# Joint Tomlinson–Harashima Source and Linear Relay Precoder Design in Amplify-and-Forward MIMO Relay Systems via MMSE Criterion

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***Abstract*—Existing minimum-mean-square-error (MMSE) transceiver designs in amplify-and-forward (AF) multiple-input–multiple-output (MIMO) relay systems all assume a linear precoder at the source. Nonlinear precoders in such a system have yet to be considered. In this paper, we study a nonlinear transceiver in AF MIMO relay systems in which a Tomlinson–Harashima (TH) precoder is used at the source, a linear precoder is used at the relay, and an MMSE receiver is used at the destination. Since two precoders and three links are involved, the transceiver design, which is formulated as an optimization problem, is difficult to solve. We first propose an iterative method to overcome the problem. In the method, the two precoders are separately optimized in an iterative step. To further improve the performance, we then propose a non-iterative method that can yield closed-form solutions for the precoders. This method uses the primal decomposition technique in which the original optimization can first be decomposed into a master and a subproblem optimization. In the subproblem, the optimum source precoder is solved as a function of the relay precoder. In the master problem, the optimization is then transferred to a relay-precoder-only problem. However, the optimization is not convex, and the primal decomposition cannot be directly applied. We then propose cascading a unitary precoder after the TH precoder so that the optimization in the subproblem and the master problem can be conducted. Furthermore, using a relay precoder structure, we can transfer the master problem to a convex optimization problem and obtain a closed-form solution by the Karuch–Kuhn–Tucker (KKT) conditions. Simulations show that the proposed transceivers can significantly outperform existing linear transceivers.**

***Index Terms*—Amplify-and-forward (AF), joint source/relay precoders, Karuch–Kuhn–Tucker (KKT) conditions, minimum-mean-square-error (MMSE), multiple-input multiple-output (MIMO), primal decomposition approach, Tomlinson–Harashima precoding (THP).**

Manuscript received February 24, 2010; revised July 15, 2010, November 7, 2010, and February 20, 2011; accepted February 27, 2011. Date of publication March 10, 2011; date of current version May 16, 2011. This work was supported in part by the National Science Council under Grant NSC-98-2221-E-009-046-MY3 and in part by MediaTek Research Center at National Chiao Tung University, Taiwan. This paper was presented in part at the 21st Annual IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications, Istanbul, Turkey, September 26–29, 2010. The review of this paper was coordinated by Prof. A. M. Tonello.

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Digital Object Identifier 10.1109/TVT.2011.2125995

## I. Introduction

Spatial diversity is a common method to overcome the multipath channel fading effect in wireless systems. The conventional way to obtain spatial diversity is the use of multiple transmit or multiple receive antennas. When both multiple transmit and multiple receive antennas are used, the system is referred to as a multiple-input–multiple-output (MIMO) system [1]. The MIMO system has been widely studied in the literature since it can enhance diversity and spectral efficiency in an efficient way [1]–[12]. However, due to shadowing, multipath fading, interference, and distance-dependent path losses, the link quality between the source and the destination in a wireless system is not always satisfactory. The fundamental linking problem greatly limits data transmission in wireless systems.

Cooperative communication, which is an emerging technique, has been developed to solve the problem. The main idea in cooperative systems is to deploy relays at strong shadowing areas [13]–[30]. In this manner, signals can be transmitted to the destination via the source-to-destination (direct link) and the source-to-relay and relay-to-destination (relay link) links. The additional links can effectively improve the link quality. Various relaying strategies have been developed such as amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [13], [14]. In AF, the relays receive the signals from the source and retransmit them to the destination with signal amplification only. Such a system is also called a nonregenerative cooperative system [15], [21]–[23], [26], [27], [29], [30]. In DF, the relays decode the received signals, re-encode the information bits, and then retransmit the resultant signals to the destination. The system is also called a regenerative cooperative system. In CF, the received signals at the relays are estimated and compressed and then re-transmitted to the destination; it acts as a compromise between AF and DF. It is simple to see that the DF protocol requires a higher computational complexity and a larger processing delay at the relays. In this paper, we only consider AF-based cooperative systems.

Recently, the MIMO technique was introduced in cooperative systems. With the multiple antennas deployed at each node, a MIMO relay system is constructed [20]–[29]. Capacity bounds for a single-relay MIMO channel were first addressed in [20]. The relay precoder in an AF MIMO relay system was then designed to enhance the overall capacity [21], [22]. Apart from the capacity, the link quality is the alternative criterion that has been considered [23], [24]. In these works, a relay precoder is designed with the minimum-mean-square-error (MMSE) criterion. Most of the MIMO relay systems previously considered only use a relay precoder. Precoding in multiple relays was investigated in [24]. Except for spatial multiplexing, beamforming and antenna selection have also been investigated [25], [26]. In most of those approaches, only the relay link is considered. It was shown that the capacity can be increased if the direct link is further taken into account [22]. More recently, joint source and relay precoders designs were investigated [27]–[30]. In [29] and [30], only the relay link is considered, whereas in [27] and [28], the direct and relay links are simultaneously considered. It is noteworthy that, in the source and relay precoder joint design, the source and relay power constraints are mutually coupled. The coupled power constraints are a main obstacle in the joint design. Most of the existing methods use iterative methods to deal with the problem [27]–[30].

Note that existing precoding designs in AF MMO relay systems all consider linear transceivers, i.e., linear precoders and linear receivers. In this paper, we consider a nonlinear transceiver design in a three-node AF MIMO relay system. In the transceiver, a Tomlinson–Harashima (TH) precoder (THP) is used at the source, a linear precoder is used at the relay, and an MMSE receiver is used at the destination. The THP is a well-known precoding scheme and has been shown to have good performance in conventional point-to-point MIMO systems [11], [12]. Similar to the precoder design in MIMO systems, the transceiver design in MIMO relay systems can also be formulated as an optimization problem. Unfortunately, the MMSE is a complicated function of the source and relay precoders, and optimization is difficult to conduct. Even with numerical methods [35], the problem is still difficult to solve. To obtain a solution, we first propose an iterative method using the TH source precoder proposed in [12] and the relay precoder in [23]. With a given TH source precoder, the optimum relay precoder can be derived with the method in [23]. Conversely, given a relay precoder, the optimum TH source precoder can be obtained with the method in [12]. Due to the iterative process, the computational complexity of this method is high. To overcome the drawback, we then propose another method obtaining closed-form solutions for the TH source and relay precoders. The method uses the primal decomposition technique [35], where the original optimization is divided into a subproblem and a master problem. In the subproblem, we derive the source precoder as a function of the relay precoder. Subsequently, we solve the relay precoder in the master problem. For our problem, however, this decomposition cannot be conducted since the source precoder cannot be solved as a function of the relay precoder. To overcome the difficulty, we propose cascading a unitary precoder after the THP. The unitary precoder can not only facilitate the formulation of the subproblem but improve the MMSE performance as well. With the unitary precoder, the joint optimization problem can finally be recast to an amenable relay-precoder-only problem. However, the simplified problem is still difficult to solve since it is not convex. We then propose a relay precoder structure that can translate the master optimization into a standard scalar-valued concave optimization problem. Using the Karash–Kuhn–Tucker (KKT) conditions, we can finally obtain closed-form solutions for the relay and source precoders. For reference convenience, we refer to the proposed iterative method as proposed Method I and the proposed non-iterative method as proposed Method II.

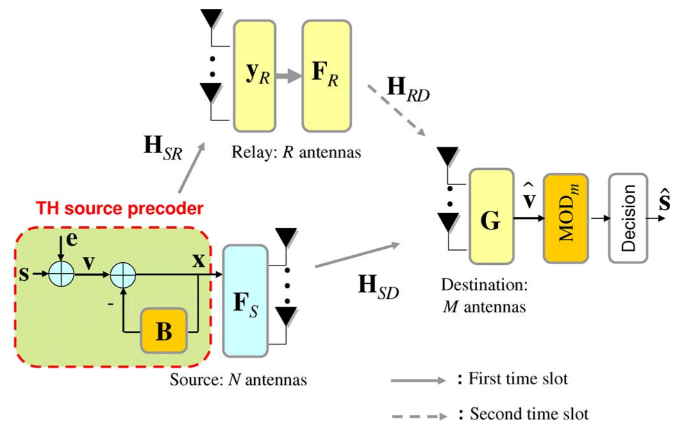


Fig. 1. TH source and linear relay precoded AF MIMO relay system with MMSE receiver.

This paper is organized as follows: In Section II, we first describe an AF MIMO relay system with a TH precoder at the source, a linear precoder at the relay, and an MMSE receiver at the destination. Then, we formulate the optimization problem for the precoder design. In Section III, we first describe proposed Method I for the optimization problem and then detail proposed Method II. In Section IV, we evaluate the performance of the proposed transceivers. Finally, we draw conclusions in Section V.

## II. System Model and Problem Formulation

### A. MMSE Receiver With TH Source and Linear Relay Precoders

We consider a three-node AF MIMO relay precoding system in which $N$, $R$, and $M$ antennas are placed at the source, the relay, and the destination, respectively, as shown in Fig. 1. In the figure, we have two precoders, i.e., a TH source precoder and a linear relay precoder, and a linear MMSE receiver. Here, we consider the general two-phase transmission protocol [21]–[27]. In the first phase, the source signal $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is first fed into the THP. The precoder conducts a successive cancellation operation characterized by a backward squared matrix $\mathbf{B}$ and a modulo operation $\mathrm{MOD}_m(\cdot)$. Each element of $\mathbf{s} = [s_1, \ldots, s_N]^T$, which is modulated by the $m$-state quadrature amplitude modulation (QAM) scheme, takes its real and imaginary values from the set $\{\pm 1, \ldots, \pm(\sqrt{m}-1)\}$. The feedback matrix $\mathbf{B}$ has a lower triangular structure, and its diagonal elements are all zeros. The modulo operation, which is conducted over the real and imaginary parts of the inputs, can be expressed as follows:

$$\mathrm{MOD}_m(x) = x - 2\sqrt{m}\cdot\left\lfloor \frac{x+\sqrt{m}}{2\sqrt{m}} \right\rfloor. \tag{1}$$

Let the signal after the modulo operation be expressed as $\mathbf{x}$. It is clear that each element of $\mathbf{x}$ is bounded between $-\sqrt{m}$ and

$\sqrt{m}$ (real and imaginary parts). With $\mathbf{B}$ and the operation in (1), the elements of $\mathbf{x}$ can be recursively expressed as [12]

$$x_k = s_k - \sum_{l=1}^{k-1} \mathbf{B}(k,l)x_l + \mathbf{e}_k \tag{2}$$

where $x_k$ denotes the $k$th elements of vector $\mathbf{x}$, $\mathbf{B}(k,l)$ is the $(k,l)$th element of matrix $\mathbf{B}$, and $\mathbf{e} = [e_1,\ldots,e_N]^T$ is the error of the modulo operation (the difference between the input and the output). From (2), we can express the transmitted signal $\mathbf{x}$ using the following matrix form:

$$\mathbf{x} = \mathbf{C}^{-1}\mathbf{v} \tag{3}$$

where $\mathbf{C} = \mathbf{B} + \mathbf{I}_N$ is a lower triangular matrix with ones in its diagonal, and $\mathbf{v} = \mathbf{s} + \mathbf{e}$. The TH-precoded $\mathbf{x}$ is further passed through a precoding matrix $\mathbf{F}_S$ and then simultaneously sent to the relay and the destination. It is worth noting that $\mathbf{F}_S$ can improve the performance in a point-to-point TH-precoded MIMO system [12]. Here, as we will see, $\mathbf{F}_S$ can also improve the performance in TH-precoded MIMO relay systems. In proposed Method II, we let $\mathbf{F}_S$ have a unitary structure, which can greatly simplify the joint precoder design and improve the bit-error-rate (BER) performance as well.

In the second phase, the received signal vector at the relay is multiplied by the relay precoder and then transmitted to the destination. Therefore, the signal received at the destination (after the two consecutive phases) can be combined into a vector form as

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{\mathrm{SD}} \\ \mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{H}_{\mathrm{SR}} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_S\mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{n}} \tag{4}$$

where $\mathbf{H}$ and $\mathbf{n}$ denote the equivalent channel matrix and the equivalent noise vector, respectively. In (4), $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the TH-precoded signal vector in (3); $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$ is the received signal vector at the destination; $\mathbf{H}_{\mathrm{SR}} \in \mathbb{C}^{R\times N}$, $\mathbf{H}_{\mathrm{SD}} \in \mathbb{C}^{M\times N}$, and $\mathbf{H}_{\mathrm{RD}} \in \mathbb{C}^{M\times R}$ are the channel matrices of the source-to-relay, the source-to-destination, and the relay-to-destination links, respectively; and $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$, $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ are the received noise vectors at the destination in the first phase, at the relay in the first phase, and at the destination in the second-phase, respectively.

Note that, if $\mathbf{v}$ can be estimated at the destination, $\mathbf{s}$ can be recovered by the modulo operation in (1). Thus, the optimum MMSE receiver $\mathbf{G} \in \mathbb{C}^{N\times 2M}$ can be found by minimizing the mean square error (MSE), which is defined as

$$J = E\left\{\|\mathbf{G}\mathbf{y}_D - \mathbf{v}\|^2\right\}. \tag{5}$$

To solve the problem in (5), we assume that the precoded signals $\mathbf{x}_k$ are statistically independent and have zero mean and the same variance. Let the variance of each element in $\mathbf{s}$ be denoted as $\sigma_s^2$. We then have $E[\mathbf{x}\mathbf{x}^H] = \sigma_s^2\mathbf{I}_N$ and $E[\mathbf{v}\mathbf{v}^H] = \sigma_s^2\mathbf{C}\mathbf{C}^H$. Note that the independent assumption is valid only for a large QAM size (e.g., $m \geq 16$) [11], [33]. For a small QAM size, the covariance matrices of $\mathbf{x}$ and $\mathbf{v}$ are difficult to derive. In this paper, we only consider the symbols with larger QAM size to ease the design problem. Then, the optimum solution of (5) can be obtained as [5]

$$\mathbf{G}_{\mathrm{opt}} = \sigma_s^2\mathbf{C}\mathbf{F}_S^H\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1} \tag{6}$$

where $\mathbf{G}_{\mathrm{opt}}$ denotes the optimum $\mathbf{G}$, and $\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^H]$ denotes the covariance matrix of the equivalent noise vector $\mathbf{n}$. In addition, note here that $\mathbf{n}$ is not white. Denoting the variance of the noise components at the destination as $\sigma_{n,d}^2$ and that at the relay as $\sigma_{n,r}^2$, as well as substituting (6) in (5), we can have the MSE matrix as

$$\begin{aligned}\mathbf{E}(\mathbf{C},\mathbf{F}_S,\mathbf{F}_R) &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \\ &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H.\end{aligned} \tag{7}$$

Then, we have

$$J_{\min} = \mathrm{tr}\{\mathbf{E}\} \tag{8}$$

where $J_{\min}$ is the MMSE, and

$$\begin{aligned}\widetilde{\mathbf{H}} &= \mathbf{R}_n^{-\frac{1}{2}}\mathbf{H} \\ &= \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{\mathrm{SD}} \\ \left(\sigma_{n,r}^2\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{\mathrm{RD}}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-\frac{1}{2}}\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{H}_{\mathrm{SR}} \end{bmatrix}\end{aligned} \tag{9}$$

is defined as the equivalent channel matrix after noise whitening. Note that the total MSE is contributed by both the direct and relay links. By ignoring the direct link and adopting a single precoder at the relay, the problem is reduced to that considered in [23] and [24]. Here, we incorporate the THP as the source precoder and further take the direct link into consideration.

### *B. Problem Formulation*

From the MMSE criterion in (5)–(9), we now can formulate our joint design problem as

$$\begin{aligned}&\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} \ \mathrm{tr}\{\mathbf{E}\} \\ &\text{s.t.} \quad \mathrm{C}_1 \text{ and } \mathrm{C}_2 \\ &\text{where} \\ &\mathrm{C}_1 \text{ and } \mathrm{C}_2 \text{ are source and relay power constraints,} \\ &\text{respectively} \\ &\mathrm{C}_1: \mathrm{tr}\left\{E\left[\mathbf{F}_S\mathbf{x}\mathbf{x}^H\mathbf{F}_S^H\right]\right\} = \sigma_s^2\mathrm{tr}\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \leq P_{S,T} \\ &\mathrm{C}_2: \mathrm{tr}\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{\mathrm{SR}}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{\mathrm{SR}}^H\right)\mathbf{F}_R^H\right\} \leq P_{R,T}\end{aligned} \tag{10}$$

with the inequalities in (10) indicating the transmitted power constraints at the source and the relay. (The maximal available power is $P_{S,T}$ and $P_{R,T}$, respectively.) Taking a close look at (10), we can observe that the cost function is a nonlinear function of $\mathbf{F}_S$ and $\mathbf{F}_R$ and is not convex. Moreover, the relay power constraint is a function of $\mathbf{F}_S$ and $\mathbf{F}_R$. As a result, directly solving the problem is very difficult if not impossible. In the next section, we propose two methods, i.e., an iterative (proposed Method I) and a non-iterative (proposed Method II) method, to solve the problem.

## III. Proposed Joint Source/Relay Precoder Design

### A. Proposed Method I

The work in [12] considers the TH source precoder design in the point-to-point MIMO system, whereas that in [23] considers the relay precoder design in two-hop systems (lacking the direct link). Both works adopt the MMSE receiver. As a result, combining the two designs can be considered as one of the possible solutions for our problem. With some minor assumptions, these two methods can be combined to yield an iterative method. From (4), we can see that, if the relay precoder is given, the MIMO relay system can be viewed as a point-to-point MIMO system. Thus, the method in [12] can be used to find the optimum TH source precoder. Conversely, if the source precoder is given, the MIMO relay system can be viewed as a relay precoded system, and the method in [23] can be used to find the optimum relay precoder. The precoders can be finally obtained when the iterative process converges.

We use the superscript "ite" to denote the parameters involved in proposed Method I. In proposed Method I, we have two processing steps in each iteration. In the first step, $\mathbf{C}^{\rm ite}$ and $\mathbf{F}_S^{\rm ite}$ are given (from the previous iteration). Let $\mathbf{H}_{\rm RD} = \mathbf{U}_{\rm RD}\boldsymbol{\Sigma}_{\rm RD}\mathbf{V}_{\rm RD}^H$ and $\mathbf{H}_{\rm SR}^{\rm ite} = \mathbf{H}_{\rm SR}\mathbf{F}_S^{\rm ite} = \mathbf{U}_{\rm SR}^{\rm ite}\boldsymbol{\Sigma}_{\rm SR}^{\rm ite}\mathbf{V}_{\rm SR}^{{\rm ite}^H}$ be the singular value decomposition (SVD) of $\mathbf{H}_{\rm RD}$ and $\mathbf{H}_{\rm SR}^{\rm ite}$, respectively. We can then solve the optimum relay precoder (with respect to the MMSE receiver) as [23]

$$\mathbf{F}_R^{\rm ite} = \mathbf{V}_{\rm RD}\boldsymbol{\Sigma}_r^{\rm ite}\mathbf{U}_{\rm SR}^{{\rm ite}^H} \tag{11}$$

where $\boldsymbol{\Sigma}_r^{\rm ite}$ is a diagonal matrix with $\sigma_{r,i}^{\rm ite}$ as its $i$th diagonal element. The value of $\sigma_{r,i}^{\rm ite}$ can be calculated as

$$\sigma_{r,i}^{\rm ite} = \sqrt{\left[\frac{\mu_r\sigma_{n,d}\sigma_{{\rm sr},i}^{\rm ite}\sigma_{{\rm rd},i}\left(\sigma_s^2\sigma_{{\rm sr},i}^{{\rm ite}^2}+\sigma_{n,r}^2\right)^{-1/2}-\sigma_{n,d}^2\sigma_s^{-2}}{\sigma_{{\rm rd},i}^2\left(\sigma_{n,r}^2\sigma_s^{-2}+\sigma_{{\rm sr},i}^{{\rm ite}^2}\right)}\right]^+} \tag{12}$$

where $[y]^+ = \max(0, y)$, $\sigma_{{\rm sr},i}^{\rm ite}$ is the $i$th diagonal element of the diagonal matrix $\boldsymbol{\Sigma}_{\rm SR}^{\rm ite}$, $\sigma_{{\rm rd},i}$ is the $i$th diagonal element of the diagonal matrix $\boldsymbol{\Sigma}_{\rm RD}$, and $\mu_r$ is the water level chosen to satisfy the relay power constraint.

In the second step, $\mathbf{F}_R^{\rm ite}$ is given, and we can solve $\mathbf{C}^{\rm ite}$ and $\mathbf{F}_S^{\rm ite}$ [12]. Note that both the source and relay power constraints are functions of $\mathbf{F}_S^{\rm ite}$. In other words, for a new $\mathbf{F}_S^{\rm ite}$, the power constraint for the relay may not be satisfied. We can ignore this problem and only consider the source power constraint. The reason is that, as the iteration proceeds, the variation of $\mathbf{F}_S^{\rm ite}$ will become increasingly smaller. As a result, the relay power constraint will be automatically satisfied when the iteration converges. Let

$$\widetilde{\mathbf{H}}^{\rm ite} = \begin{bmatrix}\mathbf{H}_{\rm SD}\\ \mathbf{H}_{\rm RD}\mathbf{F}_R^{\rm ite}\mathbf{H}_{\rm SR}\end{bmatrix} \tag{13}$$

$$\widetilde{\mathbf{H}}^{{\rm ite}^H}\widetilde{\mathbf{H}}^{\rm ite} = \mathbf{V}_{\widetilde{\mathbf{H}}}^{\rm ite}\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}}^{\rm ite}\mathbf{V}_{\widetilde{\mathbf{H}}}^{{\rm ite}^H} \tag{14}$$

be the eigendecomposition (ED) of $\widetilde{\mathbf{H}}^{{\rm ite}^H}\widetilde{\mathbf{H}}^{\rm ite}$, where $\mathbf{V}_{\widetilde{\mathbf{H}}}^{\rm ite}$ is a unitary matrix with the eigenvectors as its columns, and $\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}}^{\rm ite} = {\rm diag}\{\lambda_{\widetilde{\mathbf{H}},1}^{\rm ite}, \ldots, \lambda_{\widetilde{\mathbf{H}},N}^{\rm ite}\}$ is a diagonal matrix with the eigenvalues as its diagonal elements. The optimum $\mathbf{F}_S^{\rm ite}$ is then given by [12]

$$\mathbf{F}_S^{\rm ite} = \mathbf{V}_{\widetilde{\mathbf{H}}}^{\rm ite}\boldsymbol{\Omega}^{\rm ite}\mathbf{P}^{\rm ite} \tag{15}$$

where $\boldsymbol{\Omega}^{\rm ite}$ is a diagonal matrix with the $i$th diagonal element of $\omega_i$, and $\mathbf{P}^{\rm ite}$ is a unitary matrix. The value of $\omega_i$ can be calculated as

$$\omega_i = \sqrt{\left(\mu_s - \frac{N\sigma_s^2}{P_{S,T}\lambda_{\widetilde{\mathbf{H}},i}^{\rm ite}}\right)^+} \tag{16}$$

where $\mu_s$ is the water level chosen to satisfy the source power constraint. The matrix $\mathbf{P}^{\rm ite}$ makes the diagonal entries of a lower triangular matrix, i.e., $\mathbf{L}^{\rm ite}$, all equal. The relationship of $\mathbf{P}^{\rm ite}$ and $\mathbf{L}^{\rm ite}$ can be described through the following Cholesky factorization:

$$\mathbf{L}^{\rm ite}\mathbf{L}^{{\rm ite}^H} = \mathbf{P}^{{\rm ite}^H}\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\boldsymbol{\Omega}^{{\rm ite}^H}\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}}^{\rm ite}\boldsymbol{\Omega}^{\rm ite}\right)^{-1}\mathbf{P}^{\rm ite}. \tag{17}$$

It is found that $\mathbf{P}^{\rm ite}$ can be obtained by the geometric mean decomposition (GMD) as [12]

$$\begin{aligned}\widetilde{\mathbf{D}}^{{\rm ite}^{1/2}} &= \left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\boldsymbol{\Omega}^{{\rm ite}^H}\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}}^{\rm ite}\boldsymbol{\Omega}^{\rm ite}\right)^{-1/2}\\ &= \mathbf{Q}^{\rm ite}\mathbf{R}^{\rm ite}\mathbf{P}^{{\rm ite}^H}\end{aligned} \tag{18}$$

where $\mathbf{Q}^{\rm ite} \in \mathbb{C}^{N\times N}$ and $\mathbf{P}^{\rm ite} \in \mathbb{C}^{N\times N}$ are both unitary matrices, and $\mathbf{R}^{\rm ite}$ is an upper triangular matrix with equal diagonal elements. Using (18), we can also obtain the optimum $\mathbf{C}^{\rm ite}$ as

$$\mathbf{C}^{\rm ite} = \mathbf{D}^{\rm ite}\mathbf{R}^{{\rm ite}^{-H}} \tag{19}$$

where $\mathbf{D}^{\rm ite}$ is a diagonal matrix scaling the diagonal element to unity. Note here that $\mathbf{F}_S^{\rm ite}$ is not restricted to have a unitary structure.

### B. Proposed Method II

Since simultaneously finding the optimum $\mathbf{F}_S$ and $\mathbf{F}_R$ in (10) is almost not possible, we resort to the primal decomposition method [35] translating (10) into a subproblem and a master problem. The subproblem is to first optimize the source precoder, and then, the master problem is to optimize the relay precoder. To proceed, we reformulate (10) as

$$\begin{aligned}\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} {\rm tr}\left\{\mathbf{E}(\mathbf{C},\mathbf{F}_S,\mathbf{F}_R)\right\} &= \min_{\mathbf{F}_R}\min_{\mathbf{C},\mathbf{F}_S} {\rm tr}\left\{\mathbf{E}(\mathbf{C},\mathbf{F}_S,\mathbf{F}_R)\right\}\\ &\quad {\rm s.t.}\quad {\rm C}_1 \text{ and } {\rm C}_2.\end{aligned} \tag{20}$$

In the subproblem, the relay precoder $\mathbf{F}_R$ is assumed to be given. Then, the optimum $\mathbf{C}$ and $\mathbf{F}_S$ can first be derived as a function of $\mathbf{F}_R$. As a result, the joint precoder design can be reduced to the master optimization problem, in which the only unknown is the relay precoder. To obtain a closed-form solution, we let $\mathbf{F}_S$ have a unitary structure. The inclusion of $\mathbf{F}_S$, as we will see, has two main reasons: 1) It can decouple the source and relay power constraints, facilitating the subproblem optimization. 2) By a proper design of $\mathbf{F}_S$, the MMSE can have an amenable form, leading to a tractable optimization problem.

Let $\mathbf{F}_S = \alpha\mathbf{U}_S$, where $\mathbf{U}_S$ is a unitary matrix and $\alpha$ is a scalar designed to satisfy the power constraints. The subproblem now becomes the optimization of $\alpha$, $\mathbf{U}_S$, and $\mathbf{C}$, as given by

$$\begin{aligned}&\min_{\mathbf{C},\alpha,\mathbf{U}_S} \operatorname{tr}\{\mathbf{E}(\mathbf{C},\alpha\mathbf{U}_S,\mathbf{F}_R)\}\\ &\text{s.t.}\quad \mathrm{C}_1' \text{ and } \mathrm{C}_2\\ &\text{where } \mathrm{C}_1' \text{ is the modified power constraint}\\ &\text{for the source node;}\quad \mathrm{C}_1' : N\sigma_s^2\alpha^2 \le P_{S,T}. \end{aligned} \tag{21}$$

Let $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} = \mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{\Lambda}_{\tilde{\mathbf{H}}}\mathbf{V}_{\tilde{\mathbf{H}}}^H$ be the ED of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, where $\mathbf{V}_{\tilde{\mathbf{H}}}$ is a unitary matrix with the eigenvectors as its columns and $\mathbf{\Lambda}_{\tilde{\mathbf{H}}} = \operatorname{diag}\{\boldsymbol{\lambda}_{\tilde{\mathbf{H}},1},\ldots,\boldsymbol{\lambda}_{\tilde{\mathbf{H}},N}\}$ is a diagonal matrix with the eigenvalues as its diagonal elements.

*Theorem 1:* The optimum solutions of the subproblem (21) are

$$\mathbf{F}_{S,opt} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{P} \tag{22}$$

$$\mathbf{C}_{\mathrm{opt}} = \mathbf{D}\mathbf{L}^{-1} \tag{23}$$

where $\mathbf{L}$ is a lower triangular matrix obtained with the Cholesky factorization given by

$$\mathbf{L}\mathbf{L}^H = \mathbf{P}^H\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{\Lambda}_{\tilde{\mathbf{H}}}\right)^{-1}\mathbf{P} \tag{24}$$

$\mathbf{D}$ is a diagonal matrix scaling the diagonal elements of $\mathbf{C}_{\mathrm{opt}}$ to unity, and $\mathbf{P}$ is the unitary matrix obtained from a GMD given by $(\sigma_s^{-2}\mathbf{I}_N + (P_{S,T}/N\sigma_s^2)\mathbf{\Lambda}_{\tilde{\mathbf{H}}})^{-1/2} = \mathbf{QRP}^H$. The resultant MMSE can then be expressed as

$$J_{\min} = N\prod_{k=1}^{N}\left(\lambda_{\widetilde{\mathbf{H}},k}\frac{P_{S,T}}{N\sigma_s^2} + \sigma_s^{-2}\right)^{-1/N}. \tag{25}$$

*Proof:* See the Appendix. ■

Now, the remaining work is to solve the master problem. Considering (22) and (25), we can then formulate the master problem in (20) as

$$\min_{\mathbf{F}_R} N\prod_{k=1}^{N}\left(\lambda_{\widetilde{\mathbf{H}},k}\frac{P_{S,T}}{N\sigma_s^2} + \sigma_s^{-2}\right)^{-1/N} \quad \text{s.t. } \mathrm{C}_2. \tag{26}$$

Note that the eigenvalues are a nonlinear function of $\mathbf{F}_R$, and the cost function is not convex. To solve the master problem, let us first consider the following equivalence:

$$\max_{\mathbf{F}_R} \det\left(\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\right) \quad \text{s.t. } \mathrm{C}_2. \tag{27}$$

It is simple to see that the problem is not solvable since the utility function in (27) is still a nonconvex function of the relay precoder. To find a clue for the solution, we first consider an upper bound of (27) with the Hardamard inequality, which is described in the following lemma:

*Lemma 1 [34]:* Let $\mathbf{M} \in \mathbb{C}^{N\times N}$ be a positive definite matrix; then

$$\det(\mathbf{M}) \le \prod_{i=1}^{N}\mathbf{M}(i,i) \tag{28}$$

where $\mathbf{M}(i,i)$ denotes the $i$th diagonal element of $\mathbf{M}$. The equality in (28) holds when $\mathbf{M}$ is a diagonal matrix. This suggests that a diagonalization of the matrix in the utility function may lead to the optimum result. Indeed, in conventional MIMO systems, the diagonalization operation has been used to derive the optimum precoder [6]. In MIMO relay systems, however, the optimality of the diagonalization operation has not been rigorously proved. Nevertheless, this approach is still used in many works to simplify related designs [21]–[24]. For simplicity, we follow the same idea to diagonalize the matrix in (27). Unfortunately, from (9), we can find that $\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$ is a summation of two separated matrices, that one of them does not depend on $\mathbf{F}_R$, and that the diagonalization cannot be directly conducted. The following lemma suggests a feasible way to overcome the problem:

*Lemma 2 [34]:* Let $\mathbf{A} \in \mathbb{C}^{N\times N}$ be a positive matrix, and let $\mathbf{B} \in \mathbb{C}^{N\times N}$; then

$$\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A})\det(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}). \tag{29}$$

In the use of (29), we let $\mathbf{B} = \mathbf{H}_{\mathrm{SR}}^H\mathbf{F}_R^H\mathbf{H}_{\mathrm{RD}}^H \times (\sigma_{n,r}^2\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{\mathrm{RD}}^H + \sigma_{n,d}^2\mathbf{I}_M)^{-1}\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{H}_{\mathrm{SR}}$ and $\mathbf{A} = (N/P_{S,T})\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{\mathrm{SD}}^H\mathbf{H}_{\mathrm{SD}}$. We then have the equivalence (30), shown at the bottom of the page, where $\mathbf{H}_{\mathrm{SR}}' = \mathbf{H}_{\mathrm{SR}}(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{\mathrm{SD}}^H\mathbf{H}_{\mathrm{SD}})^{-(1/2)}$, and $\det(\mathbf{A})$ is ignored since it is not a function of $\mathbf{F}_R$. Equation (30) suggests a feasible way to diagonalize the matrix in the utility function. Consider the following SVD:

$$\mathbf{H}_{\mathrm{RD}} = \mathbf{U}_{\mathrm{RD}}\mathbf{\Sigma}_{\mathrm{RD}}\mathbf{V}_{\mathrm{RD}}^H \tag{31}$$

$$\mathbf{H}_{\mathrm{SR}}' = \mathbf{U}_{\mathrm{SR}}'\mathbf{\Sigma}_{\mathrm{SR}}'\mathbf{V}_{\mathrm{SR}}'^H \tag{32}$$

where $\mathbf{U}_{\mathrm{RD}} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}_{\mathrm{SR}}' \in \mathbb{C}^{R\times R}$ are the left singular matrices of $\mathbf{H}_{\mathrm{RD}}$ and $\mathbf{H}_{\mathrm{SR}}'$, respectively; $\mathbf{\Sigma}_{\mathrm{RD}} \in \mathbb{R}^{M\times R}$ and $\mathbf{\Sigma}_{\mathrm{SR}}' \in \mathbb{R}^{R\times N}$ are the diagonal singular value matrices of $\mathbf{H}_{\mathrm{RD}}$ and $\mathbf{H}_{\mathrm{SR}}'$, respectively; and $\mathbf{V}_{\mathrm{RD}}^H \in \mathbb{C}^{R\times R}$ and $\mathbf{V}_{\mathrm{SR}}'^H \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}_{\mathrm{RD}}$ and $\mathbf{H}_{\mathrm{SR}}'$, respectively.

$$\arg\max_{\mathbf{F}_R}\det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right) = \arg\max_{\mathbf{F}_R}\det\left(\mathbf{I}_N + \sigma_{n,d}^2\mathbf{H}_{\mathrm{SR}}'^H\mathbf{F}_R^H\mathbf{H}_{\mathrm{RD}}^H\left(\sigma_{n,r}^2\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{\mathrm{RD}}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{\mathrm{RD}}\mathbf{F}_R\mathbf{H}_{\mathrm{SR}}'\right) \tag{30}$$

TABLE I
COMPUTATIONAL COMPLEXITY OF EXISTING AND PROPOSED METHODS (MMSE RECEIVER)

| Precoders | FLOPs (General) | FLOPs ($N=R=M$) |
|---|---|---|
| Method I | $O(I_i(RN^2+N^3+R^3+I_sN+I_r\kappa))$ | $O(12N^3+8N^2)$ |
| Method II | $O(N^3+RN^2+MR^2+R^3+MN^2+I_r\kappa)$ | $O(5N^3+10N)$ |
| [23] | $O(N^3+RN^2+MR^2+R^3+I_r\kappa)$ | $O(4N^3+N^2)$ |
| [27] | $O(N^3+RN^2+MR^2+R^3+I_iI_r\kappa+I_iI_sN)$ | $O(4N^3+8N^2)$ |
| Note: in the case that $N=R=M$, $I_i$ = 4, $I_s=N$, and $I_r=N$ ($I_r=10$ in Method II). | | |

Substituting (31) and (32) into (30), the optimization problem in (27) can now be reformulated as

$$\begin{aligned}&\max_{\mathbf{F}_R} \det(\mathbf{M}')\\ &\text{s.t.}\quad \mathrm{C}_2\\ &\mathbf{M}' = \mathbf{I}_N + \sigma_{n,d}^2\boldsymbol{\Sigma}_{\mathrm{SR}}'^H\mathbf{U}_{\mathrm{SR}}'^H\mathbf{F}_R^H\mathbf{V}_{\mathrm{RD}}\boldsymbol{\Sigma}_{\mathrm{RD}}^H\\ &\quad\times\left(\sigma_{n,r}^2\boldsymbol{\Sigma}_{\mathrm{RD}}\mathbf{V}_{\mathrm{RD}}^H\mathbf{F}_R\mathbf{F}_R^H\mathbf{V}_{\mathrm{RD}}\boldsymbol{\Sigma}_{\mathrm{RD}}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\\ &\quad\times\boldsymbol{\Sigma}_{\mathrm{RD}}\mathbf{V}_{\mathrm{RD}}^H\mathbf{F}_R\mathbf{U}_{\mathrm{SR}}'\boldsymbol{\Sigma}_{\mathrm{SR}}'.\end{aligned}\tag{33}$$

From (33), we can then find a relay precoder structure diagonalizing $\mathbf{M}'$. Let

$$\mathbf{F}_{R,opt} = \mathbf{V}_{\mathrm{RD}}\boldsymbol{\Sigma}_r\mathbf{U}_{\mathrm{SR}}'^H\tag{34}$$

where $\boldsymbol{\Sigma}_r$ is a diagonal matrix with the $i$th diagonal element $\sigma_{r,i}$, $i=1,\ldots,\kappa$, $\kappa=\min\{N,R\}$, and the values of $\sigma_{r,i}$ remain to be determined. Substituting (34) into (33) and taking the natural log operation to the utility function, we can then rewrite (33) as

$$\begin{aligned}&\max_{p_{r,i},1\le i\le\kappa}\sum_{i=1}^{\kappa}\ln\left(1+\frac{p_{r,i}\sigma_{n,d}^2\sigma_{\mathrm{rd},i}^2\sigma_{\mathrm{sr},i}'^2}{p_{r,i}\sigma_{n,r}^2\sigma_{\mathrm{rd},i}^2+\sigma_{n,d}^2}\right)\\ &\text{s.t.}\sum_{i=1}^{\kappa}p_{r,i}\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{\mathrm{SR}}'(i,i)+\sigma_{n,r}^2\right)\le P_{R,T}\\ &p_{r,i}\ge 0\quad\forall i\end{aligned}\tag{35}$$

where $p_{r,i}=\sigma_{r,i}^2$, and $\mathbf{D}_{\mathrm{SR}}'=\mathbf{V}_{\mathrm{SR}}'^H(N/P_{S,T}\mathbf{I}_N+\mathbf{H}_{\mathrm{SD}}^H\mathbf{H}_{\mathrm{SD}})\mathbf{V}_{\mathrm{SR}}'$, with $\mathbf{D}_{\mathrm{SR}}'(i,i)$ being the $i$th diagonal element of $\mathbf{D}_{\mathrm{SR}}'$. Since the utility function and the inequalities are all concave for $p_{r,i}\ge 0$ [35], (35) is a standard concave optimization problem. As a result, the optimum solution of $p_{r,i}$, $i=1,\ldots,\kappa$ can be solved by means of KKT conditions given as (36), shown at the bottom of the page, where $\mu$ is chosen to satisfy the power constraint in (35). We have also proposed a water-filling algorithm to solve (36). The detailed derivation of (36) and the water-filling algorithm are given in Appendix B and C, respectively. Substituting (36) into (34), we can finally obtain the relay precoder. With the relay precoder, $\widetilde{\mathbf{H}}$ in (9) can be obtained. Subsequently, the unitary source precoder can be derived by substituting (47) into (22), and $\mathbf{C}$ can be obtained by (23).

The computations of the TH source and linear relay precoders with proposed Methods I and II mainly involve SVD, GMD, and matrix inversion operations. The precoders design in [23] and [27] involve similar operations, except that the GMD is not required. The overall computational complexity, which was measured in terms of floating-point operations, are summarized in Table I. In the table, $I_i$ denotes the iteration number in alternately solving the source and relay precoder. Note that, in all methods in Table I, each of the precoders is expressed as a series product of a unitary, a diagonal matrix, and a unitary matrix, e.g., (11), (15), and (34). The elements of the diagonal matrix are solved by an iterative water-filling algorithm. Parameters $I_s$ and $I_r$ are defined as the iteration number for the water-filling algorithms in solving the source and relay precoders, respectively. For an iterative algorithm, the iteration number directly impacts the computational complexity. The required iteration number is determined by the characteristics of the algorithm itself and the environment in which it is operated. To have a simple comparison of the proposed and existing algorithms, we let $N=R=M$ and use some typical values, which are found from simulations, for the iteration numbers. The result is also shown in Table I. From the table, we see that the computational complexity of Method I and the method in [27] is higher than that of Method II and the method in [23]. In [27], the iteration is conducted to solve the diagonal matrix in the source or the relay precoders. As a result, the computational complexity of the method in [27] is lower than that of Method I. As for the closed-form solution in [23], its computational complexity is lower than that of Method II since only a relay precoder is involved.

It is noteworthy that, in MIMO relay systems, the destination needs to know all channel matrices for the calculation of the MMSE receiver and precoders. In general, the channel estimation techniques developed in the conventional point-to-point MIMO systems, that is, either the training-based or pilot-based methods, can be applied in the estimation of the source-to-destination and relay-to-destination channels. However, for the estimation of the source-to-relay channel, the existing methods

$$p_{r,i}=\left[\sqrt{\frac{\mu}{\sigma_{\mathrm{rd},i}^2\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{\mathrm{SR}}'(i,i)+\sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma_{\mathrm{sr},i}'^{-2}+1\right)}+\frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\mathrm{rd},i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2}+1\right)^2}}-\frac{1+\frac{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2}{2\sigma_{n,r}^2}}{\sigma_{\mathrm{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2}+\sigma_{\mathrm{sr},i}'^2\right)}\right]^+\tag{36}$$

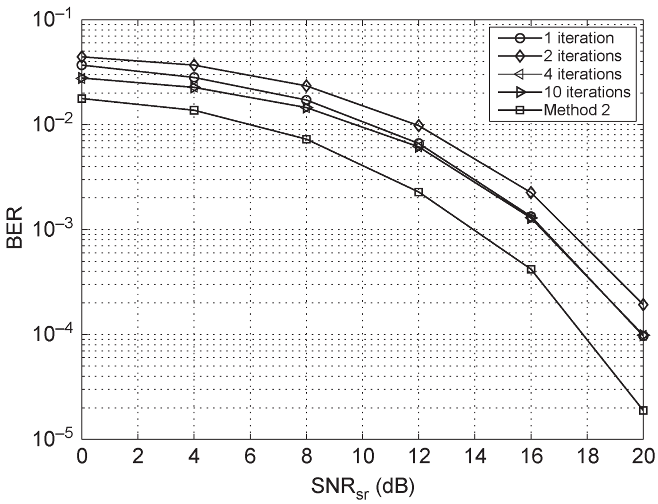


Fig. 2. BER performance comparison for Method I with different iterations.

cannot be used. This problem, which is also referred to as the far-end channel estimation, was recently studied in [31]. In this paper, we assume that perfect channel state information (CSI) are available.

## IV. Simulation Results

We consider an AF MIMO relay system with $N = R = M = 4$. The elements of each channel matrix are assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unity variance. Let $\mathrm{SNR_{SR}}$, $\mathrm{SNR_{RD}}$, and $\mathrm{SNR_{SD}}$ denote the SNR per receive antenna of the source-to-relay, relay-to-destination, and source-to-destination links, respectively. Here, transmitted symbols are modulated with the 16-QAM scheme. In the first set of simulations, we investigate the convergence behavior of Method I. Fig. 2 shows the performance of Method I with different iteration numbers. For comparison purposes, the performance of Method II is also shown. In the simulations, we let $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}} = 20$ dB and $\mathrm{SNR_{SD}}$ be varied. Examining the figure, we can have several observations. First, the performance with one iteration is superior to that with two iterations. This can be explained by the fact that the relay power constraint is not satisfied when the iteration number is odd. Thus, a larger transmit power may be used in the first iteration. Second, the performance with four/ten iterations is better than that with two iterations. This is because the power constraints are satisfied when the iteration number is even and when more iterations yield better performance. Finally, the performance with four iterations is equal to that with ten iterations, indicating that Method I converges in four iterations. From the figure, we can observe that the performance of Method I is inferior to that of Method II.

In the second set of simulations, we compared the performance of the existing and the proposed methods. Here, we let $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}} = 15$ dB and $\mathrm{SNR_{SD}}$ be varied. Figs. 3 and 4 show the MSE and BER performance comparison, respectively, for (a) an unprecoded system with the MMSE receiver, (b) the optimum relay precoded system with the MMSE receiver [23], (c) the linear source and relay precoded system with the MMSE receiver [27], (d) the TH source and linear relay precoded system obtained with proposed Method I, and (e) the TH source and linear relay precoded system obtained with proposed Method II. For proposed Method I, ten iterations are used. Note that the optimum relay precoder in [23] only considers the relay link. For fair comparison, we include the direct link when implementing the MMSE receiver. From the figures, we can see that the proposed TH precoded systems significantly outperform other methods. Although two precoders are also used in [27], the performance is inferior to the proposed algorithms. This is due to the fact that both precoders are linear in [27]. In addition, due to the joint design, proposed Method II outperforms Method I. It is noteworthy that proposed Method II is only valid with higher constellations (e.g., larger than 16-QAM), as described in the paragraph before (6). For a lower constellation, the performance is degraded even worse than that of linear transceivers.

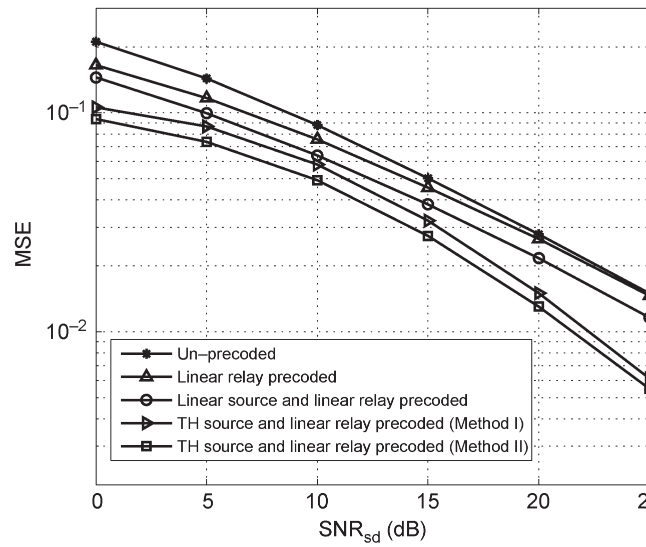


Fig. 3. MSE performance comparison for existing precoded systems and proposed TH source and linear relay precoded systems $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}} = 15$ dB (all with MMSE receiver).

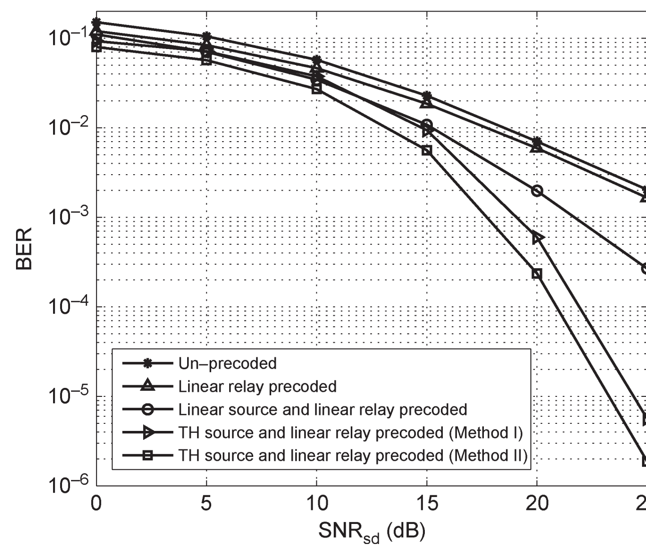


Fig. 4. BER performance comparison for existing precoded systems and proposed TH source and linear relay precoded systems with $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}} = 15$ dB (all with MMSE receiver).

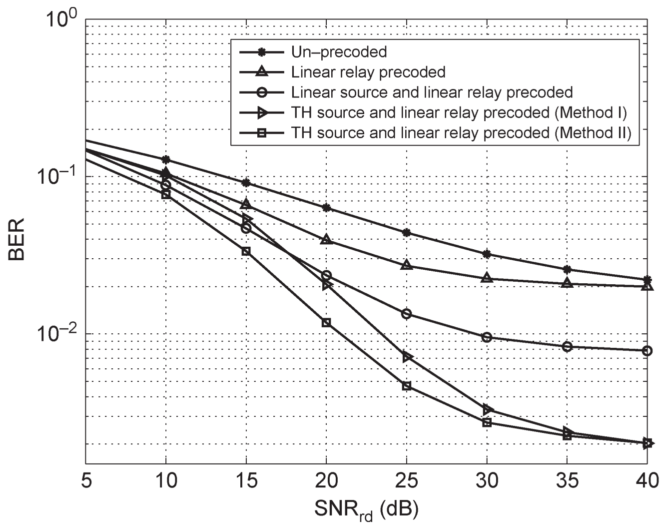


Fig. 5. BER performance comparison for existing precoded systems and proposed TH source and linear relay precoded systems with $\mathrm{SNR}_{\mathrm{SR}} = 20$ dB and $\mathrm{SNR}_{\mathrm{SD}} = 5$ dB (all with MMSE receiver).

For the last set of simulations, we let $\mathrm{SNR}_{\mathrm{SR}} = 20$ dB, $\mathrm{SNR}_{\mathrm{SD}} = 5$ dB, and $\mathrm{SNR}_{\mathrm{RD}}$ be varied. Fig. 5 shows the BER comparison for the previously mentioned systems. As we can see, the performance of the unprecoded system is inferior to that of the linear relay precoded system, and the performance of the linear relay precoded system is inferior to that of the linear source and relay precoded system. Since the proposed methods use the nonlinear TH source precoder, they outperform the other linear precoded systems. It is noteworthy that the performance of all systems saturates when $\mathrm{SNR}_{\mathrm{RD}}$ is high. This is because, when $\mathrm{SNR}_{\mathrm{RD}}$ is high, the system is degenerated to a conventional MIMO system, and the receive SNR saturates due to fixed $\mathrm{SNR}_{\mathrm{SR}}$ and $\mathrm{SNR}_{\mathrm{SD}}$. The role of the relay precoder then becomes less critical. This is also the reason the performance of proposed Method I is close to that of proposed Method II. In Figs. 3 and 4, $\mathrm{SNR}_{\mathrm{SR}}$ and $\mathrm{SNR}_{\mathrm{RD}}$ are fixed, and $\mathrm{SNR}_{\mathrm{SD}}$ is varied. The receive SNR can be increased as the increase of $\mathrm{SNR}_{\mathrm{SD}}$; so does the performance.

## V. Conclusion

In this paper, we have studied a nonlinear transceiver design in an AF MIMO relay system consisting of a TH source precoder, a linear relay precoder, and a linear MMSE receiver. Since the direct optimization of the nonlinear transceiver is very difficult, we have then proposed two suboptimal methods: One is iterative, and the other is non-iterative. The iterative method is conceptually simple, but the computational complexity is high. The non-iterative method, having a closed-form solution, can reduce the complexity and, at the same time, perform better. The approach relies on the use of primal decomposition. By cascading a unitary precoder after the original TH precoder and using a relay precoder structure, the orignal nonconvex matrix-valued optimization problem can be simplified and transferred into a scalar-valued concave problem. A closed-form solution for the precoders can then be derived by the KKT conditions. Simulations have shown that the proposed systems significantly outperform the existing precoded systems. In this paper, we have only considered suboptimal solutions with perfect CSI. Finding the optimum solution and design for precoders with imperfect CSI can serve as interesting topics for further research.

## Appendix A
## Optimal Solutions of (21)

To find the solution of (21), we can first find the optimum $\alpha$, which is denoted as $\alpha_{\mathrm{opt}}$, with $\mathbf{U}_S$ and $\mathbf{C}$ fixed. For any feasible $\alpha \leq \sqrt{P_{S,T}/N\sigma_s^2}$, we can always have the optimum relay precoder $\mathbf{F}'_R$ such that

$$\mathrm{tr}\left\{\mathbf{F}'_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\alpha^2\mathbf{H}_{\mathrm{SR}}\mathbf{H}_{\mathrm{SR}}^H\right)\mathbf{F}'^H_R\right\} = P_{R,T}. \tag{37}$$

This is because, if the equality does not hold, we can always adjust the scale of $\mathbf{F}'_R$, yielding another precoder $\mathbf{F}''_R = \beta\mathbf{F}'_R$ such that the equality can hold. Now, if we let $\alpha_1 = \gamma_1\alpha$ and $\alpha_2 = \gamma_2\alpha$, with $\gamma_1 \geq \gamma_2 \geq 1$, the optimum relay precoders then become $\beta_1\mathbf{F}'_R$ and $\beta_2\mathbf{F}'_R$, respectively, where $\beta_1$ and $\beta_2$ are the scalars designed to satisfy (37). Substituting $\alpha_1$ and $\alpha_2$ into the cost function in (21), we can show that

$$\mathrm{tr}\left\{\mathbf{E}\left(\mathbf{C}, \alpha_1\mathbf{U}_S, \beta_1\mathbf{F}'_R\right)\right\} \leq \mathrm{tr}\left\{\mathbf{E}\left(\mathbf{C}, \alpha_2\mathbf{U}_S, \beta_2\mathbf{F}'_R\right)\right\}. \tag{38}$$

This is to say that, under the relay power constraint, the optimum relay precoder will decrease the cost function along with the increase of $\alpha$. From (38), we can then find the optimum $\alpha$ as

$$\alpha_{\mathrm{opt}} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}. \tag{39}$$

This is because the cost function is a strictly decreasing function of $\alpha$, and we can use a maximum value of $\alpha$ as long as we can satisfy the source power constraint. Note that $\alpha_{\mathrm{opt}}$ in (39) can also maximize the signal-to-interference-plus-noise ratio at the relay, reduce the noise enhancement at the relay node, and thus minimize the MSE value. Substituting (39) in (21), we find that the resultant relay power constraint $\mathrm{tr}\{\mathbf{F}_R(\sigma_{n,r}^2\mathbf{I}_R + (P_{S,T}/N)\mathbf{H}_{\mathrm{SR}}\mathbf{H}_{\mathrm{SR}}^H)\mathbf{F}_R^H\} \leq P_{R,T}$ is not a function of the source precoder. As a result, it only has to be considered in the master problem. The subproblem thus becomes

$$\min_{\mathbf{C},\alpha,\mathbf{U}_S} \mathrm{tr}\left\{\mathbf{E}(\mathbf{C}, \alpha\mathbf{U}_S, \mathbf{F}_R)\Big|_{\alpha=\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}}\right\}. \tag{40}$$

With a known relay precoder and the source power constraint, the problem in (40) is similar to the THP scheme in the conventional point-to-point MIMO systems, and the optimum solution $\mathbf{C}$, which is denoted as $\mathbf{C}_{\mathrm{opt}}$, has been solved in [11]. Let $\mathbf{L}$ be a lower triangular matrix in a Cholesky factorization and

$$\mathbf{L}\mathbf{L}^H = \left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}. \tag{41}$$

Then, $\mathbf{C}_{\text{opt}}$ is given by

$$\mathbf{C}_{\text{opt}} = \mathbf{D}\mathbf{L}^{-1} \tag{42}$$

where $\mathbf{D}$ is a diagonal matrix scaling the diagonal elements of $\mathbf{C}_{\text{opt}}$ to unity. Substituting (42) in (40), we then have the cost function as

$$\text{tr}\left(\mathbf{C}_{\text{opt}}\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}_{\text{opt}}^H\right) = \sum_{k=1}^{N}\mathbf{L}(k,k)^2 \tag{43}$$

which is a function of $\mathbf{U}_S$.

The next step is to find $\mathbf{U}_S$ such that (43) is minimized. To start with, we first decompose $\mathbf{U}_S$ as

$$\mathbf{U}_S = \mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S \tag{44}$$

where $\mathbf{V}_{\widetilde{\mathbf{H}}} \in \mathbb{C}^{N\times N}$ is the left singular matrices of $\widetilde{\mathbf{H}}$, and $\mathbf{U}'_S \in \mathbb{C}^{N\times N}$ is an another unitary matrix. Note that this decomposition is always possible for any unitary matrix. Substituting (44) into (41), we then have

$$\begin{aligned}\mathbf{L}\mathbf{L}^H &= \left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\left(\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^{-1}\\ &= \mathbf{U}'^H_S\underbrace{\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}}\right)^{-1}}_{:=\widetilde{\mathbf{D}}}\mathbf{U}'_S\end{aligned} \tag{45}$$

where $\boldsymbol{\Lambda}_{\widetilde{\mathbf{H}}} = \text{diag}\{\lambda_{\widetilde{\mathbf{H}},1},\ldots,\lambda_{\widetilde{\mathbf{H}},N}\}$ is a diagonal matrix with the eigenvalues of $\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$. It is simple to see that $\widetilde{\mathbf{D}}$ is a diagonal matrix.

Applying arithmetic-geometric inequality, we can find that the minimum value in (43) can be achieved when $\mathbf{L}(i,i) = \mathbf{L}(j,j)$, $i \neq j$. Thus, the remaining work is to find a proper $\mathbf{U}'_S$, so that $\mathbf{L}(i,i) = \mathbf{L}(j,j)$. Let us start with the following decomposition:

$$\widetilde{\mathbf{D}} = \widetilde{\mathbf{D}}^{\frac{1}{2}}\widetilde{\mathbf{D}}^{\frac{1}{2}} \tag{46}$$

where $\widetilde{\mathbf{D}}^{1/2}$ is the square-root matrix of $\widetilde{\mathbf{D}}$. Applying GMD [8] on $\widetilde{\mathbf{D}}^{1/2}$, we can have

$$\widetilde{\mathbf{D}}^{1/2} = \mathbf{Q}\mathbf{R}\mathbf{P}^H \tag{47}$$

where $\mathbf{Q}$ and $\mathbf{P}$ are some unitary matrices, and $\mathbf{R}$ is an upper triangular matrix with equal diagonal elements. Substituting (47) in (45), we then have

$$\mathbf{L}\mathbf{L}^H = \mathbf{U}'^H_S\widetilde{\mathbf{D}}\mathbf{U}'_S = \mathbf{U}'^H_S\mathbf{P}\mathbf{R}^H\mathbf{R}\mathbf{P}^H\mathbf{U}'_S. \tag{48}$$

Let $\mathbf{U}'_S = \mathbf{P}$, and we can rewrite (48) as

$$\mathbf{L}\mathbf{L}^H = \mathbf{R}^H\mathbf{R}. \tag{49}$$

From (49), it is clear that $\mathbf{L} = \mathbf{R}^H$, and the diagonal elements of $\mathbf{L}$ are all equal. Therefore, the optimal $\mathbf{F}_S$, which is denoted as $\mathbf{F}_{S,opt}$, can then be expressed as (22). From (43), the resultant MSE can be expressed as

$$\begin{aligned}J_{\min} &= \sum_{k=1}^{N}\mathbf{L}(k,k)^2\\ &= \sum_{k=1}^{N}\mathbf{R}(k,k)^2\\ &= N\prod_{k=1}^{N}\left(\lambda_{\widetilde{\mathbf{H}},k}\frac{P_{S,T}}{N\sigma_s^2} + \sigma_s^{-2}\right)^{-1/N}.\end{aligned} \tag{50}$$

Note here that the subproblem includes two power constraints, which make the problem difficult to tackle. To make the problem tractable, we propose cascading a unitary precoder facilitating the application of the primal decomposition. Although the THP design in [12] also cascades a precoder, its structure is not restricted to be unitary. The purpose is different from ours.

## APPENDIX B
## OPTIMUM SOLUTION IN (35)

The Lagrangian function in (35) can be expressed as

$$\begin{aligned}L = &\sum_{i=1}^{\kappa}\ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{\text{rd},i}^2\sigma'^2_{\text{sr},i}}{p_{r,i}\sigma_{n,r}^2\sigma_{\text{rd},i}^2 + \sigma_{n,d}^2}\right)\\ &+ \lambda\left[\sum_{i=1}^{\kappa}p_{r,i}\left(\frac{P_{S,T}}{N}\sigma'^2_{\text{sr},i}\mathbf{D}'_{\text{SR}}(i,i) + \sigma_{n,r}^2\right) - P_{R,T}\right]\\ &- \sum_{i=1}^{\kappa}v_{r,i}p_{r,i}\end{aligned} \tag{51}$$

where $\lambda \geq 0$. $v_{r,i} \geq 0$, with $i = 1,\ldots,\kappa$. By the KKT conditions for all $i$, we have

$$\frac{\partial L}{\partial p_{r,i}} = -\frac{\frac{\sigma_{n,d}^2\sigma_{n,r}^2\sigma_{\text{rd},i}^2\sigma'^2_{\text{sr},i}}{\left(p_{r,i}\sigma_{n,r}^2\sigma_{\text{rd},i}^2+\sigma_{n,d}^2\right)^2}}{1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{\text{rd},i}^2\sigma'^2_{\text{sr},i}}{p_{r,i}\sigma_{n,r}^2\sigma_{\text{rd},i}^2+\sigma_{n,d}^2}} + \lambda\left(\frac{P_{S,T}}{N}\sigma'^2_{\text{sr},i}\mathbf{D}'_{\text{SR}}(i,i) + \sigma_{n,r}^2\right) - v_{r,i} = 0 \tag{52}$$

$$\lambda, v_{r,i}, p_{r,i} \geq 0 \tag{53}$$

$$v_{r,i}p_{r,i} = 0 \tag{54}$$

$$\lambda\left[\sum_{i=1}^{\kappa}p_{r,i}\left(\frac{P_{S,T}}{N}\sigma'^2_{\text{sr},i}\mathbf{D}'_{\text{SR}}(i,i) + \sigma_{n,r}^2\right) - P_{R,T}\right] = 0. \tag{55}$$

Substituting (52) into (54) and noting the fact that $p_{r,i} > 0$, we have (56)–(59), shown at the bottom of the next page. After some straightforward manipulations and the use of (53), we can have the optimum $p_{r,i}$ given as (60), shown at the bottom of the next page, where $\mu = 1/\lambda$ is chosen to satisfy the power constraint in (35).

## APPENDIX C
## WATER-FILLING ALGORITHM FOR (36)

For convenience, we let

$$a_i = \frac{1}{\sigma_{\mathrm{rd},i}^2 \left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2 \mathbf{D}_{\mathrm{SR}}'(i,i) + \sigma_n^2\right)\left(\sigma_{n,r}^2 \sigma_{n,d}^{-2} \sigma_{\mathrm{sr},i}'^{-2} + 1\right)} \tag{61}$$

$$b_i = \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\mathrm{rd},i}^4 \left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2 \sigma_{\mathrm{sr},i}'^2} + 1\right)^2} \tag{62}$$

$$c_i = \frac{1 + \frac{\sigma_{n,d}^2 \sigma_{\mathrm{sr},i}'^2}{2\sigma_{n,r}^2}}{\sigma_{\mathrm{rd},i}^2 \left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma_{\mathrm{sr},i}'^2\right)} \tag{63}$$

$$d_i = \left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2 \mathbf{D}_{\mathrm{SR}}'(i,i) + \sigma_{n,r}^2\right). \tag{64}$$

We can rewrite (36) in a general water-filling form

$$p_{r,i} = \left[\sqrt{a_i\left(\mu + \frac{b_i}{a_i}\right)} - c_i\right]^+. \tag{65}$$

An easy way to solve (65) and, at the same time, satisfy the power constraint in (35) is the bisection method summarized in Table II. In the table, $\mu_{M,0}$ and $\mu_{L,0}$ denote the maximum and minimum initial $\mu$, respectively; $\epsilon$ is the tolerate error determining the numbers of iterations. We use a simple method to determine $\mu_{M,0}$ and $\mu_{L,0}$. Let $D_i = \min\{b_i/a_i\}$, $i = 1, 2, \ldots, \kappa$. We ignore the operation of $[.]^+$ in (65), replace $(b_i/a_i)$ with $D_i$ in (65), and solve $p_{r,i}$ for $i = 1, 2, \ldots, \kappa$. Using the power constraint, we can then obtain an upper bound of $\mu$, which can serve as $\mu_{M,0}$, and a mathematical expression given by

$$\left(\sqrt{\mu + D_i}\right)\left(\sum_{i=1}^{\kappa}\sqrt{a_i}\right) - \sum_{i=1}^{\kappa} c_i \leq P_{R,T}. \tag{66}$$

From (66), we have

$$\mu \leq \left(\left[\frac{\sum_{i=1}^{\kappa} c_i + P_{R,T}}{\sum_{i=1}^{\kappa}\sqrt{a_i}}\right]^2 - D_i\right) := \mu_{M,0}. \tag{67}$$

TABLE II
PROPOSED WATER-FILLING ALGORITHM SOLVING (36)

$\mu_M = \mu_{M,0}$, $\mu_L = \mu_{L,0}$, $\delta_\mu$
while $\delta_\mu > \epsilon$
  $\mu = \frac{\mu_M + \mu_L}{2}$
  if $\sum_{i=1}^{\kappa}\left[\sqrt{a_i\left(\mu + \frac{b_i}{a_i}\right)} - c_i\right]^+ d_i \leq P_{R,T}$
    $\mu_L = \mu$
  else
    $\mu_M = \mu$
  end
  $\mu' = \frac{\mu_M + \mu_L}{2}$, $\delta_\mu = |\mu' - \mu|$
end

$$\frac{1}{p_{r,i}^2 \underbrace{\sigma_{\mathrm{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2} + 1\right)}_{:=A_i} + 2p_{r,i}\underbrace{\left(\sigma_{\mathrm{sr},i}'^{-2} + \frac{\sigma_{n,d}^2}{2\sigma_{n,r}^2}\right)}_{:=B_i} + \underbrace{\frac{\sigma_{n,d}^2}{\sigma_{n,r}^2\sigma_{\mathrm{rd},i}^2\sigma_{\mathrm{sr},i}'^2}}_{:=C_i}} = \lambda\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{\mathrm{SR}}'(i,i) + \sigma_{n,r}^2\right) \tag{56}$$

$$P_{r,i} = \sqrt{\frac{1}{\lambda\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{\mathrm{SR}}'(i,i) + \sigma_{n,r}^2\right)A_i} + \frac{B_i^2}{A_i^2} - \frac{C_i}{A_i}} - \frac{B_i}{A_i} \tag{57}$$

$$\frac{B_i^2 - A_iC_i}{A_i^2} = \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\mathrm{rd},i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2} + 1\right)^2} \tag{58}$$

$$\frac{B_i}{A_i} = \frac{1 + \frac{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2}{2\sigma_{n,r}^2}}{\sigma_{\mathrm{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma_{\mathrm{sr},i}'^2\right)} \tag{59}$$

$$p_{r,i} = \left[\sqrt{\frac{\mu}{\sigma_{\mathrm{rd},i}^2\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{\mathrm{SR}}'(i,i) + \sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma_{\mathrm{sr},i}'^{-2} + 1\right)} + \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\mathrm{rd},i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2} + 1\right)^2}} - \frac{1 + \frac{\sigma_{n,d}^2\sigma_{\mathrm{sr},i}'^2}{2\sigma_{n,r}^2}}{\sigma_{\mathrm{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma_{\mathrm{sr},i}'^2\right)}\right]^+ \tag{60}$$

For $\mu_{L,0}$, we can simply use the minimum $\mu$ such that $p_{r,i}$ is nonnegative in (65). Since $p_{r,i}$ is nonnegative, we must have

$$\mu \geq \frac{c_i^2 - b_i}{a_i} \geq 0 \qquad \forall i. \tag{68}$$

From (61)–(63), we can see that $(c_i^2 - b_i/a_i) \geq 0$. Thus, we can let $\mu_{L,0} = \min\{c_i^2 - b_i/a_i \;\forall i\}$.

## ACKNOWLEDGMENT

The authors would like to thank the editor and anonymous reviewers for their helpful comments.

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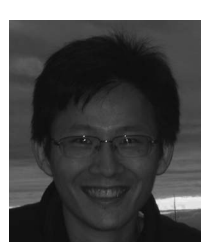

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