行政院國家科學委員會專題研究計畫成果報告

應用次空間法從微動、自由振動及地震反應量測識別結構系統特性
The Modal Identification of Structures from Ambient Vibration, Free Vibration, and Seismic Response Data by Using a Subspace Approach

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中文摘要

本研究旨在發展一套可同時應用於處理微動量測、自由振動量測、及地震反應數據之統一系統識別程序。該程序架構於狀態空間模式，並利用次空間法結合工具變數法求取結構系統之動態特性。因此，使用者只需要熟悉一套理論，即可同時解決從微動量測、自由振動量測、及地震反應進行系統識別之問題。

本研究中，首先利用數值模擬確認所提系統識別程序之可行性及準確性。然後，為驗證實際應用之可行性，進行以下之試驗數據處理：(1) **進行一五層樓鋼構之微動量測分析，並與多變數** ARMA **模式所得者比較；**(2) **進行一三跨連續預力混凝土橋之衝擊試驗分析，並與** Ibrahim **時間域識別法所得者比較；**(3) **進行一五層樓鋼構之振動台試驗分析，並與有限元素法所得者比較。**

關鍵詞：次空間法；微動量測；自由振動量測；地震反應；系統識別

1. Abstract

The main purpose of this research is to develop a unifying procedure for identifying the dynamic characteristics of structures from their ambient vibration, free vibration, and earthquake response data. The proposed procedure will use a state-space model cooperating with a subspace approach and instrumental variable concept. Hence, the users only need to know a procedure and the corresponding theoretical background in order to determine the dynamic characteristics of a structure from different field tests.

*The feasibility and accuracy of the proposed procedure will be demonstrated through numerical simulation. Then, the procedure will be applied to real observed data: (1) ambient vibration measurement of a five-story frame; (2) free vibration measurement of a three-span continuous pre-stressed concrete bridge; (3) earthquake response data from shaking table test. The identified results will be compared with those obtain*ed *from other methods.*

Keywords: subspace-based approach, ambient vibration measurement, free vibration measurement, earthquake responses, system identification

2. Motive and Goal

Investigating the dynamic characteristics of an existing structure system based on field tests is essential in confirming the construction quality, validating or improving analytical finite element structural models, or conducting damage assessment. To accomplish this task, the popular field tests are ambient vibration tests, forced vibration tests, free vibration tests, and earthquake response measurement. Notably, excluding forced vibration tests due to their periodic characteristics of input, identifying the dynamic characteristics of a structural system from the other three tests can be accomplished in time domain.

Even in the time domain analysis, various schemes are often applied to process the data from various field tests. For example, to determine the dynamic characteristics of a structural system from free vibration test, Ibrahim time domain system identification (ITD) technique is often applied [1]. However, it cannot be directly applied to process either the ambient vibration test or the earthquake response measurement. That is, to process the ambient vibration measurement, ITD technique has to comply with random decrement technique [2]. There is no a rigorous procedure to extract free vibration responses from earthquake responses such that ITD technique can be applied to determine the dynamic characteristics from the resultant responses. Based on the assumption of stationary process for observed data, time series models, *i.e.* AR and ARMA models, are also often employed for ambient vibration measurements [3-6]. Apparently, however, this assumption is not valid for free vibration measurement and earthquake response measurement. Consequently, to analyze the observed data from different tests, various techniques as well as the corresponding theoretical backgrounds must be understand, which becomes burdensome for the users. Therefore, this study develops a system identification procedure capable of processing the measurement from various tests.

The proposed procedure is based on state-space model cooperating with a subspace approach. Rao and Arun [7] provided a comprehensive review on the data processing by using state space approaches, while Van DerVeen et al. [8] collected more than one hundred articles on signal analysis by subspace-based approaches. Viberg [9] also reviewed and compared numerous subspace-based schemes. He classified these schemes into two categories:(1) realization-based subspace methods [10-12], which estimate the coefficient matrices of a state-space model via measured impulse response functions; (2) direct subspace-based methods [13-16], which estimate the coefficient matrices via observed input and output signals. Apparently, even in the subspace-based approach, varying schemes were applied to the data from varying tests.

To identify the dynamic characteristics of structures from the ambient vibration, free vibration, and earthquake response data, this study develops a unified procedure by extending, with some modification, the direct subspace-based method with the instrumental variables proposed by Viberg et al. [17], who developed a procedure to estimate the observability matrix for the state space model with measured inputs. Furthermore, to demonstrate the feasibility of the proposed procedure, the procedure is applied to process an ambient vibration measurement of a five-story steel frame, a free vibration measurement of a three-span continuous pre-stressed concrete bridge, and simulated earthquake responses of two five-story steel frames from the shaking table test. The dynamic characteristics identified herein are compared with those obtained from other methods.

3. Contents of the Research

3.1 Methodology

The observed responses of a linear system with a n degrees of freedom subjected to m input can be expressed by the following state-space model:

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}\boldsymbol{z}_k + \boldsymbol{B}\,\boldsymbol{f}_k, \qquad (1)$$

$$\boldsymbol{y}_k = \boldsymbol{E}\boldsymbol{z}_k + \boldsymbol{D}\boldsymbol{f}_k + \boldsymbol{a}_k, \qquad (2)$$

where $\boldsymbol{z}_k$ is a $2n$-dimensional state vector, $\boldsymbol{f}_k$ is a m-dimensional input vector, $\boldsymbol{y}_k$ is a l-dimensional observed response vector, $\boldsymbol{a}_k$ is a l-dimensional white-noise vector. The subscript "k" denotes the signal at $t=k\,\Delta t$, where Δt is time increment. Substituting Eq. (1) into Eq. (2) with careful arrangement yields

$$\boldsymbol{Y}_k = \tilde{\boldsymbol{A}}_{\sim}\boldsymbol{Z}_k + \ddot{\boldsymbol{O}}_{\sim}\boldsymbol{F}_k + \ddot{\boldsymbol{A}}_k, \qquad (3)$$

where the definitions and lengthy expressions for $\mathbf{Y}_k$, $\mathbf{Z}_k$, $\mathbf{F}_k$, $\ddot{\mathbf{A}}_k$, $\tilde{\mathbf{A}}_{\sim}$, and $\ddot{\mathbf{O}}_{\sim}$ are given in the writer's recent papers [18-19].

From linear algebra, one can define a orthogonal projection matrix, $\boldsymbol{Đ}_f^{\perp}$, onto the null-space of $\boldsymbol{F}_k$ as

$$\boldsymbol{Đ}_f^{\perp} = \boldsymbol{I} - \boldsymbol{F}_k^T(\boldsymbol{F}_k\boldsymbol{F}_k^T)^{-1}\boldsymbol{F}_k. \qquad (4)$$

Introduce instrumental variables $\boldsymbol{P}$ defined as

$$\boldsymbol{P} = \begin{bmatrix}\boldsymbol{F}_p^T & \boldsymbol{Y}_p^T\end{bmatrix}^T, \qquad (5)$$

where $p<k$. Then, multiplying $\boldsymbol{Đ}_f^{\perp}$ to both sides of Eq. (3) to eliminate the second term in the right-hand side of Eq. (3) and multiplying $\boldsymbol{P}^T/N$ to the both sides of resulting equation yield [17]

$$\frac{1}{N}\boldsymbol{Y}_k\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T = \frac{1}{N}\tilde{\boldsymbol{A}}_{\sim}\boldsymbol{Z}_k\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T. \qquad (6)$$

It should be noted that the fact has been used in deriving Eq. (6) that white-noise vector $\boldsymbol{a}_k$ is not correlated with input force $\boldsymbol{f}_t$ for all t, and with observed response vector $\boldsymbol{y}_q$ for $k>q$.

To reduce the variance of the estimated coefficient matrices for the state-space model due to noise and the bias of the estimation due to under-modeling, the weight matrices suggested by Verhaegen [15]: $\boldsymbol{W}_r = \boldsymbol{I}$ and $\boldsymbol{W}_c = (\boldsymbol{P}\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T/N)^{-1/2}$ were multiplied to Eq. (6). Then, we have

$$\frac{1}{N}\boldsymbol{W}_r\boldsymbol{Y}_k\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T\boldsymbol{W}_c = \frac{1}{N}\boldsymbol{W}_r\tilde{\boldsymbol{A}}_{\sim}\boldsymbol{Z}_k\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T\boldsymbol{W}_c \qquad (7)$$

Define a matrix $\overline{\boldsymbol{H}}$ equal to the left-hand side of Eq. (7). By using singular value decomposition, the following relation holds:

$$\overline{\boldsymbol{H}} \approx \boldsymbol{Q}_{\overline{n}}\,\Sigma_{\overline{n}}\,\boldsymbol{V}_{\overline{n}}^T, \qquad (8)$$

where $\Sigma_{\overline{n}}$ is a diagonal matrix containing the $\overline{n}$ largest singular values, the columns of $\boldsymbol{Q}_{\overline{n}}$ and $\boldsymbol{V}_{\overline{n}}$ are the corresponding left and right singular vectors, respectively. It should be noted that $\overline{n}$ is equal to $2n$ in the perfect data case. If the data contain noises, $\overline{n}$ is typically larger than $2n$.

From Eqs. (7) and (8), one has

$$\tilde{\boldsymbol{A}}_{\sim} = \hat{\tilde{\boldsymbol{A}}}_{\sim}\boldsymbol{T}_{\overline{n}} \qquad (9)$$

where $\hat{\tilde{\boldsymbol{A}}}_{\sim} = \boldsymbol{W}_r^{-1}\boldsymbol{Q}_{\overline{n}}$ and $\boldsymbol{T}_{\overline{n}} = \Sigma_{\overline{n}}\boldsymbol{V}_{\overline{n}}(\boldsymbol{Z}_k\boldsymbol{Đ}_f^{\perp}\boldsymbol{P}^T\boldsymbol{W}_c/N)^{-1}$. It can be easily proved the following relations

$$\hat{\tilde{\boldsymbol{A}}}_{\sim} = \begin{bmatrix}\hat{\boldsymbol{E}}^T & (\hat{\boldsymbol{E}}\hat{\boldsymbol{A}})^T & \text{i} & (\hat{\boldsymbol{E}}\hat{\boldsymbol{A}}^{s-1})^T\end{bmatrix}^T,$$

$$\hat{\boldsymbol{A}} = \boldsymbol{T}_{\overline{n}}\boldsymbol{A}\boldsymbol{T}_{\overline{n}}^{-1}, \text{ and } \hat{\boldsymbol{E}} = \boldsymbol{E}\boldsymbol{T}_{\overline{n}}^{-1}. \qquad (10a\text{-}c)$$

(4i)

From the measured responses and input, one can easily estimate matrix $\hat{\tilde{\boldsymbol{A}}}_{\sim}$. Then, a least-squares approach was applied to estimate $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{E}}$ from the relationship given in Eq. (10a).

The dynamic characteristics of the structural system can be determined from the eigenvalues and eigenvectors of $\hat{\boldsymbol{A}}$. Let's denote the j^{th} eigenvalue and the corresponding eigenvector by $\hat{a}_j$ and $\tilde{\ }_j$, respectively. Furthermore, let $\hat{a}_j = a_j + ib_j$. Then, the corresponding undamped circular natural frequency μ_j and modal damping ratio $¶_j$ are determined by the following relations:

$$\mu_j = \sqrt{\sim_j^2 + \ _j^2} \text{ and } ¶_j = -\sim_j / \mu_j \qquad \text{(11a,b)}$$

where $\sim_j = \ln(a_j^2 + b_j^2)/(2\text{Ä}t)$ and $\ _j = \tan^{-1}(b_j / a_j)/\Delta t$. The modal shape for the observed degrees of freedom $\ddot{o}_j$ is equal to $\hat{E}\tilde{\ }_j$.

3.2 Aplications

To demonstrate the feasibility of the proposed procedure on processing the data in real applications, the proposed procedures were applied to identify the dynamic characteristics of structures from their ambient vibration measurement, simulated response data from shaking table tests, and free vibration measurement.

Table 1 shows the identified modal parameters for five-story steel structure by using present procedure to process its ambient vibration measurement in long-span direction. Table 1 also lists the identified results obtained by employing the multivariate ARMA model in combination with a two-stage least-squares approach [20]. Comparing the present results with those published reveals an excellent agreement between them. Strictly speaking, the agreement for the frequencies and modal shapes is superior to that for the damping ratios.

The proposed identification procedure was also applied to determine the modal parameters of a five-story steel frame from its acceleration responses subjected to base excitation in shaking table tests. The results for the first three modes were well correlated with those from finite element analysis. As a third example, to identify the dynamic characteristics of a highway bridge in vertical direction from its impulse test, the proposed procedure was applied. This test was conducted on an elevated highway bridge before it was opened to the public. Because of the limitation of pages, the results will not be shown here. The readers who are interesting these results may refer to [19].

4. Discussion

A unified procedure to identify the dynamic characteristics of the structural system from ambient vibration, free vibration, and earthquake response data has been presented herein. This procedure was established through a space-state model to describe the measured responses. The coefficient matrices related to the dynamic characteristics were evaluated by a subspace method cooperating with the concept of instrumental variable. Then, the dynamic characteristics were evaluated from the eigenvalues and eigenvectors of a coefficient matrix. One of the primary advantages of the procedure is that it proposes a suitable order for the space-state model from the singular values of $\overline{H}$ in Eq. (8).

To demonstrate its feasibility for actual applications, the proposed procedure has been applied to process in situ ambient vibration measurement of a five-story steel frame and free vibration measurement of a three-span continuous highway bridge. The identified dynamic characteristics from the ambient vibration data closely corresponded to those obtained from the multivariate ARMA model with a two-stage least-squares approach. The identified results from the free vibration data also display an excellent agreement with those from ITD technique, thus validating the applicability of the proposed approach.

5. Comment and Conclusion

We have achieved the goals of the project given in the proposal. Based on the results in this work, a paper has been accepted for publishing in *Earthquake Engineering and Structure Dynamics.* Furthermore, some of the results has also been reported in *the First International Conference on Structural Stability and Dynamics* held in Taipei 2000.

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Table 1. Identified results from ambient vibration measurement

Mode	Frequencies (Hz)		Mode Shapes		Modal Damping (%)	
	Present	ARMAV	Present	ARMAV	Present	ARMAV
1	0.88	0.88	{1.0, .85, .68, .42, .15, .002}	{1.0, .85, .68, .42, .16, /}	0.22	0.27
2	2.93	2.93	{−.88, −.059, .78, 1.0, .53, .009}	{−.88, −.059, .78, 1.0, .53, /}	0.20	0.22
3	5.54	5.55	{.71, −.85, −.81, .74, 1.0, .019}	{.71, −.86, −.80, .75, 1.0, /}	0.51	0.46
4	8.37	8.37	{−.36, .98, −.61, −.41, 1.0, .012}	{−.35, .96, −.58, −.41, 1.0, /}	0.49	0.52
5	10.4	10.4	{−.15, .62, −.96, 1.0, −.77, −.018}	/	0.46	0.51

Note: / means no data available.