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Complexity Reduction by Using QR-Based Scheme in Computing Capacity for Optimal Transmission

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Abstract Multiple-input-multiple-output (MIMO) technique is often employed to increase capacity in comparing to systems with single antenna. However, the computational complexity in evaluating channel capacity or transmission rate (data rate) grows proportionally to the number of employed antennas at both ends of the wireless link. Recently, the QR decomposition (QRD) based detection schemes have emerged as a low-complexity solution. After conducting QRD on a full channel matrix that results in a triangular matrix, we claim that computational complexity can be simplified by the following ways. First, to simplify channel capacity calculation, we prove that eigenvalues of the full channel matrix multiplication equals eigenvalues of the triangular channel matrix multiplication. Second, to simplify the calculation of the optimal transmission rate constrained constellation, we propose a simplistic multiplication of the resulted simple triangular matrix and a transmitted signal vector. Then, we also propose a modified mutual information calculation (MMIC) to achieve a quite low-complexity via the divided calculation. By using computer simulation and field-programmable gate array (FPGA) implementation, simulation results show that the proposed QRD-based schemes are capable of achieving conventional performance, but at a low-complexity level.

Keywords Multiple-input multiple-output · Channel capacity · QR decomposition · Eigenvalues · Mutual information · Field-programmable gate array

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1 Introduction

Multiple-input-multiple-output (MIMO) technique [1–3] is often employed to increase capacity in comparing to systems with single antenna [4,5]. In wirless MIMO communications, the design of an optimal transmission is essential in order to meet demands from a large number of simultaneous data transmissions [1–3]. Bertrand et al. [6] shows the mutual information constrained constellation to achieve the maxim achievable rate at various signal-to-noise ratio (SNR) conditions in MIMO system. However, with full MIMO channel matrix (**H**) [6–10], the computational complexity in calculating channel capacity or transmission rate (data rate) increases proportionally when the number of antennas and/or modulation orders are higher [6–10]. More specifically, the full MIMO channel matrix multiplication [6] leads to complicated very-large-scale integration (VLSI) [11], field-programmable gate array (FPGA) implementation [12] and real-time design [1–3]. To simplify the computational complexity, the QR-decomposition (QRD) scheme [13–15] with restraining complexity has been devised to reduce the MIMO channel matrix multiplication due to involving a lot of zeros in a triangular matrix [13–15].

In this paper, we propose a simple way to realize Bertrand's scheme [6]. In order to achieve a simply way, the MIMO channel matrix can be uniquely represented a triangular matrix (**R**) and a unitary matrix after QRD [13–15], provided that the number of receiving antennas (*M*) is larger than or equal to the number of transmitting antennas (*N*). Thus, computational complexity can be simplified by the following ways. First, to simplify channel capacity calculation, we prove that eigenvalues of the full channel matrix multiplication ($\mathbf{HH}^H$) [6] equals eigenvalues of the triangular channel matrix multiplication ($\mathbf{RR}^H$) [13–15]. Based on this equivalence, we can reduce the eigenvalue processes because $\mathbf{RR}^H$ with *N*-by-*N* has fewer dimensions than $\mathbf{HH}^H$ with *M*-by-*M* [16–18]. Second, to design an optimal transmission rate via evaluating mutual information constrained constellation [6], we propose a simplistic multiplication of the resulted simple triangular matrix and a transmitted signal vector. We also propose a modified mutual information calculation (MMIC) to achieve a quite low-complexity via the divided calculation. By using computer simulation, simulation results show that the proposed QRD-based schemes are capable of achieving conventional performance (Bertrand's scheme), but at a low-complexity level [16–18]. Additionally, the proposed QRD-based schemes have also been implemented in FPGA to verify the functional correctness and the complexity advantage.

This paper is organized as follows. In Sect. 2, we give conventional MIMO equations in computing channel capacity. Section 3, The proposed QRD-based scheme in computing channel capacity and mutual information of constrained constellation are developed. In Sect. 4, we analyze the computational efficiency (CE) in the proposed QRD-based scheme. In Sect. 5, we conduct computer simulation and FPGA implementation to confirm the effectiveness of the proposed QRD-based schemes. Finally, we conclude the paper and suggest future work in Sect. 6.

2 Conventional MIMO Capacity Equations

We consider a communication system with *N* transmitting and *M* receiving antennas ($M \geq N$) over a MIMO channel. The sampled basedband received signals are given by [9]

$$\mathbf{y} = \mathbf{Hx} + \mathbf{v}, \tag{1}$$

where $\mathbf{y} \in C^{M\times 1}$ is the received signal vector, $\mathbf{x} \in C^{N\times 1}$ is the transmitted signal vector and $\mathbf{H} \in C^{M\times N}$ is the MIMO channel matrix and the noise vector $\mathbf{v} \in C^{M\times 1}$ has an *i.i.d.* complex Gaussian entries and noise power is σ_v^2. Then, the MIMO technique promises to become the technology of future wireless communication when high spectral efficiency is required. Therefore, the capacity of a random MIMO channel can be expressed as follows [6–9]:

$$C = E\left\{\max_{p(\mathbf{x}):tr(\mathbf{R}_{xx})=N} \mathrm{I}(\mathbf{x};\mathbf{y})\right\}, \tag{2}$$

where $\mathbf{R}_{xx} := E\{\mathbf{x}\mathbf{x}^H\}$, is the covariance matrix of the transmitted symbol vector $\mathbf{x}$ and $p(\mathbf{x})$ denotes all possible transmitter statistical distribution and $(\cdot)^H$ is the Hermitian transpose. By using a MIMO channel, the mutual information between $\mathbf{x}$ and $\mathbf{y}$ which can be given as [6–9]

$$I(x;y) = E\left\{\log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_V^2 N}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right)\right]\right\} \tag{3}$$

where total power P_T is limited, irrespective of the number of transmitting antennas. Substituting (3) into (2), we have

$$C = \max_{tr(\mathbf{R}_{xx})\le N} E\left\{\log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_v^2 N}\mathbf{H}\mathbf{R}_{xx}(\mathbf{H})^H\right)\right]\right\}. \tag{4}$$

In this work, we assume that the channel coefficient is unknown to the transmitter and hence the uniform power distribution is considered in transmitter. The covariance matrix of $\mathbf{x}$ is then given by $\mathbf{R}_{xx} = \mathbf{I}_M$, which implies that the transmitted symbol x is an *i.i.d.* random variable with zero-mean and unit-variance. As a result, the ergodic capacity for a spatially white MIMO channel can be given as [6–9]

$$C = E\left\{\log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_v^2 N}\mathbf{H}(\mathbf{H})^H\right)\right]\right\} = E\left\{\log_2\left[\det\left(\mathbf{I}_M + \frac{\rho}{N}\mathbf{H}(\mathbf{H})^H\right)\right]\right\}, \tag{5}$$

where $\rho := \frac{P_T}{\sigma_v^2}$ is the average SNR at each receiver branch. By using the eigenvalue decomposition (EVD) [16,17], we can get

$$\mathbf{H}\mathbf{H}^H = \mathbf{E}\boldsymbol{\Lambda}_{\mathbf{H}}\mathbf{E}^H, \tag{6}$$

where $\mathbf{E}$ is an $M \times M$ matrix which $\mathbf{E}\mathbf{E}^H = \mathbf{E}^H\mathbf{E} = \mathbf{I}_M$ and $\boldsymbol{\Lambda}_{\mathbf{H}} = \mathrm{diag}\{\Lambda_{\mathbf{H},1}, \Lambda_{\mathbf{H},2}, \ldots \Lambda_{\mathbf{H},M}\}$ is a diagonal matrix with $\Lambda_{\mathbf{H},i} \ge 0$ employed in full MIMO channel matrix. Assuming $\Lambda_{\mathbf{H},i}$'s are ordered so that $\Lambda_{\mathbf{H},i} \ge \Lambda_{\mathbf{H},i+1,}$ then we have $\Lambda_{\mathbf{H},i} = 0$ if $d+1 \le i \le M$, where d is given as

$$d = \mathrm{rank}(\mathbf{H}) \le N, \tag{7}$$

therefore, the capacity of a MIMO channel can be rewritten as [6–9]

$$C = \sum_{i=1}^{d} E\left\{\log_2\left(1 + \frac{\rho}{N}\right)\Lambda_{\mathbf{H},i}\right\}. \tag{8}$$

In outage analysis, we consider the $q\%$ outage capacity $C_{out,q}$ as the information rate that is guaranteed for $(100-q)\%$ of the channel realization as

$$Prob(C \le Cout, q) = q\%, \tag{9}$$

in upper bound, the outage capacity is larger than the channel capacity when a finite probability q is considered. Furthermore, when the channel knowledge is known at the transmitter, the capacity of a MIMO channel is the sum of the capacities associated with the parallel SISO channels given by [7–9]

$$C = \sum_{i=1}^{r} E\left\{\log_2\left(1 + P_i \frac{\rho}{N} \Lambda_{\mathbf{H},i}\right)\right\}, \tag{10}$$

where transmitting power in the ith sub-channel is $P_i := E\{|x_i|^2\}$ for $i = 1, 2, \ldots, r$ and total power satisfies $P_1 + P_2 + \cdots + P_r = N$. Therefore, the transmitter can access the spatial sub-channels, it can allocate variable power across the sub-channels to maximize the channel capacity. The channel capacity with power loaded optimization is

$$C = \max_{P_T = N} \sum_{i=1}^{r} E\left\{\log_2\left(1 + P_i \frac{\rho}{N} \Lambda_{\mathbf{H},i}\right)\right\}. \tag{11}$$

Hence, the solution can be obtained by using the Lagrangian methods [7–9]. The optimal power allocation of the ith sub-channel is denoted as

$$P_i^o = \left(\mu - \frac{N}{\rho \Lambda_{\mathbf{H},i}}\right)_+, \tag{12}$$

where μ is a constant chosen to satisfy the power constraint of (12) and $(x)_+$ denotes

$$(x)_+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}. \tag{13}$$

The optimal power allocation in (13) is found iteratively through the "waterfilling" algorithm [7–9].

3 Reduced-Complexity for MIMO Capacity Equations

In this section, the QRD-based scheme is proposed to reduce matrix multiplication in computing channel capacity and mutual information of constrained constellation as follows.

3.1 Channel Capacity

In this subsection, we consider the triangular matrix multiplication to achieve low computational complexity via the QRD-based scheme. Thus, the full MIMO channel matrix can be expressed as [13–15]

$$\mathbf{H} = [\mathbf{Q}_1,\ \mathbf{Q}_2] \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} = \mathbf{QR}, \tag{14}$$

where $\mathbf{Q} = [\mathbf{Q}_1 \in C^{N\times N}, \mathbf{Q}_2 \in C^{N\times(M-N)}]$ is an $M \times N$ unitary matrix so $\mathbf{Q}^H\ \mathbf{Q} = \mathbf{I}$ and $\mathbf{R}$ is an $N \times N$ upper triangular matrix as [13–15]

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ 0 & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & r_{N,N} \end{bmatrix} \in C^{N\times N}, \tag{15}$$

where $\mathbf{R}$ involves a lot of zeros and hence matrix multiplication is reduced. Based on (10), considering low-complexity in the matrix multiplication, we thus prove that the eigenvalues of full channel matrix multiplication are equivalent to the eigenvalues of triangular channel matrix multiplication as follows.

Theorem 1 *Assuming* $\mathbf{H} \in C^{M\times N}$*, let* $\mathbf{Q}$ *and* $\mathbf{R}$ *be the* $\mathbf{QR}$ *decomposition of* $\mathbf{H}$ (*i.e.* $\mathbf{H} = \mathbf{QR}$). *Then, we have* $\mathbf{\Lambda}(\mathbf{HH}^H) = \mathbf{\Lambda}(\mathbf{RR}^H)$ *where* $\mathbf{\Lambda}(\cdot)$ *is the set of eigenvalues.*

Proof We first consider the "⊆" case, let $\mathbf{\Lambda_H} \in \mathbf{\Lambda}(\mathbf{HH}^H)$. There is a $\mathbf{a} \neq \mathbf{0}$, such that

$$\mathbf{HH}^H\mathbf{a} = \mathbf{\Lambda_H a}. \tag{16}$$

So $\mathbf{H} = \mathbf{QR}$ implies

$$\mathbf{Q}^H(\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a}) = \mathbf{Q}^H(\mathbf{\Lambda_H a}) = \mathbf{\Lambda_H Q}^H\mathbf{a}. \tag{17}$$

Since $\mathbf{Q}^H\mathbf{a} \neq \mathbf{0}$, we have $\mathbf{\Lambda_H} \in \mathbf{\Lambda}(\mathbf{RR}^H)$ and hence proof is done. Next, for the "⊇" case, let $\mathbf{\Lambda_R} \in \mathbf{\Lambda}(\mathbf{RR}^H)$. For a vector $\mathbf{b} \neq \mathbf{0}$, $\mathbf{a} = \mathbf{Qb}$ implies

$$\mathbf{Q}^H\mathbf{a} = \mathbf{b}, \quad \forall\, \mathbf{a} \neq \mathbf{0}. \tag{18}$$

So $\mathbf{RR}^H\mathbf{b} = \mathbf{\Lambda_R b}$ implies

$$\mathbf{RR}^H\mathbf{Q}^H\mathbf{a} = \mathbf{\Lambda_R Q}^H\mathbf{a}. \tag{19}$$

By multiplying $\mathbf{Q}$ on left of (19), we have

$$\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a} = \mathbf{\Lambda_R QQ}^H\mathbf{a} = \mathbf{\Lambda_R a}. \tag{20}$$

Notice that because of (16), we have $\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a} = \mathbf{HH}^H\mathbf{a} = \mathbf{\Lambda_R a}$. Thus, proof is done.

Based on the Theorem 1, to reduce the eigenvalue processes, the capacity of a MIMO channel of (8) can be rewritten as

$$C = \sum_{i=1}^{d} E\left\{\log_2(1 + \frac{\rho}{N})\Lambda_{\mathbf{R},i}\right\}, \tag{21}$$

where eigenvalues $\mathbf{\Lambda_R}$ are obtained by $\mathbf{RR}^H = \mathbf{U\Lambda_R U}^H$ and $\mathbf{\Lambda_R} = \mathbf{\Lambda_H}$, depicted in Theorem 1. Similarly, the channel capacity with power loaded optimization of (11) can be rewritten as

$$C = \max_{P_T = N} \sum_{i=1}^{r} E\left\{\log_2\left(1 + P_i\frac{\rho}{N}\Lambda_{\mathbf{R},i}\right)\right\}. \tag{22}$$

Thus, by using $\mathbf{RR}^H$, we can reduce the eigenvalue processes as the capacity value of (21) and (22) are equivalent to the capacity value of (5) and (8). In this paper, channel capacity calculation will be developed in computer and FPGA implementation as follows. For computer implementation, the symmetric QR (A) and divide-and-conquer (B) algorithm [16 pp. 421–444] are proposed to process the symmetric eigenvalue problem in Fig. 1. For FPGA implementation, we consider the Jacobi method to realize symmetric eigenvalue problem as follows [16]

$$\mathbf{U}^H\mathbf{AU} = U_n^H U_{n-1}^H \ \dots \ U_1^H \mathbf{A} U_1 \ \dots \ U_{n-1}U_n = \mathbf{\Lambda}, \tag{23}$$

where **U,** the jacobi matrix, performs a sequence of orthogonal two sided plan rotations to the symmetric matrix $\mathbf{A}$ ($\mathbf{A} = \mathbf{RR}^H$). Specifically, assuming $U_i (1 \leq i \leq n)$ is an orthonormal

(S.1) Symmetric QR algorithm [16 p.421] is applied to

$$\mathbf{RR}^H = \mathbf{P}^H\mathbf{TP} = \mathbf{P}^H \begin{bmatrix} a_1 & b_1 & & \\ b_1 & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & b_n & a_n & b_{n-1} \end{bmatrix} \mathbf{P}$$; // **P** is the Givens processes

(D.1) **procedure** *DC*(**T**, **Q**, **Λ**, n : input size) // *DC*: divide-and-conquer
(D.2) **begin**
(D.2) **if** $n <= n_0$ **then**
(D.3) Solve problem without sub-division;
(D.4) **else**
(D.5) sub-division: $\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{T}_k \end{bmatrix} + b_m \mathbf{vv}^H$; // $1 \le m \le n$
(D.6) Recursive: call *DC*($\mathbf{T}_i$, $\mathbf{Q}_i$, $\mathbf{\Lambda}_i$, n/k) *for i* =1 *to k*; // *k*: positive integer;
(D.7) *Dcompose* $\mathbf{D}_i + \rho_i \mathbf{uu}^H = \mathbf{V}_i \Lambda_i \mathbf{V}_i^H$; // $\mathbf{v}^H$=[**0** 1 1 **0**] as k =2
(D.8) Update the eigenvector: $\mathbf{Q}_i = \mathbf{Q}_i \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$; // $\mathbf{u} = \begin{bmatrix} \mathbf{Q}_1^H & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Q}_k^H \end{bmatrix} \mathbf{v}$
(D.9) Return **Q** and **Λ**;
(D.10) **endif**
(D.11) **end** *DC*.

Fig. 1 Pseudo code for EVD in computer implementation

plane rotation via an angle θ, we thus have $U_{pp} = \cos\theta$, $U_{pq} = \sin\theta$, $U_{qp} = -\sin\theta$, and $U_{qq} = \cos\theta$. The 2-by-2 EVD algorithm is presented as follows [16]

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^H \begin{bmatrix} a_{pp} & a_{pq} \\ a_{qp} & a_{qq} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} b_{pp} & b_{pq} \\ b_{qp} & b_{qq} \end{bmatrix}, \tag{24}$$

where $a_{pp}, a_{qq}, a_{pq}, a_{qp} \in \mathbf{A}_i$ is $i^{th}(1 \le i \le n)$ iteration in processing EVD of $\mathbf{A}$. To compute θ in (24), the algorithm gives symmetric $\mathbf{A}$, p and q to satisfy $b_{pq} = b_{qp} = 0$, these processes can be depicted in Fig. 2. Based on (23) and (24), we use the coordinate rotation digital computer (CORDIC) algorithm to develop channel capacity evaluation in the FPGA implementation [19]. In Fig. 3, the three main blocks of the EVD process are depicted as follows: (1) The CORDIC blocks work in vector rotation and arctangent process. (2) The memory block stores the eigenvalue and eigenvector temporarily when data bus is busy. (3) The random access memory (RAM) stores the eigenvectors and eigenvalues via the CORDIC processes as well as the variable data with various iterations. Especially, in this paper, the EVD process can be calculated easily due to invoking symmetric characteristic [16–18].

3.2 Mutual Information of Constrained Constellation

In this section, to design an optimal transmission rate, the mutual information of constrained constellation is essentially evaluated. To achieve the effort of various constellations on achieving maximum rate in (1), the mutual information between received signals and transmitted signals can be denoted as [6]

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}), \tag{25}$$

(A.1) **if** $\mathbf{A}(p, q) \neq 0$

(A.2) $\tau = (\mathbf{A}(q, q) - \mathbf{A}(p, p))/2\mathbf{A}(p, q)$;

(A.3) **if** $\tau \geq 0$

(A.4) $\theta = \tan^{-1}\left(\dfrac{1}{\tau + \sqrt{1+\tau^2}}\right)$; // $\tan^{-1}(\cdot)$ is an arctangent operation

(A.5) **else**

(A.6) $\theta = \tan^{-1}\left(\dfrac{-1}{-\tau + \sqrt{1+\tau^2}}\right)$;

(A.7) **endif**

(A.8) **else**

(A.9) $\theta = 0$;

(A.10) **endif.**

Fig. 2 Pseudo code for achieving the rotation angle in FPGA implementation

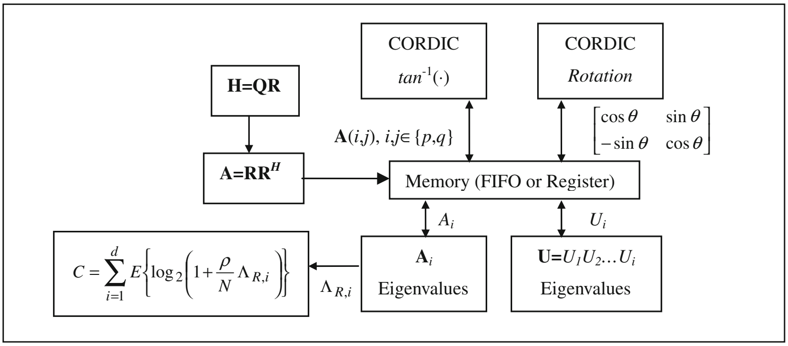

Fig. 3 Channel capacity calculation for FPGA implementation

where $H(\mathbf{y}|\mathbf{x}) = N \cdot \log(2\pi\sigma^2 e)$ is standard division assuming Gaussian channel for any symbol constellation. $H(\mathbf{y})$ is entropy function as

$$H(\mathbf{y}) = -E \log p(\mathbf{y}) = -E \log \sum_{\mathbf{x}} p(\mathbf{y}|\mathbf{x}) p(\mathbf{x}), \tag{26}$$

where probability of event $\mathbf{x}$ is given as $p(\mathbf{x}) = 2^{-mN}$ and m is an modulation order. The condition probability $p(\mathbf{y}|\mathbf{x})$ can be given as [6]

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{(2\pi\sigma_v^2)^N} \exp\left(-\frac{1}{2\sigma_v^2} \|\mathbf{Y} - \mathbf{Hx}\|^2\right), \tag{27}$$

therefore, mutual information of (25) can be written as

$$I(\mathbf{x}, \mathbf{y}) = -E \log\left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N} \sum_{\mathbf{x}\in C^N} \exp\left[-\frac{1}{2\sigma_v^2} \|\mathbf{Y} - \mathbf{Hx}\|^2\right]\right) - N \log 2\pi\sigma_v^2 e. \tag{28}$$

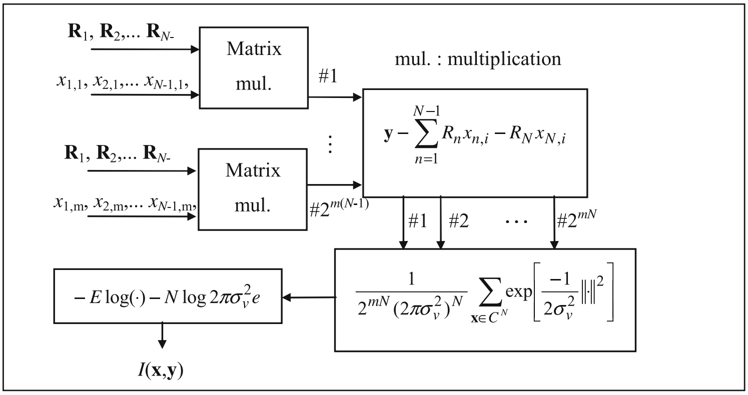

Fig. 4 The block diagram of the mutual information calculation

In this paper, to reduce the multiplication processes, we will simplify the full matrix (**H**) in (28) to the upper triangular matrix (**R**) by using QRD [13–15] as follows.

$$||\mathbf{y} - \mathbf{Hx}||^2 = ||\mathbf{Q}(\mathbf{z} - \mathbf{Rx}||^2 = (\mathbf{z} - \mathbf{Rx})^H \mathbf{Q}^H \mathbf{Q}(\mathbf{z} - \mathbf{Rx}) = ||\mathbf{z} - \mathbf{Rx}||^2, \tag{29}$$

where $\mathbf{y} = \mathbf{Qz}$ and $\mathbf{Q}$ is orthogonal with $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$. By applying this simplistic multiplication ($\mathbf{Rx}$) of (29), the mutual information of (28) can be rewritten as

$$I(\mathbf{x}, \mathbf{y}) = -E \log \left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N} \sum_{\mathbf{x} \in C^N} \exp\left[-\frac{1}{2\sigma_v^2} \|\mathbf{z} - \mathbf{Rx}\|^2 \right] \right) - N \log 2\pi\sigma_v^2 e. \tag{30}$$

Therefore, we can simplify multiplication as the mutual information value of (30) because it is equivalent to the mutual information value of (28). To reduce iterations in evaluating (30), we have

$$I(\mathbf{x}, \mathbf{y}) = -E \log \left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N} \sum_{\mathbf{x} \in C^N} \exp\left[-\frac{1}{2\sigma_v^2} \left\| \left(\mathbf{z} - \sum_{n=1}^{N-1} \mathbf{R}_n x_{n,i} \right) - \mathbf{R}_N x_{N,i} \right\|^2 \right] \right) \\ -N \log 2\pi\sigma_v^2 e, \tag{31}$$

where $i(1 \leq i \leq m)$ is a constellation index. Specifically, to reduce m^N iterations to m^{N-1} iterations in (30), the terms calculation inside the norm of (31) is divided into brackets and $\mathbf{R}_N x_{N,i}$. The brackets values are determined after calculating all combinational constellation values with $(x_{1,i}, x_{2,i}, \ldots, x_{N-1,i})$, which all combinational calculations involve m^{N-1} iterations. Then, we consider that the brackets (determined) values minus $\mathbf{R}_N x_{N,i}$ when all constellations in $x_{N,i}$ are employed. Based on (31), the block diagram of the modified mutual information calculation (MMIC) is proposed in Fig. 4.

4 Computational Complexity

In this subsection, we analyze the computational efficiency (CE) in evaluating channel capacity and mutual information of constrained constellation as follows. To evaluate channel capacity, the symmetric QR and divide-and-conquer algorithms [16, pp. 421–444] are proposed to process the symmetric eigenvalue problem. For computational complexity, the symmetric QR algorithm and the divide-and-conquer algorithm are about $O(4M^3/3)$ and $O(M^2)$ [17, 18], respectively. Considering the multiplication of the full channel matrix ($\mathbf{HH}^H \in C^{M\times M}$) [6–9], the computational complexity of (8) is given as [16, 17]

$$Compl_{\mathbf{H}} \cong \frac{4}{3}M^3 + M^2 + NM^2 \text{ flops.} \tag{32}$$

To reduce the eigenvalue processes in computing channel capacity, the QR-based method is employed where QR complexity is about $O(2MN^2 - 2N^3/3)$ [16]. Considering the multiplication of triangular channel matrix ($\mathbf{RR}^H \in C^{N\times N}$), the computational complexity of (21) is given as [16, 17]

$$Compl_{\mathbf{R}} \cong \frac{4}{3}N^3 + N^2 + \sum_{i=1}^{N} i^2 + MN^2 - \frac{N^3}{3}\text{flops.} \tag{33}$$

Based on (31) and (32), the computational complexity requirements of (8) and (21) for evaluating channel capacity are compared by the CE ratio:

$$CE_c := \frac{Complex_{\mathbf{R}}(\text{capacity })}{Complex_{\mathbf{H}}(\text{capacity})} \cong \frac{\frac{4}{3}N^3 + N^2 + \sum_{i=1}^{N} i^2 + MN^2 - \frac{N^3}{3}}{\frac{4}{3}M^3 + M^2 + NM^2}. \tag{34}$$

To compute mutual information under various constellations, by using a full MIMO channel matrix multiplication [6], the computational complexity of mutual information of (28) is given as

$$Complex_{\mathbf{H}}(\text{mutual}) \cong 2^{mN} \cdot M \cdot N + 2^{mN}(M+1) \text{ flops,} \tag{35}$$

where m is an modulation order. By using the triangular channel multiplication, the computational complexity of mutual information of (31) is given as [13–15]

$$Complex_{\mathbf{R}}(\text{mutual}) \cong 2^{m(N-1)} \cdot \left(\frac{N^2+N}{2}\right) + 2^{m} \cdot N + 2^{mN}(M+1) + 2MN^2 - 2N^3/3\text{flops.} \tag{36}$$

Based on (35) and (36), assuming m and/or N is large enough, the computational complexity requirements of (28) and (31) for computing mutual information is compared by the CE ratio:

$$CE_m := \frac{Complex_{\mathbf{R}}(\text{mutual })}{Complex_{\mathbf{H}}(\text{mutual})} \cong \frac{\frac{N^2+N}{2} + M + 1}{2^m NM + M + 1} \leq 1, \quad \forall M \geq N. \tag{37}$$

Clearly, by using (37), the proposal QRD-based scheme in (30) is less computational complexity than the conventional scheme in (28) [6] because of the triangular matrix multiplication. Beside, analyzing channel capacity evaluation and mutual information evaluation in FPGA implementations, the complexity of the proposed QR-based schemes are analyzed by using the number of logic elements (LEs) in next.

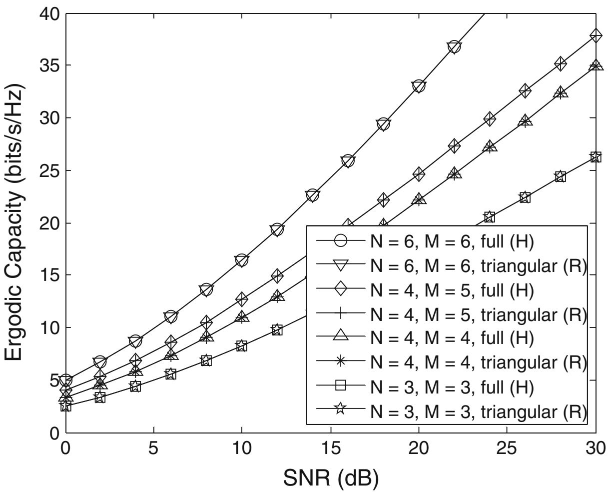

Fig. 5 Ergodic capacity versus SNR performance for various antenna configurations (s denotes second)

5 Simulation Results

In this section, first, we present the calculated channel capacity by using the proposed QRD-based scheme [13–15]. Then, we demonstrate the results by using our proposed QRD-based method which can reduce computational complexity in achieving mutual information under various constellations. In this MIMO system, the channel coefficient of (1) is obtained from the transmitting antenna n ($n = 1, 2, \ldots, N$) to the receiving antenna $m(m = 1, 2, \ldots, M)$ as [13–15]

$$h_{m,n} = h_{\mathrm{R}} + jh_{\mathrm{I}}, \tag{38}$$

where h_R and h_I are complex Gaussian random variables with a zero mean that denote the real part and the image part, respectively. Each point on the curves was obtained with averaging over 10^5 trials in the Monte-Carlo simulation. Assuming channel is perfectly known at receiver, the proposed QRD-based schemes evaluate the effects of channel capacity and optimal power, mutual information of constrained various constellations and reduced-complexity analysis as follows.

5.1 Channel Capacity and Optimal Power

Based on (10) and (21), Figs. 5 and 6, respectively show the ergodic capacity and 10% outage capacity for different SNR conditions and numbers of antennas. As expected, the ergodic capacity and outage capacity increases when SNR or the number of antennas was increased, respectively. For optimal power in (11) and (22), Fig. 7 shows the complementary cumulative distribution function (ccdf) versus capacity for various antenna configurations at SNR = 10 dB. As observed, assuming the same antenna configuration, the figure shows that the capacity is larger with channel knowledge known at the transmitter than without channel knowledge at the transmitter. That is, this capacity gains because the transmitter involves power allocation when the channel knowledge is given at transmitter. In Fig. 7, K and U denote that the channel

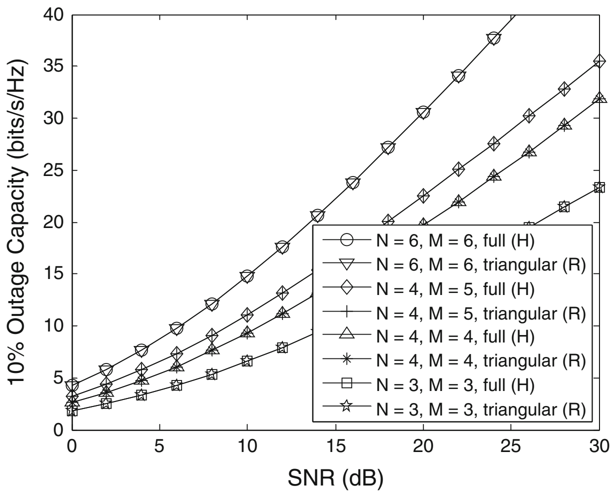

Fig. 6 Outage capacity versus SNR performance for various antenna configurations

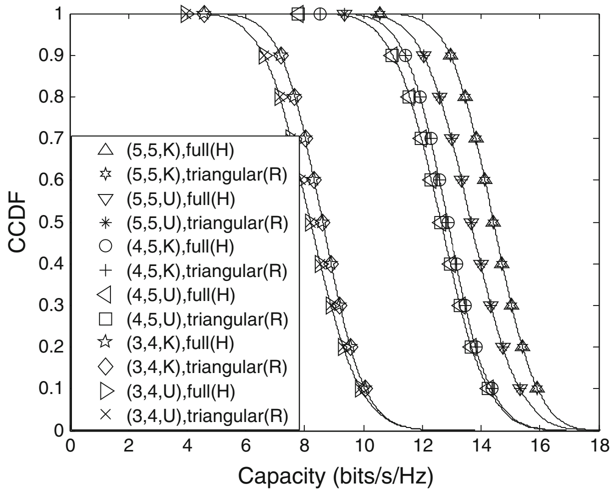

Fig. 7 CCDF versus capacity for various antenna configurations at SNR = 10 dB

knowledge is known and unknown at transmitter. Thus, Figs. 5, 6 and 7 demonstrate that the proposed triangular matrix (**R**) in (21) and (22) can be employed to realize the full MIMO channel matrix (**H**) in (10) and (11) [6–9], where it involves less computational complexity by using the triangular matrix multiplication.

5.2 Mutual Information of Constrained Various Constellations

In Fig. 8a, with $N = M = 3$, the theoretic channel capacity of (5) is represented by the highest curve. Based on (28) and (31), the remaining curves indicate mutual information of constrained various constellations when various SNR conditions are employed. That is,

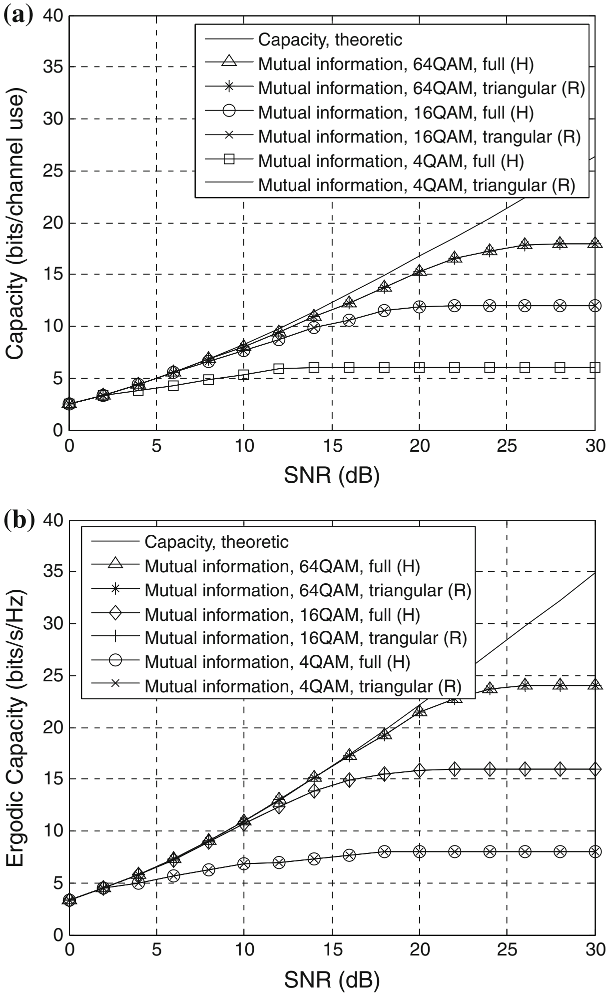

Fig. 8 Ergodic capacity versus SNR performance for various modulation orders employing. **a** $N = 3, M = 3$, **b** $N = 4$ and $M = 4$

the curves show maximum transmission rate variability achieved with 64-QAM, 16-QAM and 4-QAM. As a result, the design of a suitable transmitted data rate (C) can be ensured by channel code rate (R), modulation order (m) and the number of transmitting antennas (N). For example, considering an error-free transmission rate possibly for $R \cdot N \cdot m \leq C$ in Fig. 8a, we can be sure $R = 1$ at SNR $\geq$ 24 dB when $C = 18$ (bits/channel use), $N = 3$ and $m = 6$ are employed. Beside, we can ensure $R = 0.5$ at about SNR = 10 dB when $C = 9$ (bits/channel use), $N = 3$ and $m = 6$ are employed. Similarly, with increasing number of antennas, Fig. 8b shows the capacity increase when modulation order and/or SNR is increased. As expected, Fig. 8a and Fig. 8b demonstrate that the proposed triangular matrix (**R**) in (31) can be employed to realize the full MIMO channel matrix (**H**) in (28 [6], but at a low complexity level.

Table 1 Evaluating channel capacity complexity with computational impelmentation

Empolying in (34)	CE_c (Mul./Mul.) with QPSK
$M = 30, N = 30$	81131/81153 = 0.9997
$M = 30, N = 26$	56811/76581 = 0.7418
$M = 30, N = 22$	37398/72009 = 0.5193
$M = 30, N = 18$	22841/67437 = 0.3387
$M = 30, N = 14$	12491/62856 = 0.1987
$M = 30, N = 10$	5695/45900 = 0.0977

Table 2 Evaluating channel capacity complexity with FPGA impelmentation

Empolying in (34)	CE_c (LEs/LEs) with QPSK
$M = 20, N = 20$	23279/24473 = 0.9512
$M = 20, N = 19$	21476/23707 = 0.9
$M = 20, N = 18$	18726/23199 = 0.8072
$M = 20, N = 17$	16214/22691 = 0.7146
$M = 20, N = 16$	13929/22183 = 0.6279
$M = 20, N = 15$	7421/12237 = 0.5473

Table 3 Evaluating mutual information complexity with computational impelmentation

Empolying in (37)	CE_m (Mul./Mul.)
$M = 10, N = 10$ (QPSK)	25954107/116391936 = 0.223
$M = 10, N = 9$ (QPSK)	5833874/26476544 = 0.2203
$M = 10, N = 8$ (QPSK)	1311726/5963776 = 0.1992
$M = 5, N = 5$ (16QAM)	8729701/39007121 = 0.2238
$M = 5, N = 4$ (16QAM)	2044726/521235 = 0.2549

5.3 Reduced-Complexity Analysis in the Proposed QRD-Based Scheme

In this subsection, we investigate (1) channel capacity evaluation and (2) mutual information evaluation in both computer simulation and FPGA implementation. First, regarding channel capacity evaluation in computer simulation, considering M is fixed, Table 1 demonstrates that the CE ratio of (34) decreases when N is decreased in practical. Specifically, in Table 1, the proposed QRD-based scheme can reduce the computational complexity by about 91% over [6–9], when $N = 10$ and $M = 30$. That is, we can reduce the eigenvalue processes because $\mathbf{RR}^H$ with N-by-N has fewer dimensions than $\mathbf{HH}^H$ with M-by-M. In Fig. 3, channel capacity evaluation has been designed in Verilog and implemented with FPGA via Xilinx Virtex-5 (XC5VSX240T) [19]. Table 2 demonstrates that the CE ratio of (34) decreases when N is decreased. Second, regarding mutual information evaluation in our computer simulation, Table 3 demonstrates that the proposed QRD-based scheme in (31) has less computational complexity than conventional scheme in (28). For FPGA implementation in Fig. 4, Table 4 shows that the proposed QRD-based scheme can reduce the computational complexity than conventional scheme [6]. Thus, the CE ratio of (34) and (37) are claimed via Tables 1, 2, 3, 4 in computer and FPGA implementation.

Table 4 Evaluating mutual information complexity with FPGA impelmentation

Empolying in (37)	CE_m (LEs/LEs) with QPSK
$M = 6, N = 6$ (QPSK)	$54488/187128 = 0.292$
$M = 6, N = 5$ (QPSK)	$38283/132457 = 0.289$
$M = 6, N = 4$ (QPSK)	$2587/8967 = 0.2887$
$M = 3, N = 3$ (16QAM)	$74547/25206 = 0.3381$
$M = 3, N = 2$ (16QAM)	$1571/3584 = 0.4385$

6 Conclusions

In this paper, considering low-complexity, the triangular matrix multiplication is proposed to achieve channel capacity, optimal power and mutual information of constrained constellation for MIMO communications. Beside, we also propose a modified mutual information calculation (MMIC) to achieve a quite low-complexity via the divided calculation. To evaluate low-complexity, the MIMO channel capacity and mutual information was analyzed by using the proposed CE in simulation results. Our future work will investigate MIMO techniques in accordance with precoder and STBC for mobile broadband wireless access applications.

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