) = floor(\mathbf{r}(k)) - \text{sign}(z)\sum_{j=1}^{|z|} e_{i_j} \quad \text{and}$$
$$\mathbf{r}_f(k) = \mathbf{r}(k) - \mathbf{r}_i(k) \tag{29}$$

where z is defined as the sum of elements in $floor(\mathbf{r}(k))$, *i.e.*, $z = \sum_{j=1}^{N} floor(\mathbf{r}(k))_j$. $e_{i_j} \in R^N$ is the unit vector with the i_jth element equal to 1 and other elements 0, and i_j corresponds to the index of the jth minimum (maximum) value in the vector $\mathbf{r}(k) - fix(\mathbf{r}(k))$ when $z > 0$ ($z < 0$). One can then easily verify that the sum of elements in $\mathbf{r}_i(k)$ is zero

$$\begin{aligned}\sum_{k=1}^{N} \mathbf{r}_i(k)_k &= \sum_{k=1}^{N}\left(floor(\mathbf{r}(k)) - \text{sign}(z)\sum_{j=1}^{|z|} e_{i_j}\right)_k \\ &= \sum_{k=1}^{N} floor(\mathbf{r}(k))_k \\ &\quad - \text{sign}(z)\sum_{k=1}^{N}\left(e_{i_1} + e_{i_2} + \cdots + e_{i_{|z|}}\right)_k \\ &= z - z = 0.\end{aligned}$$

Similarly, the sum of elements in $\mathbf{r}_f(k)$ is also zero. Further, because elements of $\mathbf{r}_f(k)$ are within $[-1\ 1]$, one can synthesize $\mathbf{r}_f(k)$ using only two levels, i.e., the problem is reduced to a two-level multiphase problem

$$\mathbf{H}_1\mathbf{u} = \mathbf{r}_f(k) + \lambda_1\mathbf{d}. \tag{30}$$

The aforementioned equation is similar to (22). By applying the minimum total conduction time criterion in Section III-C, switching commands $\hat{\mathbf{H}}_s = [\,\mathbf{h}_{s1} \quad \mathbf{h}_{s2} \quad \cdots \quad \mathbf{h}_{s(N-1)}\,]$ and

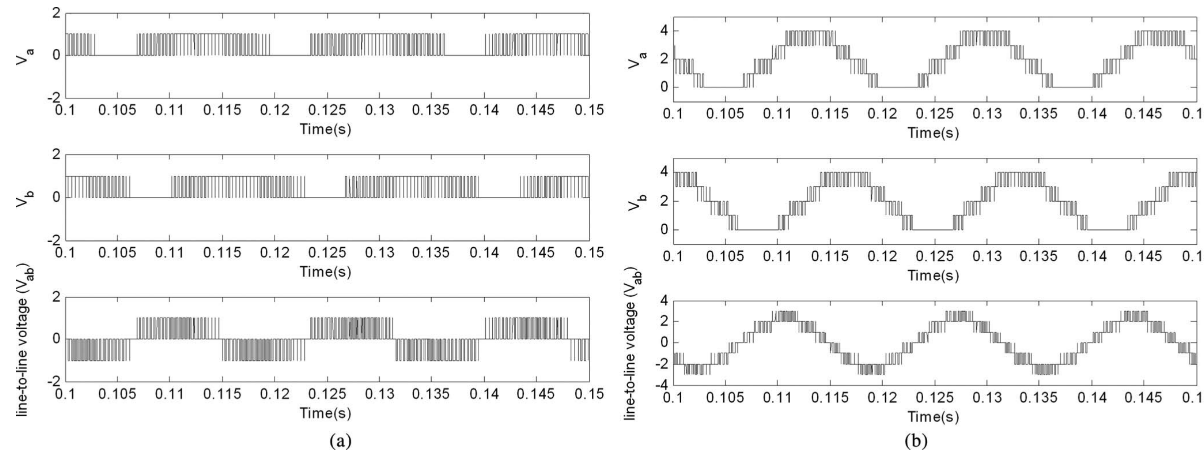


Fig. 3 Switching states and line-to-line voltage of the five-level five-phase system. (a) Input amplitude 0.51. (b) Input amplitude 2.0.

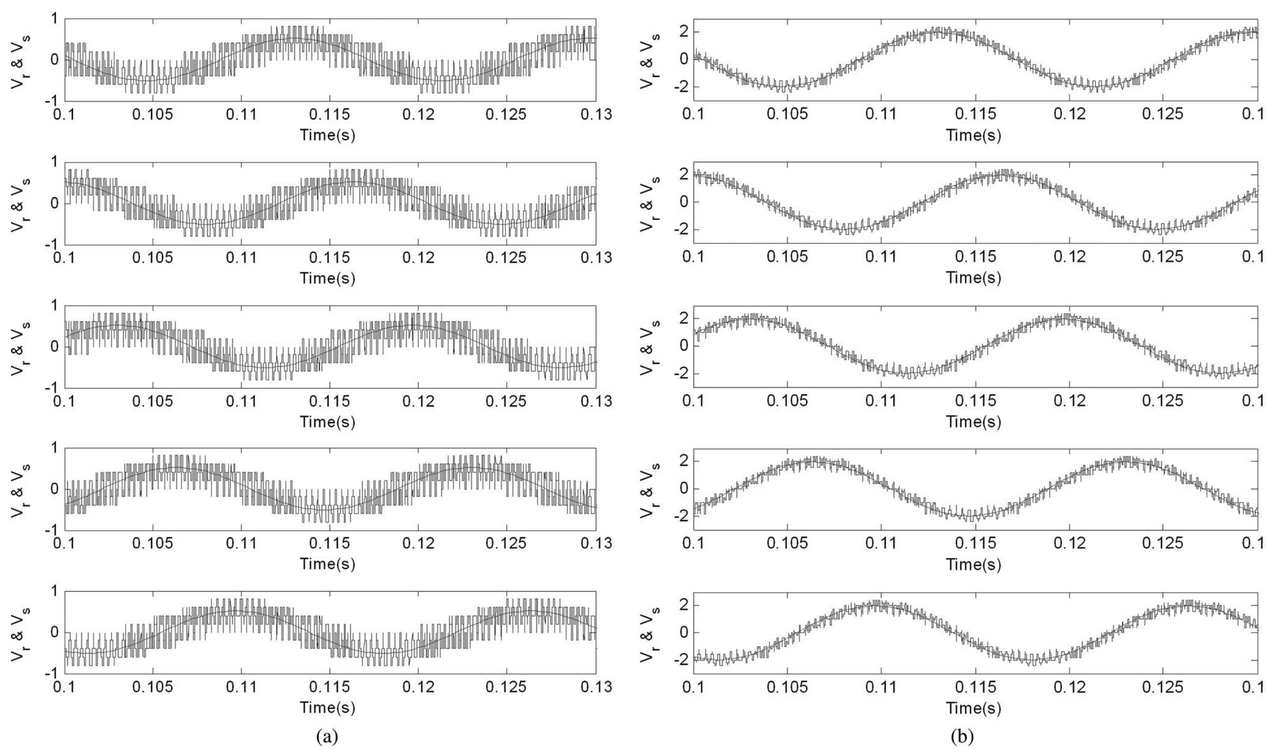


Fig. 4 Phase voltages of the five-level five-phase system. (a) Input amplitude 0.51. and (b) Input amplitude 2.0.

corresponding duties u_1 of (30) are written as follows:

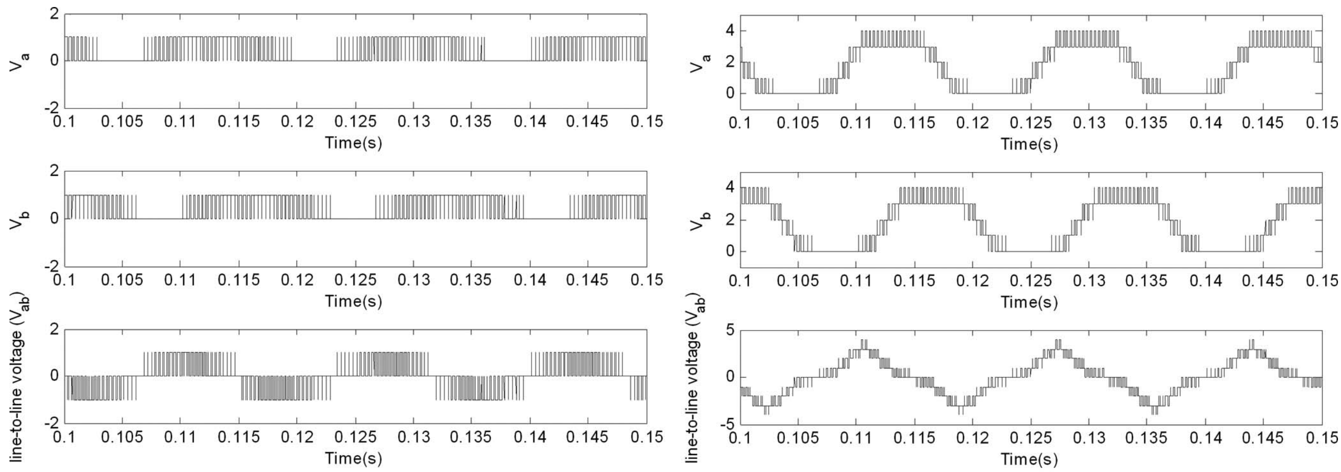
$$\hat{\mathbf{H}}_s = \mathbf{P}_M^T \begin{bmatrix} \mathbf{H}_d \\ \mathrm{o} \end{bmatrix} \text{ where } \mathbf{H}_d = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 1 \end{bmatrix},$$

$$\mathbf{u}_1 = \begin{bmatrix} \rho_1 - \rho_2 \\ \rho_2 - \rho_3 \\ \vdots \\ \rho_{N-1} - \rho_N \end{bmatrix} \tag{31}$$

and $\mathbf{P}_M \mathbf{r}_f\,(k) = \boldsymbol{\rho}_f\,(k)$, such that elements of the vector $\boldsymbol{\rho}_f\,(k)$ satisfy $\rho_1 \geq \rho_2 \geq \cdots \geq \rho_{N-1} \geq \rho_N$. Consequently

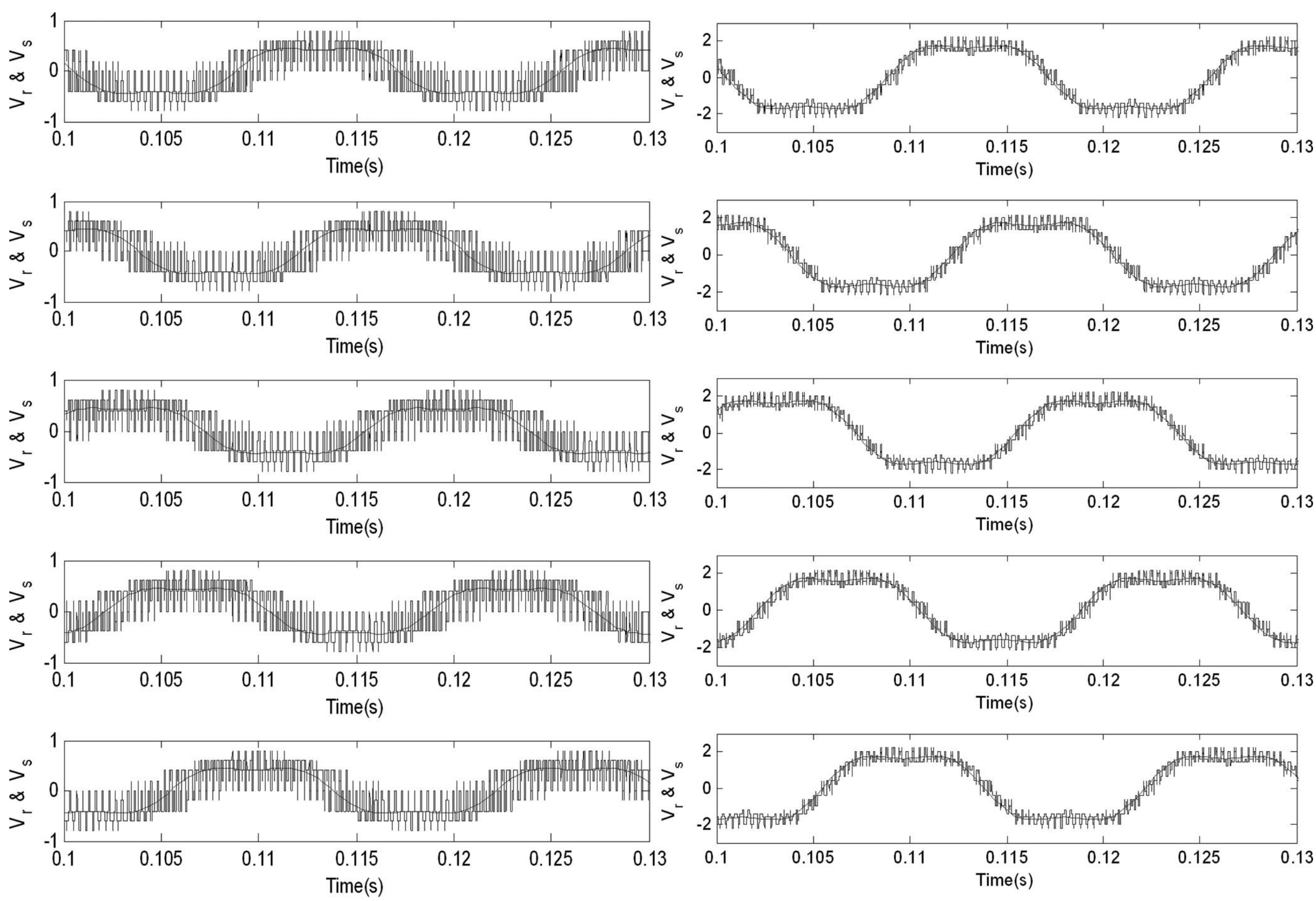
$$\begin{bmatrix} \mathbf{h}_{s0} & \hat{\mathbf{H}}_s \end{bmatrix} \begin{bmatrix} \mathbf{1} - \rho_1 + \rho_N \\ \mathbf{u}_1 \end{bmatrix} = \mathbf{P}_M^T \begin{bmatrix} \mathbf{o}^T & \mathbf{H}_d \\ 0 & \mathbf{o} \end{bmatrix} \times \begin{bmatrix} \mathbf{1} - \rho_1 + \rho_N \\ \mathbf{u}_1 \end{bmatrix} = \mathbf{r}_f\,(k) + \lambda_1 \mathbf{d} \tag{32}$$

where $\mathbf{h}_{s0} \in R^N$ is a zero vector. This means that (N–1) nonzero space vectors are used to synthesize the signal $\mathbf{r}_f\,(k) + \lambda_1 \mathbf{d}$ and the system stays at a zero state during the rest of the duty, say,



(a) (b)

Fig. 5 Switching states and line-to-line voltage of the five-level five-phase system. (a) Input amplitude 0.51 with third harmonic. and (b) Input amplitude 2.0 with third harmonic.



(a) (b)

Fig. 6 Phase voltages of the five-level five-phase system, input amplitude 2 with 0.2 third harmonic.

$\mathbf{1} - \rho_1 + \rho_N$. To consider $\mathbf{r}_i\,(k)$, (32) is written as follows:

$$\begin{bmatrix} \mathbf{h}_{s0} & \hat{\mathbf{H}}_s \end{bmatrix} \begin{bmatrix} \mathbf{1} - \rho_1 + \rho_N \\ \mathbf{u} \end{bmatrix} + \mathbf{r}_i = (\mathbf{r}_f + \lambda_1 \mathbf{d}) + \mathbf{r}_i. \quad (33)$$

Observe that $\mathbf{r}_i = [\, \mathbf{r}_i \quad \cdots \quad \mathbf{r}_i \,] \begin{bmatrix} \mathbf{1} - \rho_1 + \rho_N \\ \mathbf{u} \end{bmatrix}$, (33) is equivalent to

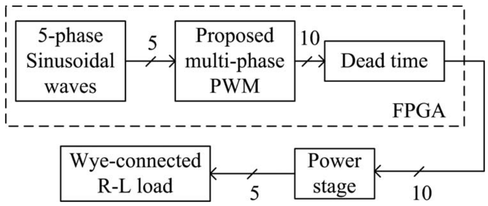


Fig. 7 Block diagram of the experimental platform.

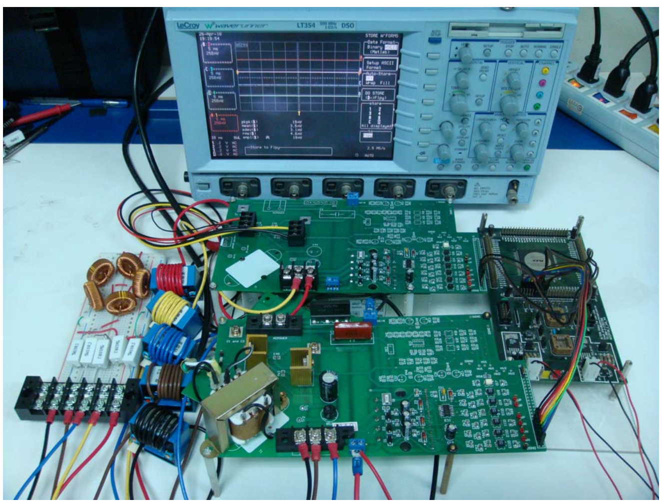


Fig. 8 Experimental platform.

$$\begin{bmatrix} \mathbf{h}_{s0}+\mathbf{r}_i & \mathbf{h}_{s1}+\mathbf{r}_i & \cdots & \mathbf{h}_{s(N-1)}+\mathbf{r}_i \end{bmatrix} \times \begin{bmatrix} \mathbf{1}-\rho_1+\rho_N \\ \mathbf{u} \end{bmatrix} = (\mathbf{r}_f+\lambda_1\mathbf{d})+\mathbf{r}_i. \quad (34)$$

This implies that if the elements of $(\mathbf{h}_{sj}+\mathbf{r}_i), j = 0$ to $(N-1)$ are valid, i.e., elements of $(\mathbf{h}_{sj}+\mathbf{r}_i)$ belong to the set $\mathbf{L}$, the control strategy of the multilevel multiphase system uses columns of $[\mathbf{h}_{s0}+\mathbf{r}_i \quad \mathbf{h}_{s1}+\mathbf{r}_i \quad \cdots \quad \mathbf{h}_{s(N-1)}+\mathbf{r}_i]$ as switching states and the corresponding duties are $\begin{bmatrix} \mathbf{1}-\rho_1+\rho_N \\ \mathbf{u}_1 \end{bmatrix}$. However, the elements of the vector $(\mathbf{h}_{sj}+\mathbf{r}_i)$ may be negative and cannot be switching states. Further computations are needed. One simple method without changing the output phase voltage of VSI is to add an integer $-\eta_j$ to each element of the vector $(\mathbf{h}_{sj}+\mathbf{r}_i)$, such that all elements are non-negative, thereby yielding a switching state $\mathbf{h}'_{sj} = (\mathbf{h}_{sj}+\mathbf{r}_i) - \eta_j\mathbf{d}$. The selected η_j should not be larger than the minimum value among $(\mathbf{h}_{sj}+\mathbf{r}_i)$

$$\eta_j \le \min(\mathbf{h}_{sj}+\mathbf{r}_i). \quad (35)$$

$$\text{One selection is, } \eta_j = \min(\mathbf{h}_{sj}+\mathbf{r}_i). \quad (36)$$

At the last step, the order of switching sequences $(\mathbf{h}_{sj}+\mathbf{r}_i), j = 0$ to $(N-1)$, is reorganized to have minimum switching number. First, we simply sum up the elements in the vectors $(\mathbf{h}_{sj}+\mathbf{r}_i), j = 0 \sim (N-1)$, i.e., N numbers are obtained and each corresponds to a switching state. The sequence of switching is obtained by sorting these N numbers in an increasing order.

B. Example

A five-phase five-level VSI system with sinusoidal reference inputs is used as an illustrative example, i.e., the reference signal for phase P is

$$r_p = R\sin\left(\omega t + \frac{2\pi}{5}(P-1)\right) \quad \text{for } P = 1, 2, \ldots, 5. \quad (37)$$

When speed is $\omega = 2\pi \times 60$ (rad/s) and normalized amplitude is $R = 2$, the reference vector at time $t = 0.051$ s is written as follows:

$$\mathbf{r} = [0.74 \quad 2.00 \quad 0.50 \quad -1.69 \quad -1.55]^T.$$

Therefore, $floor(\mathbf{r}(k)) = [0 \quad 2 \quad 0 \quad -2 \quad -2]^T$ and $z = -2$. Because $z < 0$, we have to find indices i_1 and i_2 that correspond to the maximum and second maximum values in $\mathbf{r}(k) - \text{fix}(\mathbf{r}(k))$.

From $\mathbf{r}(k) - \text{fix}(\mathbf{r}(k)) = [0.74 \quad 0 \quad 0.5 \quad 0.31 \quad 0.45]^T$

$i_1 = 1$, $i_2 = 3$, and $e_{i_1} = e_1 = [1 \quad 0 \quad 0 \quad 0 \quad 0]^T$, $e_{i_2} = e_3 = [0 \quad 0 \quad 1 \quad 0 \quad 0]^T$.

Thus,

$$\mathbf{r}_i(k) = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

$$\text{and} \quad \mathbf{r}_f = \begin{bmatrix} -0.26 \\ 0 \\ -0.5 \\ 0.31 \\ 0.45 \end{bmatrix}.$$

The minimum conduction-time solution of $\mathbf{H}_1\mathbf{u} = \mathbf{r}_f(k) + \lambda_1\mathbf{d}$ is obtained from (31)

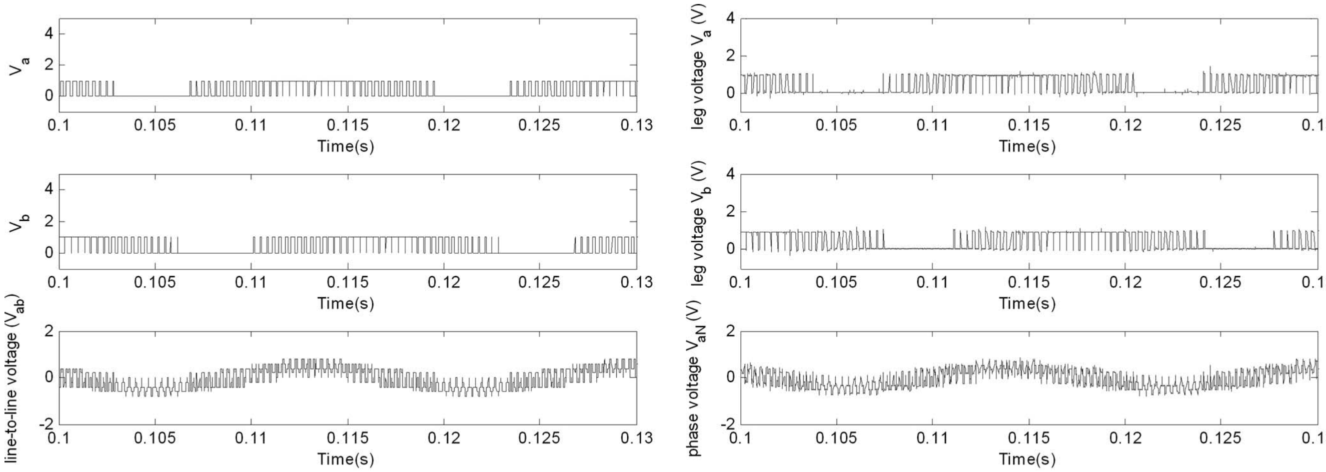
$$\hat{\mathbf{H}}_s = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{u}_1 = \begin{bmatrix} 0.14 \\ 0.31 \\ 0.26 \\ 0.24 \end{bmatrix}.$$

$$\left(Note{:}\mathbf{P}_M = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \right).$$

Therefore, the N switching state candidates and corresponding duty ratios are [see (32) and (33)]

$$\mathbf{h}_{s0}+\mathbf{r}_i = [1 \quad 2 \quad 1 \quad -2 \quad -2]^T,$$

$$u_0 = 1 - (0.14 + 0.31 + 0.26 + 0.24) = 0.05 \quad (38a)$$



(a) (b)

Fig. 9 Top and middle: leg voltages of the first and the second legs; bottom: line-to-line voltages. (a) Simulation results. (b) Experimental results (values are normalized to the dc supply voltage).

$$\mathbf{h}_{s1} + \mathbf{r}_i = [1 \quad 2 \quad 1 \quad -2 \quad -1]^T, \qquad u_1 = 0.14 \tag{38b}$$

$$\mathbf{h}_{s2} + \mathbf{r}_i = [1 \quad 2 \quad 1 \quad -1 \quad -1]^T, \qquad u_2 = 0.31 \tag{38c}$$

$$\mathbf{h}_{s3} + \mathbf{r}_i = [1 \quad 3 \quad 1 \quad -1 \quad -1]^T, \qquad u_3 = 0.26 \tag{38d}$$

$$\mathbf{h}_{s4} + \mathbf{r}_i = [2 \quad 3 \quad 1 \quad -1 \quad -1]^T, \qquad u_4 = 0.24 \tag{38e}$$

To derive reasonable switching states, (36) is used to find the level shift for $\mathbf{h}_{sj}$s as

$$\eta_0 = -2, \quad \eta_1 = -2, \quad \eta_2 = -1, \quad \eta_3 = -1, \quad \eta_4 = -1.$$

Therefore, the valid switching states and corresponding duties are

$$\mathbf{h}'_{s0} = [3 \quad 4 \quad 3 \quad 0 \quad 0]^T, \qquad u_0 = 0.05 \tag{39a}$$

$$\mathbf{h}'_{s1} = [3 \quad 4 \quad 3 \quad 0 \quad 1]^T, \qquad u_1 = 0.14 \tag{39b}$$

$$\mathbf{h}'_{s2} = [2 \quad 3 \quad 2 \quad 0 \quad 0]^T, \qquad u_2 = 0.31 \tag{39c}$$

$$\mathbf{h}'_{s3} = [2 \quad 4 \quad 2 \quad 0 \quad 0]^T, \qquad u_3 = 0.26 \tag{39d}$$

$$\mathbf{h}'_{s4} = [3 \quad 4 \quad 2 \quad 0 \quad 0]^T, \qquad u_4 = 0.24 \tag{39e}$$

To decide the switching sequence for minimum switching number, the summations of elements in $\mathbf{h}'_{sj}$, $j = 0 \sim 4$, are calculated: 10, 11, 7, 8, and 9. Then the switching states are applied according to the increasing order of these five summations, i.e., switching sequence is $\mathbf{h}'_{s2} \to \mathbf{h}'_{s3} \to \mathbf{h}'_{s4} \to \mathbf{h}'_{s0} \to \mathbf{h}'_{s1} \to \mathbf{h}'_{s0} \to \mathbf{h}'_{s4} \to \mathbf{h}'_{s3} \to \mathbf{h}'_{s2}$ and the corresponding duties are 0.155, 0.13, 0.12, 0.025, 0.14, 0.025, 0.12, 0.13, and 0.155. Notably, only one phase leg changes its states during each state transition yielding the least switching number for one state transition. Further, with the fewest space vector usage that implies the minimum transition occurrence, the switching sequence of the proposed scheme has the minimum switching number.

Remark 4: The switching states obtained in (39a) and (39e) correspond to different gate drive signals of switches for different inverter types. To have a union presentation without losing the focus of this study, it is not discussed here.

Remark 5: The maximum modulation index, $m_{\max}$, of the multilevel multiphase system for sinusoidal reference signals is the same as the two-level multiphase system [5]. By definition, the maximum modulation index is the maximum applicable ratio of reference amplitude to maximum supplied voltage (*L–1*); thus, the allowable amplitude R for the *L*-level system is written as follows:

$$R \le (L-1)\, m_{\max} = \frac{(L-1)}{2\cos\left(\frac{\pi}{2N}\right)}$$

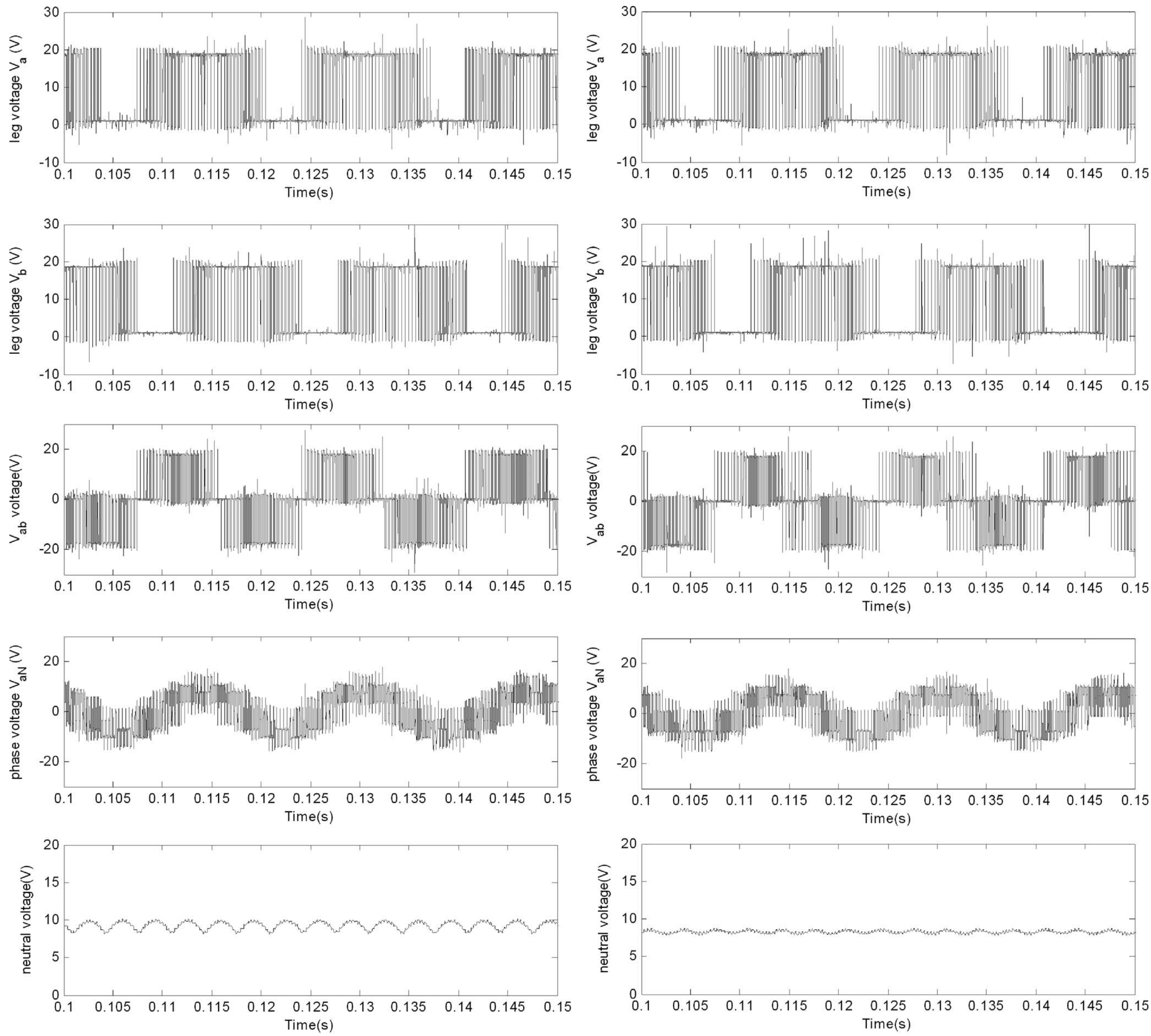
for odd *N* and

$$R \le (L-1)\, m_{\max} = \frac{(L-1)}{2}$$

for even *N*.

IV. Simulation Results

Two 60 Hz five-phase sinusoidal waves with normalized amplitudes of 0.51 and 2.0 are applied to the five-level five-phase system in the simulation. The sampling frequency of the reference input is 3 kHz and the resolution in one input period is 8 bits, yielding a minimum pulse width of $1/(3k \times 2^8)$ s. Switching states for the first two phases and the corresponding line-to-line voltage V_{ab} are shown in Fig. 3. Fig. 4 shows the reference voltage and phase voltage applied by the VSI. To have nonsinusoidal references, a third harmonic with amplitude 1/5



(a) (b)

Fig. 10 Experimental results: leg voltage a, leg voltage b, phase voltage, and neutral voltage (from top to bottom): (a) Sinusoidal reference input. (b) Sinusoidal reference with third harmonic.

of the fundamental amplitudes is added to the reference. The corresponding waveforms are shown in Figs. 5 and 6.

V. Experimental Results

A five-phase two-level *R–L* system is built to verify the effectiveness of the proposed scheme. Fig. 7 shows the implementation block diagram. The proposed space vector PWM solution for minimum conduction time is employed to produce 10 control signals for the five-phase VSI. A wye-type R–L circuit with the resistance of 10 Ω and inductance of 0.5 mH is connected to the output of the VSI as the load. The dc supply voltage of the VSI is 20 V. Fig. 8 shows a picture of the experimental platform.

Five sinusoidal waves with $2\pi/5$ phase shift are applied as the reference signals. The frequency is 60 Hz and the normalized amplitude is 0.51. A comparison between experimental and simulation results shown in Fig. 9 is in a good agreement. A third harmonics with amplitude 1/5 of the fundamental amplitude is further added to the reference signals. Fig. 10 shows various signals measured from the first phase of the *R–L* circuit under the sinusoidal references with/without third harmonics. Leg voltage is the output voltage of one inverter leg with respect to ground, and the corresponding phase voltage is the difference between leg voltage and neutral voltage. The filtered neutral voltage (with respect to ground) is also shown in Fig. 10. Five-phase current signals are measured as shown in Fig. 11. The five-phase current signals with and without third harmonic are

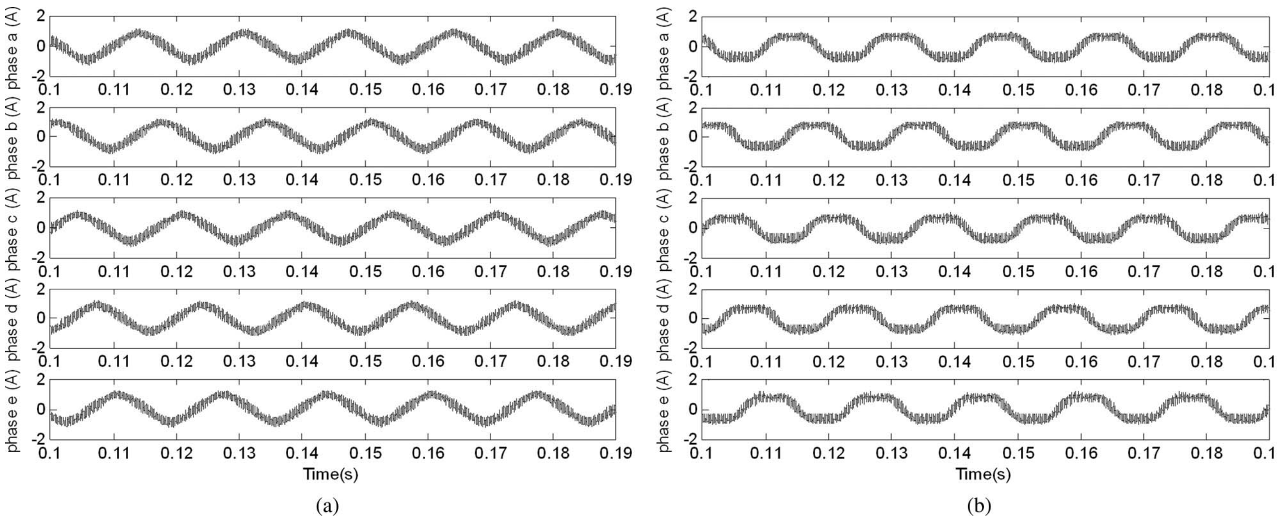


Fig. 11 Phase currents: (a) Sinusoidal reference input. (b) Sinusoidal reference with third harmonic.

reproduced accordingly at the load indicates that the proposed scheme selects switching control commands effectively.

VI. Conclusion

This study adopts an algebraic analysis viewpoint of space-vector PWM technology as opposed to a vector analysis one, which is commonly adopted. This generates a concise and effective treatment of the problem. By applying eigenspace decomposition, a general solution to match reference signals under limited space vectors can be derived for a two-level multiphase system. The free design parameter in the solution is chosen, such that total conduction time for the power stage is minimized. This yields a switching scheme that also uses the fewest space vectors. Extensions to the multilevel multiphase system are given using two-level results. The results show that the computation for space vectors and corresponding duty ratios is simple and the computation complexity is independent of the level number. We believe that this study will increase the current understanding of the multilevel multiphase space-vector PWM in aspects such as harmonic analysis or waveform modifications for efficiency enhancement. For future extension, one can consider noise effect due to switching. For instance, ideas such as noise shaping [24], [25], which generates a different matching conditions, can be investigated.

To demonstrate the proposed strategy, simulation of the five-level five-phase system is shown. The experimental platform of a two-level five-phase system is constructed and experimental results are compared to the simulation to validate the theory.

Appendix A

For a three-phase two-level system, the resulting line-to-neutral voltage vectors, $\mathbf{s}_i \in \mathbb{R}^{N\times 1}$ and the corresponding bitwise expansion vectors $\mathbf{\Gamma}_i$, $i = 0$ to $M - 1$, are

$$\mathbf{s}_0 = \mathbf{s}_{000} \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{s}_1 = \mathbf{s}_{100} \triangleq \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\mathbf{s}_2 = \mathbf{s}_{010} \triangleq \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}, \qquad \mathbf{s}_3 = \mathbf{s}_{110} \triangleq \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$\mathbf{s}_4 = \mathbf{s}_{001} \triangleq \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \qquad \mathbf{s}_5 = \mathbf{s}_{101} \triangleq \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\mathbf{s}_6 = \mathbf{s}_{011} \triangleq \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}, \qquad \mathbf{s}_7 = \mathbf{s}_{111} \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\Gamma_0 \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma_1 \triangleq \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma_2 \triangleq \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \Gamma_3 \triangleq \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\Gamma_4 \triangleq \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \Gamma_5 \triangleq \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \Gamma_6 \triangleq \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \Gamma_7 \triangleq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

It is straightforward to know that three vectors ($\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_4$) belong to group 1, i.e., $\sigma(i) = 1$ for $i = 1, 2$, and 4 and the matrix $\mathbf{S}_c$ in (2) is in the form.

$$\mathbf{S}_c = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}.$$

Therefore, we can write $\mathbf{s}_i = \mathbf{S}_c \mathbf{\Gamma}_i$. By observing $\mathbf{s}_{100} = -\mathbf{s}_{011}$, $\mathbf{s}_{010} = -\mathbf{s}_{101}$ and $\mathbf{s}_{001} = -\mathbf{s}_{110}$, another expression of $\mathbf{s}_i$ is obtained: $\mathbf{s}_i = -\mathbf{s}_{\bar{i}} = -\mathbf{S}_c \mathbf{\Gamma}_{\bar{i}}$ where $\bar{i}$ is the bitwise complement of i. Then the general expression of phase voltages for three-phase system is derived as shown in (7). By eigenvalue decomposition, $\mathbf{S}_c$ is written as follows:

$$\mathbf{S}_c = \begin{bmatrix} \mathbf{V}_c \\ \mathbf{v}_0^T \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_c \\ \mathbf{v}_0^T \end{bmatrix}$$

where

$$\mathbf{V}_c = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}$$

and $\mathbf{v}_0 = \left[(1/\sqrt{3}) \quad (1/\sqrt{3}) \quad (1/\sqrt{3})\right]^T$. Therefore, the matching (9) is transformed into (22)

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{u}(k) = \mathbf{H}_1 \mathbf{u}(k) = \mathbf{r}(k) + \lambda_1 \mathbf{d}. \tag{A1}$$

According to the minimal conduction time discussed in Section II-C, $\lambda_1 = -\rho_3$ and apply the permutation matrix $\mathbf{P}_M$, (A1) becomes

$$\mathbf{P}_M \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{u}(k) = \mathbf{P}_M \mathbf{H}_1 \mathbf{u}(k) = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \\ 0 \end{bmatrix}. \tag{A2}$$

Notably, $\mathbf{P}_M$ will permute the row vectors of $\mathbf{H}_1$, i.e., it is easy to observe that column vectors of $\mathbf{P}_M \mathbf{H}_1$ are the same as that of $\mathbf{H}_1$ but with different order. Therefore, solving (A2) is the same as solving

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{u}(k) = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \\ 0 \end{bmatrix}. \tag{A3}$$

Because the last element of the right-hand-side vector in (A3) is always zero, the problem is reduced to

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{u}(k) = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \\ 0 \end{bmatrix}$$

$$\text{or} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \end{bmatrix} \tag{A4}$$

To have least state transition for minimum switching number, only two column vectors of $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ are used. Further, to ensure the existence of the solution, two vectors are selected in the form $\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$ where c is either 0 or 1. The selection of c depends on the minimum conduction time

$$\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \end{bmatrix}$$

which implies $u_2(k) = \rho_2 - \rho_3$ and $u_1(k) = \rho_1 - \rho_3 - c(\rho_2 - \rho_3)$, yielding the total conduction time $u_1(k) + u_2(k) = \rho_1 - \rho_3 + (1 - c)(\rho_2 - \rho_3)$. For $\rho_2 - \rho_3 \geq 0$, select $c = 1$ will result in minimum conduction time $\rho_1 - \rho_3$. Therefore, we can write

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} \rho_1 - \rho_3 \\ \rho_2 - \rho_3 \\ 0 \end{bmatrix} \quad \text{with} \quad u_1(k) = \rho_1 - \rho_2 \ \text{ and } \ u_2(k) = \rho_2 - \rho_3. \tag{A5}$$

Comparing (A1) and (A2) with (A5), we obtain

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \mathbf{P}_M (\mathbf{r}(k) + \lambda_1 \mathbf{d}).$$

Further apply $\mathbf{P}_M^{-1} = \mathbf{P}_M^T$

$$\mathbf{P}_M^T \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \mathbf{r}(k) + \lambda_1 \mathbf{d}$$

implying that the selected switching states are $\mathbf{P}_M^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{P}_M^T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and the corresponding duties are $u_1(k) = \rho_1 - \rho_2$ and $u_2(k) = \rho_2 - \rho_3$. The total conduction time is $\rho_1 - \rho_3$. Notably, when total conduction time is less than 1 ($\rho_1 - \rho_3 < 1$), then zero switching state will be applied during the rest of the duty, i.e., with the duty of $1 - \rho_1 + \rho_3$.

APPENDIX B

From (1), the vector $\mathbf{s}_{i_n}$ in group *n* is represented as follows:

$$\mathbf{s}_{i_n j} = \gamma_j(i_n) - \frac{n}{N}. \tag{B1}$$

There are *n* bits in value i_n that equal 1, and the bit numbers are denoted as $j_1, j_2, \ldots, j_n$. The other $(N - n)$ bits are represented as $j_{n+1}, j_{n+2}, \ldots, j_n$, i.e.,

$$\gamma_{j_1}(i_n) = \gamma_{j_2}(i_n) = \cdots, = \gamma_{j_n}(i_n) = 1 \text{ and}$$

$$\gamma_{j_{n+1}}(i_n) = \gamma_{j_{n+2}}(i_n) = \cdots, = \gamma_{j_N}(i_n) = 0.$$

Now consider the vectors, $\mathrm{s}_{i_{1,m}}, m = 1 - n$, in group 1. They are represented as $\mathrm{s}_{i_{1,m} j} = \gamma_j(i_{1,m}) - (1/N), m = 1 - n$. Suppose the only bit that equals 1 in the value $i_{1,m}$ is j_m, i.e., $\gamma_{j_m}(i_{1,m}) = 1$. Then the sum of these *n* space vectors in group 1, $\mathrm{s}_{i_{1,m} j}, m = 1 - n$ is written as follows:

$$\sum_{m=1}^{n} \mathbf{s}_{i_{1,m} j} = \sum_{m=1}^{n} \left(\gamma_j(i_{1,m}) - \frac{1}{N}\right) = \sum_{m=1}^{n} \gamma_j(i_{1,m}) - \frac{n}{N}. \tag{B2}$$

Notably, the term $\sum_{m=1}^{n} \gamma_j(i_{1,m})$ is 1 when $j = j_m$ and 0 otherwise, indicating that the right side of (B1) is equivalent to that of (B2). Therefore, when the union of bit numbers that correspond to a bit value 1 in *n FSV*s matches the set of bit numbers with a value 1 in the vector belongs to group *n*, the group-*n*-vector can be represented by a linear combination of these *n FSV*s.

Appendix C

From (13) and (20), we obtain that

$$\mathbf{u}_c(k) = \mathbf{H}\mathbf{u}(k) = \mathbf{r}(k) + \lambda\mathbf{d}$$

From (12) and the fact that $\mathbf{H}_2 = \mathbf{\Lambda} - \mathbf{H}_1$ where $\mathbf{\Lambda} \in \mathbf{R}^{N\times M}$ and all elements in $\mathbf{\Lambda}$ are 1, we can write $(\mathbf{H}_1\mathbf{B} - (\mathbf{\Lambda} - \mathbf{H}_1)(\mathbf{I} - \mathbf{B}))\,\mathbf{u}(k) = \mathbf{r}(k) + \lambda\mathbf{d}$ or equivalently, $\mathbf{H}_1\mathbf{u}(k) - \mathbf{\Lambda}(\mathbf{I} - \mathbf{B})\,\mathbf{u}(k) = \mathbf{r}(k) + \lambda\mathbf{d}$, which implies

$$\mathbf{H}_1\mathbf{u}(k) = \mathbf{r}(k) + (\lambda + b)\,\mathbf{d}$$

where $b = \sum_{i=1}^{M}(1 - \beta_i)\,\alpha_i$.

Appendix D

First, $\mathbf{H}_d$ should be chosen such that its rank is *N*–1. Second, because the elements in the RHS vector in (24) are positive numbers in descending order and elements of $\mathbf{u}_1$ must be positive, the sum of row vectors in $\mathbf{H}_d$ should also be in a descending order to ensure the existence of feasible solutions. Notably, elements of $\mathbf{H}_d$ are either 0 or 1. To unify the expression of $\mathbf{H}_d$, this study permutes the columns of $\mathbf{H}_d$, such that $\mathbf{H}_d$ can be written as an upper triangular matrix [see (D1)]. Notably, the columns of $\mathbf{H}_d$ are the selected switching states, i.e., permuting the columns will not alter the solution

$$\mathbf{H}_d = \begin{bmatrix} 1 & h_{12} & \cdots & h_{1\times(N-1)} \\ 0 & 1 & & \vdots \\ \vdots & \ddots & \ddots & h_{(N-2)\times(N-1)} \\ 0 & \cdots & 0 & 1 \end{bmatrix}. \tag{D1}$$

The corresponding duties of (24), $\mathbf{u}_1 = [\bar{u}_1 \quad \bar{u}_2 \quad \cdots \quad \bar{u}_{N-1}]^T$, are then written as follows:

$$\bar{u}_{N-1} = \rho_{N-1} - \rho_N$$

and

$$\bar{u}_i = \rho_i - \rho_N - \sum_{j=i+1}^{N-1} h_{ij}\bar{u}_j$$

for

$$i = (N-2), (N-3), \ldots, 2, 1,$$

yielding total conduction time of

$$\sum_{i=1}^{N-1}\bar{u}_i = \sum_{i=1}^{N-2}\left(\rho_i - \rho_N - \sum_{j=i+1}^{N-1} h_{ij}\bar{u}_j\right) + \rho_{N-1} - \rho_N = -\rho_N - \sum_{i=1}^{N-2}\left(\sum_{j=i+1}^{N-1} h_{ij}\bar{u}_j\right). \tag{D2}$$

To minimize the value in (D2), h_{ij}, when $i = 1, 2, \ldots, (N-2)$ and $j > i$ are selected to be one because for all *j*, $u_j > 0$, indicating that $\mathbf{H}_d$ is the upper triangular matrix with all upper elements equal to one.

Appendix E

The maximum modulation index, $m_{\max}$, of the *N*-phase system with a sinusoidal reference signal is obtained by solving the minimization problem

$$m_{\max} = \underset{\substack{\theta\in\mathbb{S} \\ t\in[0,2\pi/\omega]}}{\mathrm{Min}} \frac{1}{|\sin(\omega t) - \sin(\omega t - \theta)|}$$

where

$$\mathbb{S} = \left\{\frac{2\pi}{N}, \frac{2\pi}{N}\times 2, \frac{2\pi}{N}\times 3, \cdots, \frac{2\pi}{N}\times(N-1)\right\}$$

or equivalently

$$m_{\max} = \underset{\substack{n\in\mathbb{Z} \\ t\in[0,2\pi/\omega]}}{\mathrm{Min}} \frac{1}{\left|\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{N}n\right)\right|} \quad \text{where}$$

$$\mathbb{Z} = \{1, 2, 3, \ldots, (N-1)\}. \tag{E1}$$

Observe that

$$\left|\sin(\omega t) - \sin\left(\omega t - \frac{2\pi}{N}n\right)\right| = \left|2\sin\left(\frac{\pi}{N}n\right)\right| |\cos(\omega t + \phi)| \tag{E2}$$

Using the simplified representation in (E2), (E1) can be written as (E3)

$$m_{\max} = \frac{1}{\underset{n\in\mathbb{Z}}{\mathrm{Max}}\left|2\sin\left(\frac{\pi}{N}n\right)\right|}. \tag{E3}$$

Because of symmetry, one can only consider the values of *n* that result in the phases that are in the first and second quadrants, i.e., $n \in \mathbb{Z}$ and $n \leq (N/2)$. Further, as the phase difference between $(\pi/N)n$ and $(\pi/2)$ decreases, the value of $|2\sin(\pi/N(n))|$ increases. Therefore, finding $m_{\max}$ is equivalent to finding *n* ($n \in \mathbb{Z}$ and $n \leq (N/2)$), such that $|(\pi/N(n)) - (\pi/2)|$ attains the minimum. One can straightforwardly show that $n = (N \pm 1)/2$ for odd N and $n = (N \pm 1)/2$ for even N results in $m_{\max}$, as indicated in (E4)

$$m_{\max} = \frac{1}{2\sin\left(\frac{\pi}{2}\frac{N\pm 1}{N}\right)} = \frac{1}{2\sin\left(\frac{\pi}{2} \pm \frac{\pi}{2N}\right)} = \frac{1}{2\cos\left(\frac{\pi}{2N}\right)}, \quad \text{odd} \quad N \tag{E4a}$$

$$m_{\max} = \frac{1}{2}, \quad \text{even} \quad N. \tag{E4b}$$

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