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亞洲大學與商學院評比系統設計----達爾菲排序分組法

(2/3)

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行政院國家科學委員會專題研究計畫期中進度報告

亞洲大學與商學院評比系統設計－達爾菲排序分組法(2/3)

Ranking and Grouping on World Business Schools (2/3)

計畫編號：NSC 94-2213-E-009-023

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本研究為三年期計畫，目的在於發展亞洲大學與商學院評比系統。時下許多雜誌如 Time、U.S. News 和 Financial Times 等，利用不同數學公式出版全美或全球大學評鑑刊物，然而卻被許多人在評比方法之確切性及資料正確性上遭受批評。本論文所提出的方法能利用決策者所提供之偏好自行計算出各評比指標之權重，並以表格及 3D Ball 之視覺工具呈現排序與分群結果。以此結果為參考，決策者能再次加入偏好或是在決策單位〔DMU〕間作修正，以獲得想要之結果。此方法能根據決策者多次加入之偏好來計算與其邏輯相似之評比結果，以達成協助決策者做明確的決策選擇。

關鍵詞：排序、分群、商學院、階層分析法、偏好

Abstract

Companies like Time, U.S. News, and Financial Times use different ranking models to publish university ranking guides. However, many critics say the ranking formulas are constantly changing and the data is highly manipulable. In the proposed model, the decision makers can rank universities based on their preferences. Based on the preferences, this model will automatically generate a set of weightings for criteria in the ranking process. The ranking and the grouping result will be displayed using both tables and 3D ball visualization tool. The decision makers can further specify the relationships between DMUs or add more preferences to obtain desired outcome. Providing decision makers various chances and means to add their opinions through out the ranking process, this model can ensure that the result are consistent with what decision makers had in mind and can ,hence, help them in the decision making process.

Keywords: Business School, Ranking, Grouping, Pairwise Comparison, AHP, Preference

1. Introduction

Every year, many high school graduates and university graduates purchase University Ranking Guides to help them select the right undergraduate program or graduate program that is best suited for them. Although among the quarter million freshmen who participated in the survey done by the Higher Education Research Institute, only 8.6% responded that the rankings were very important to them when selecting colleges or universities (Crissey, 1997). The reasons may lie on the question of ranking methodology. How do we know these rankings are right for the students and rank universities in the way the students needed? How do we know the criteria participated in the ranking system are what the ones students consider important? These are some of the key concerns which should be solved.

Currently, there are many publishers which release various kinds of ranking each year. US News and World Report, for example, started releasing university ranking in with the October issue in late 1980's. They have realized that in the subsequent years, the October issue had sold many more copies than any other issues. Hence, they decided to start publishing an independent issue for university ranking. In the 1990's, many other publishers like Time, Newsweek, Money Magazine, and many more have also realized that the market for university ranking is enormous and have started to create their own rankings and publish them. Similarly, Canada, Asia, and Europe all have magazines that do rankings for universities in different regions.

The ranking guides currently in the market are heavily criticized by many people ranging from educational field to people in the publishing industry. Some of these criticisms are as follow:

(1) To increase the sales, publishers may introduce new measures or change the weightings of measures from year to the next (Gater, 2003).

(2) Some of the factors are highly manipulable, and, as a result, the ranking outcome is meaningless (Leiter, 2003).

(3) Ranking formula and factors participated in the ranking process are constantly changing, so the results are high in variation (Levin, 1997).

In this study, we propose a new ranking method that can help the Decision Makers (DM) rank Decision Making Units (DMUs). The characteristics are listed below:

(1) The model can automatically generate weightings with minimal human influence.

(2) Ranking can still be done with minimum information from Decision Makers, i.e. preferences.

(3) 3D ball representation gives clear view on the correlations.

(4) This model allows DM to add preferences through out the ranking process.

(5) DM can specify groupings for DMUs.

2. Literature Review

There are several rankings published in the market. Each of them has different methodology to rank universities. They vary in criteria selection, assignment of weightings, and raw data, just to name a few. Let us look at few of the more popular ranking systems and their methodology.

2.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a method for evaluating the activity performance, especially for organizations such as business firms, government agencies, hospitals, educational institutions, and etc (Cooper etc. 1999). A commonly used measure for efficiency is the output-input ratio. Number of items sold in a store will be an example of the output; number of sales clerk in the store will be the input. Hence, the efficiency of this store, basing on only these two criteria, will simply be NumberOfGoodsSold / NumberOfClerk. These comparable entities are often called Decision Making Units (DMUs).

The purpose of DEA is to empirically estimate the efficient frontier based on the set of available DMUs and assumes that each performance measure can be categorized as either an input or an output (Schrage, 1997). It provides the user information about both efficient and inefficient units along with the efficiency scores and reference sets for inefficient units (Halme etc, 1999). An Efficient Frontier is a line that has at least one DMU point touching it. The DMUs, who touch the EF line, are the most efficient DMUs. The idea of Production Frontier is first discussed by Farrell in 1975 which has three assumptions. The attractive feature of DEA is that it produces efficiency score between 0 and 1.

In 1978, Charnes, Cooper, and Rhodes proposed a DEA model called the CCR model basing on Farrell's single input-output model in 1975. CCR model is designed to measure the cases of multi input and multi output. The following is the pseudo-code for the CCR model. *Ur* represents the weighting for r^{th} output criterion and *Vi* represents the weighting for i^{th} input criterion. They are automatically generated when the score of k^{th} DMU is maximized. *Yr* and *Xi* are the output and input criteria.

For each DMU *k*

$$MAX \quad Score_k = \frac{\sum_{r=1}^{s} U_r Y_r}{\sum_{i=1}^{m} V_i X_i}$$

such that

$$Score_k \leq 1$$

$$U_r > 0$$

$$V_i > 0$$

Where

Yr is the rth output of DMU

X_i is the i^{th} input of DMU
U_r is the weighting for r^{th} output
V_i is the weighting for i^{th} input

In this CCR model, it will calculate the score of each DMU based on the weightings that can maximize the score of current DMU, which means that the n^{th} DMU can obtain the best score with n^{th} set of weightings. Hence, if there are n numbers of DMUs, then there will have n set of weightings. k^{th} set of weighting is determined under the condition that they can maximize the $Score_k$. All the scores have to be between 0 and 1. Once score of each DMU is determined, it then compares all of them again with their score. The DMU with highest score is the most efficient one.

2.2 Analytic Hierarchy Process

The Analytical Hierarchy Process (AHP) was proposed by Saaty in 1980 and his collaborators as a method for establishing priorities in multi-criteria decision making contexts based on variables that do not have exact numerical consequences (Genest, 1996). It also helps people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best.

AHP can be conducted in three steps:

Setp 1: Perform pairwise comparisons between each DMU on every criterion

In this step, the goal is to obtain the priorities between DMUs for each criterion. To do so, a pairwise comparison has to take place between each DMU with respect to each criterion. For each criterion, a m by m matrix, where m is the number of DMUs, will be generated and the priority vector will be calculated from this matrix. Priority vector displays the preference orders for each DMU with respect to criteria. Since there are n numbers of criteria, n number of priority vector will be generated at the end.

Step 2: Perform pairwise comparison between each criterion

In the decision making process, not every criterion is quantitatively measurable, so a pairwise comparison between each criterion has to take place in order to specify the importance between each criterion. From the comparison, a set of weightings can be found for score calculation at the last step.

Step 3: Compute final scores for DMUs

With the priority vectors and the weightings for criteria, DM can now calculate the score for each DMU. DMU with the higher score should be the better alternative for the Decision Maker.

Following is an illustration of an example of a student, John, wanting to purchase a car. Due to his financial limitation, John can only buy a second hand car, and only has few things that he really car. He wants to buy a car that is cheap, nice out look, and comfortable. However, among the three cars he has in mind, none of them has best score on each of these criteria. He has decided to use AHP to help him select a car from these three. Table 2.1 lists all the data he gathered about these three cars.

Table 2.1 Hard data provided by John on cars.

	Price	Look	Comfort
Car 1	13100	Good	Very good
Car 2	12000	Fair	Good
Car 3	9800	Good	Fair

To perform pairwise comparison between each car with respect to each criterion, a priority score has to be assigned to each comparison. The scores can range from 1 to 9, where 9 is the most satisfactory score. Notice that if a DM compare A_1 to A_2 and assigns a score of 4, then the score between comparison of A_2 and A_1 will be the inverse of A_1 and A_2's, which will be 1/4. This property can ensure the logical consistency for each comparison.

1 Choice i and j are equally important
3 Choice i is weakly more important than j
5 Choice i is strongly more important than j
7 Choice i is very strongly more important than j
9 Choice i is absolutely more important than j
2, 4, 6, 8 are intermediate values

After finishing pairwise comparisons, matrixes with these priority scores will be generated (Table 2.2).

Table 2.2 Comparison score for each car with respect to each criterion

Criteria	**Price**			**Look**			**Comfort**		
	Car1	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**
Car1	1	1/3	1/8	1	3	1	1	3	6
Car2	3	1	1/6	1/3	1	1/4	1/3	1	4
Car3	8	6	1	1	4	1	1/6	1/4	1

From these matrixes, normalization has to be done before the priority vectors can be calculated (Table 2.3). Normalization is simply divides each value by the sum of corresponding column. For example, the normalized value between car2 and car3 with respect to price is calculated by
(1/6) / (1/8 + 1/6+ 1) = 0.1290.

Table 2.3 Normalized comparison table

Criteria	Price			Look			Comfort		
	Car1	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**
Car1	0.0833	0.0454	0.0967	0.4286	0.375	0.4444	0.6666	0.7059	0.5454
Car2	0.250	0.1363	0.1290	0.1428	0.125	0.1111	0.2222	0.2352	0.3636
Car3	0.6666	0.8182	0.7742	0.4286	0.5	0.4444	0.1111	0.0588	0.0909

Each criterion has its own priority vector and the values in the vector can be seen as the score of each DMU on corresponding criterion. The values in the priority vectors are the sum of rows from the normalized pairwise comparison matrix and divided by the number of DMUs, as in Table 2.4. The values in priority vector for price is calculated as follow:

(0.0833 + 0.0454 + 0.0976) / 3 = 0.2254

(0.2500 + 0.1363 + 0.1290) / 3 = 0.5153

(0.6666 + 0.8182 + 0.7742) / 3 = 2.2590

Table 2.4 Priority vectors with respect to each criterion

	Priority Vector for Price	**Priority Vector for Look**	**Priority Vector for Comfort**
Car1	0.0751	0.4160	0.6393
Car2	0.1717	0.1263	0.2736
Car3	0.7530	0.4576	0.0869

After the values of priority vector is calculated, pairwise comparison has to perform on criteria to obtain the weightings for each criterion. Similar to previous steps, a 3 by 3 matrix, with criteria on both row and column, will be created. Using the same calculation method for priority vector, the weighting for each criterion can also be found (Table 2.5).

Table 2.5 Comparison tables and weightings for criteria

	Comparison Matrix			Normalized Comparison Matrix			
	Price	Look	Comfort	Price	Look	Comfort	Weighting
Price	1	1/5	3	0.1579	0.1489	0.2727	0.1931
Look	5	1	7	0.7894	0.7447	0.6363	0.7234
Comfort	1/3	1/7	1	0.0526	0.1064	0.0909	0.0833

The weightings on Table 2.5 suggest that Look is the most important criterion for John. Price is the next concern and comfort is the last. With the weightings on the criteria and the priority vectors on each criterion, the score for each car can now be calculated as follow:

Car 1: (0.0751 * 0.1931)+(0.4160 * 0.7234)+(0.6393 * 0.0833) = 0.3687
Car 2: (0.1717 * 0.1931)+(0.1263 * 0.7234)+(0.2736 * 0.0833) = 0.1473
Car 3: (0.7530 * 0.1931)+(0.4576 * 0.7234)+(0.0896 * 0.0833) = 0.4839

From the calculation, Car 3 has the highest score and should be the best choice for John to consider.

3. Ranking and Grouping Models

In this section, the ranking and grouping process can be break down into two major parts. First part will deal with the actual ranking and score calculation. The second part is mapping each school onto a 3D ball and clustering these data points. Figure 3.1 shows the entire process of proposed ranking and grouping model.

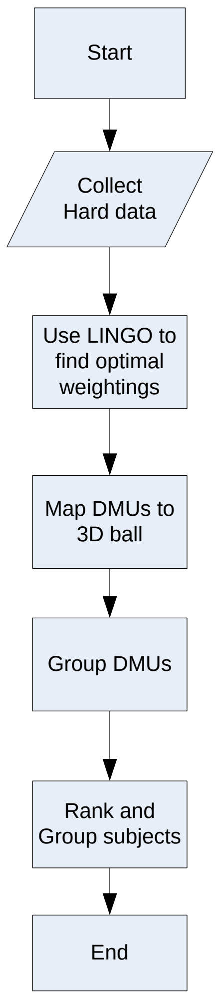

Figure 0.1Flowchart

3.1 Common Weight Model

As discussed in chapter 2, DEA is mainly used for efficiency measurement. The concept of DEA is to calculate the ratio between inputs and outputs, and rank each DMU (Data Making Unit) by their maximized scores. In this ranking objective, however, DEA is not the perfect tool for the ranking process because the most efficient DMU might not be the best choice for DM (Decision Maker). Moreover, , sometimes criteria are hard to distinguish from input or output, the proposed method has modified the traditional DEA method to meet the DMs' requirement without the need to identify inputs and outputs for criteria. This model will automatically ranks and groups the DMUs based on the absolute dominance relationships found in the hard data, so the DMs do not need to worry about assigning weightings for each criterion. This is a big improvement from the traditional ranking systems, which often have controversy on weighting settings.

In the experiments, Lingo8.0 is used as the optimization tool. Given the correct model and inputs, the system will calculate the ideal weights for each criterion, which will allow us to rank the DMUs and map each DMU to a coordinate on 3D ball to help DM visualize the relationships between DMUs, as well as the correlation between DMUs. In this section, the mathematical model and the concept behind it will be discussed in detail and the model will be applied on an example of 20 universities. Before the mathematical model is being discussed, Table 3.1 lists and describes the variables, following is the model.

Table 3.1 Variables for Common Weight Model

Variables	**Descriptions**
m	Total number of DMUs
n	Total number of criteria
$t_{i,j}$	$t_{i,j} = 1$ if DMU *j* is better than DMU *i, else* $t_{i,j} = 0$
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
M	A large constant number

Common Weight Model (Model 1):

$$Min \quad \sum_{i=1}^{m}\sum_{j\neq i}^{m} t_{i,j} \qquad \textbf{(3.1)}$$

Subject to

$$\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right)+\left(M * t_{i,j}\right) \quad \geq \quad \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right). \quad \forall\ i, j\ and\ j \neq i \qquad \textbf{(3.2)}$$

$$\sum_{k=1}^{n} w_k \quad = \quad 1 \qquad \textbf{(3.3)}$$

$$w_k \geq \varepsilon, \quad \forall \quad k \qquad \textbf{(3.4)}$$

$$t_{i,j} \in \{0,1\} \qquad \textbf{(3.5)}$$

$$t_{i,j} + t_{j,i} \leq 1, \quad \forall \quad i, j < i \qquad \textbf{(3.6)}$$

In this model, Lingo will generate a set of weightings for the ranking process. This model ranks the DMUs without DMs worrying about the numbers (weightings). Moreover, these weightings could be more convincing for some DM because these numbers are generated by the system automatically based only on the absolute dominance relationships.

After this model is run by Lingo, Lingo will return a matrix with the size of m by m. This matrix will consist values of only 0 and 1. For t_{ij}, if $t_j > t_i$, then t_{ij} will be set to 1. The sum of each row will represent their rank correspondingly. The objective function (3.1) is trying to maximize the rank of each DMU by minimizing the sum of *t* for each row. Note that the DMU with lower the sum of *t*, the higher rank it will get. Constraint 3.2 is for determining the values of $t_{i,j}$. If $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right)$ is greater than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right)$, then $t_{i,j}$ will be 0, since we are minimizing the sum of $t_{i,j}$. On the other hand, if $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right)$ is smaller than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k}-\underline{C}_k}{\overline{C}_k-\underline{C}_k}\right)\right)$, in order to satisfy constraint 3.2, the value of $M * t_{i,j}$ must not be 0, so $t_{i,j}$ will be set to 1.

Constraint 3.3 is to make sure that the sum of weights of all the criteria will be equal to 1. Also, constraint 3.4 ensures that the weights are all non-zero, so every criterion will be taken into account in this ranking process. Constraint 3.5 specifies that $t_{i,j}$ is a binary variable, which can only be 0 or 1. The last constraint is to insure that if *i* is better than *j*, then j can not be better than *i* at the same time.

Once the weights for each criterion are automatically generated by the model, score of each DMU will be calculated by equation 3.7 for future ranking purposes. This score function ensures that the scores are all between 0 and 1 by normalizing the hard data. This will help DM to see the differences in the scores.

$$SCORE_i = \sum_{k=1}^{n} \left(w_k * \frac{\left(C_{i,k} - \underline{C_k}\right)}{(\overline{C_k} - \underline{C_k})} \right) \qquad \textbf{(3.7)}$$

Table 3.2 shows the original hard data of the first twenty universities listed on the Financial Times' 2004 Global MBA Ranking. The data has been normalized so that 1 is the maximum score and 0 is the minimum score. Notice that we have only chosen six criteria that have the heaviest weightings.

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Table 3.2 Normalized hard data from Financial Times' 2004 Ranking

Rank in 2004	School name	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT doctoral rank	FT research rank
1	University of Pennsylvania:	0.836865335	0.855670103	0.74157303	1	1	0.987805
2	Harvard Business School	1	0.525773196	0	0.888889	0.92	1
3	Columbia Business School	0.696863457	1	0.39325843	0.888889	0.88	0.939024
4	Insead	0.553465223	0.257731959	1	0.888889	0.373333	0.890244
4	London Business School	0.42117949	0.680412371	0.87640449	0.888889	0.56	0.780488
4	University of Chicago GSB	0.658188819	0.855670103	0.6741573	0.888889	0.773333	0.963415
7	Stanford University GSB	0.814405559	0.402061856	0.35955056	0.944444	0.866667	0.97561
8	New York University: Stern	0.408235773	0.886597938	0.4494382	0.944444	1	0.865854
9	MIT: Sloan	0.645918112	0.463917526	0.17977528	0.777778	0.973333	0.902439
10	Dartmouth College: Tuck	0.725693358	0.773195876	0.30337079	0.777778	0	0.829268
11	Northwestern University: Kellogg	0.640330558	0.494845361	0.78651685	0.833333	0.746667	0.95122
12	IMD	0.694437488	0	0.47191011	0.722222	0	0.097561
13	Iese Business School	0.018985162	0.907216495	0.97752809	0.944444	0.346667	0.146341
13	Yale School of Management	0.485553747	0.979381443	0.04494382	0.888889	0.12	0.560976
15	Instituto de Empresa	0	0.515463918	0.95505618	0	0	0.04878
16	Cornell University: Johnson	0.490624804	0.618556701	0.28089888	0.666667	0.16	0.743902
17	Georgetown Uni: McDonough	0.359716396	0.824742268	0.53932584	0.5	0	0.402439
17	Uni of N Carolina: Kenan-Flagler	0.303355663	0.659793814	0.69662921	0.555556	0.64	0.853659
19	University of Virginia: Darden	0.606570463	0.742268041	0.23595506	0.888889	0.12	0
20	Duke University: Fuqua	0.375430414	0.505154639	0.68539326	0.555556	0.453333	0.878049

After applying the hard data to the Common-Weight Model, Tables 3.3a and 3.3b displays the results. Table 3.3a shows the new score and the new rankings for these twenty universities along with the original rankings and Table 3.3b shows the new weightings. Please note that due the number of the original criteria, only five were selected from the original twenty criteria. Hence the result varied greatly.

Table 3.3 Results from Common-Weight Model

(a) New scores and rankings

Schools	score	Original Ranking	New Ranking	Change in Rankings
University of Pennsylvania:	0.845614	1	1	0
Harvard Business School	0.594397	2	11	-9
Columbia Business School	0.726394	3	3	0
Insead	0.668107	4	6	-2
London Business School	0.692608	5	4	0
University of Chicago GSB	0.758528	6	2	2
Stanford University GSB	0.615344	7	10	-3
New York University: Stern	0.6338	8	7	1
MIT: Sloan	0.50519	9	17	-8
Dartmouth College: Tuck	0.6338	10	8	2
Northwestern University: Kellogg	0.688126	11	5	6
IMD	0.435994	12	19	-7
Iese Business School	0.632058	13	9	4
Yale School of Management	0.543776	14	16	-3
Instituto de Empresa	0.390719	15	20	-5
Cornell University: Johnson	0.501724	16	18	-2
Georgetown Uni: McDonough	0.543776	17	13	4
Uni of N Carolina: Kenan-Flagler	0.562208	18	12	5
University of Virginia: Darden	0.543776	19	13	6
Duke University: Fuqua	0.543776	20	13	7

(b) New weightings obtained from Common-Weight Model

	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT research rank
Original Weightings	0.2	0.2	0.06	0.05	0.1
Normalized original	0.303030303	0.303030303	0.09090909	0.07575758	0.151515
New weightings	0.291382783	0.243472234	0.27496259	0.13661036	0.053572
Change (%)	-1.16%	-5.96%	18.41%	6.09%	-9.80%

By studying both tables, it is clear that the criterion “International Mobility Rank” has increased its weighting by more than double of its original weightings and criteria other than “Weighted Salary” has changed about 6% to 10% each. These changes have effected the new extremely. In the new ranking, half of the universities have shifted their rankings for more than 4 spots. Harvard and MIT have shifted 9 spots and 8 spots accordingly. Harvard has dropped 9 spots in ranking due to the fact that it has the lowest value in “International Mobility Rank”, which is accounted for 27.50% of the total score. MIT has dropped 8 spots because it has the second lowest score on “International Mobility Rank” and fourth lowest score on “Salary Increase %”, which accounted for 24.35%.

After applying the statistical t-test, the P value was found to be 0.8919, which means the differences between the original rankings and the new rankings are considered to be not statistically significant. Hence the result from the Common-Weight Model is acceptable statistically.

3.2 3D Spherical Model

In last section, the weights for each criterion were generated by the model, as well as the rankings. The model will calculate the coordinates of each DMU based on the weightings and project them onto a 3D ball. To insure the correctness of the mapping and the correlations between each DMU, the concept of dissimilarity is used in the calculation of the coordinates. Dissimilarity is the degree of difference between subjects. The general calculation method for dissimilarity will be discussed later in this section.

Table 3.4 lists the variables used in 3D Spherical Model and their meanings. Note that all the radius of the 3D balls is set to 1, and an ideal solution will be projected onto the North Pole. Ideal solution is an imaginary DMU that has the maximum value for each of its criterion. The purpose of this ideal DMU, as the standard, is to help the comparison process.

Table 3.4 Variables and descriptions

Variables	Descriptions
m	Total number of DMUs
n	Total number of criteria
S_i	Score of i^{th} DMU
$D_{i,j}$	The dissimilarity between DMU *i* and DMU *j*
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
X_i, Y_i, Z_i	The X,Y, and Z coordinates of DMU *i*

The X_i, Y_i, and Z_i are the actual coordinates of the DMUs on the 3D ball. Also, because the distances between DMUs on the 3D ball are not exactly the same as the values of dissimilarities, we minimize the error between these two values to obtain the closest solution (Equation 3.8). With this solution, the projection of the points on the ball will be able to represent the relationships of the DMUs.

3D Spherical Model (Model 2):

$$MIN \quad \sum_{i=1}^{m}\sum_{j>i}^{m}\left|(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2 - D_{i,j}^2\right| \tag{3.8}$$

Subject to:

$$S_i = \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right) \tag{3.9}$$

$$D_{i,j} = \sqrt{2} * \sum_{k=1}^{n}\left(w_k * \left(\frac{\left|C_{i,k} - C_{j,k}\right|}{\overline{C_k} - \underline{C_k}}\right)\right) \tag{3.10}$$

$$X_i^2 + Y_i^2 + Z_i^2 = 1 \ , \ \forall \ i \tag{3.11}$$

$$Y_i = 2S_i - S_i^2 \ , \ \forall \ i \tag{3.12}$$

The objective of this model is to let the dissimilarity between two DMUs represents the distance between two DMUs. This is accomplished by minimizing the difference between the straight line distance of two DMUs and their dissimilarity value.

Equation 3.9 is the function to calculate score, which is the same as equation 3.7. Equation 3.10 calculates the dissimilarity between DMU *i* and DMU *j*. The largest possible value for $D_{i,j}$ is $\sqrt{2}$, because when one DMU is the ideal solution, which have all the maximum value for each criterion, and the other DMU is the worst possible DMU, which must have minimum value for each criterion. Since the ideal solution will be at the North Pole and the worst possible solution will be on the equator. The straight line distance from the North Pole to the Equator on a ball with radius of 1 will be $\sqrt{2}$. Similarly, if two DMUs are exactly the same, thought it is not likely to happen, the numerator will become 0, and so the $D_{i,j}$ will be 0.

Equation 3.11 is to ensure that every point is on the surface of the ball. And equation 3.12 defines the relationship between the Y coordinates and the score. To explain this equation, there is a proposition to discuss, as stated below.

Proposition 1:

$$Y_i = 2 * S_i - S_i^2 \quad , \quad \forall \ i \qquad \textbf{(3.13)}$$

Proof:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = (\sqrt{2} * D_{i,*})^2 = 2(1 - S_i)^2 \qquad \textbf{(3.14)}$$

$$2 - 2Y_i = 2(1 - 2S_i + S_i^2) \qquad \textbf{(3.15)}$$

$$Y_i = 2S_i - S_i^2 \qquad \textbf{(3.16)}$$

In this proposition, $D_{i,*}$ in equation 3.14 represent the dissimilarity between DMU *i* and the ideal solution. The original equation that calculates the distance between two points was changed to the current form, $(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$, since the ideal solution has the coordinate of (0, 1, 0). Equation 3.14 can be verified with (ideal solution, worst possible solution) pair and (ideal solution, best possible solution) pair. When these two pairs of DMUs are plugged in 3.165, they both hold. Hence, equation 3.14 is further simplified to 3.15 and finally 3.16. The simplification processes are shown as below.

LHS:	**RHS:**
$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$	$2(1 - S_i)^2$
$\Rightarrow X_i^2 + Y_i^2 - 2Y_i + 1 + Z_i^2$	$\Rightarrow 2(1 - 2S_i + S_i^2)$
$\Rightarrow (X_i^2 + Y_i^2 + Z_i^2) - 2Y_i + 1$	$\Rightarrow 2 - 4S_i + 2S_i^2$
$\Rightarrow 1 - 2Y_i + 1$	
$\Rightarrow 2 - 2Y_i$	

LHS = RHS:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = 2(1 - S_i)^2$$

$$\Rightarrow \quad 2 - 2Y_i = 2 - 4S_i + 2S_i^2$$

$$\Rightarrow \quad Y_i = 2S_i - S_i^2$$

By applying the model to the example from section 3.1, we obtain the result shown in Table 3.5.

Table 3.5 Coordinates for each universities

Schools	score	New Ranking	x	y	z
Ideal Solution	1		0	1	0
University of Pennsylvania: Wharton	0.845613979	1	-0.21658096	0.97616496	0.013952
Harvard Business School	0.594397421	11	-0.41597168	0.83548655	0.359068
Columbia Business School	0.726394479	3	-0.36376656	0.92514002	0.108581
Insead	0.66810701	6	-0.32914496	0.88984704	-0.31597
London Business School	0.692608172	4	-0.32199465	0.90551026	-0.27635
University of Chicago GSB	0.758528351	2	-0.33056332	0.94169144	-0.06281
Stanford University GSB	0.615343904	10	-0.49832929	0.85203969	0.160301
New York University: Stern	0.633799985	7	-0.45945342	0.86589755	-0.1978
MIT: Sloan	0.505189925	17	-0.65293543	0.75516299	0.058345
Dartmouth College: Tuck	0.633799985	8	-0.49305753	0.86589755	0.084355
Northwestern University: Kellogg	0.688125846	5	-0.41447314	0.90273451	-0.11525
IMD	0.435994334	19	-0.73131613	0.68189761	0.0139
Iese Business School	0.63205834	9	-0.12768982	0.86461893	-0.48593
Yale School of Management	0.543776095	16	-0.58187571	0.79185975	0.185415
Instituto de Empresa	0.390719143	20	-0.36325638	0.62877684	-0.68752
Cornell University: Johnson	0.501723624	18	-0.65438051	0.75172065	-0.08187
Georgetown Uni: McDonough	0.543776095	13	-0.53405605	0.79185975	-0.29621
Uni of N Carolina: Kenan-Flagler	0.562207931	12	-0.49102552	0.8083381	-0.32478
University of Virginia: Darden	0.543776095	13	-0.60957381	0.79185975	0.037127
Duke University: Fuqua	0.543776095	13	-0.52498427	0.79185975	-0.31201

As previously mentioned, the ideal point is a point formed by setting the value of each of its criterion to the maximum value found from hard data. This point will lie on the North Pole with coordinates of (0, 1, 0) and score of 1. The worst point will be A_4, with coordinates of (0.99127, 0, 0) and score of 0. With this example, it is coincident that the ideal solution is same as A_1 and the worst point A_4 is lying on the equator. Despite these facts, the distances between each point are shown in Table 3.6. These numbers also represent the dissimilarity between each DMU.

Table 3.6   Dissimilarity matrix

	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20
A0	0	0.22	0.57	0.39	0.47	0.43	0.34	0.54	0.52	0.7	0.52	0.44	0.8	0.52	0.65	0.86	0.7	0.65	0.62	0.65	0.65
A1	0.22	0	0.49	0.27	0.45	0.32	0.12	0.33	0.32	0.48	0.3	0.26	0.58	0.52	0.51	0.81	0.49	0.43	0.4	0.43	0.43
A2	0.57	0.49	0	0.45	0.67	0.65	0.52	0.27	0.56	0.27	0.35	0.48	0.59	0.99	0.42	1.03	0.41	0.7	0.68	0.4	0.6
A3	0.39	0.27	0.45	0	0.55	0.42	0.18	0.28	0.2	0.31	0.15	0.36	0.47	0.61	0.26	0.91	0.32	0.37	0.47	0.26	0.49
A4	0.47	0.45	0.67	0.55	0	0.26	0.38	0.42	0.5	0.45	0.55	0.22	0.44	0.52	0.67	0.57	0.48	0.57	0.43	0.55	0.35
A5	0.43	0.32	0.65	0.42	0.26	0	0.25	0.48	0.26	0.47	0.41	0.21	0.59	0.34	0.47	0.49	0.33	0.31	0.2	0.41	0.23
A6	0.34	0.12	0.52	0.18	0.38	0.25	0	0.35	0.22	0.36	0.23	0.19	0.49	0.47	0.39	0.74	0.36	0.3	0.3	0.3	0.31
A7	0.54	0.33	0.27	0.28	0.42	0.48	0.35	0	0.38	0.2	0.23	0.29	0.34	0.8	0.5	0.86	0.31	0.53	0.51	0.34	0.43
A8	0.52	0.32	0.56	0.2	0.5	0.26	0.22	0.38	0	0.38	0.26	0.39	0.53	0.43	0.25	0.74	0.25	0.2	0.29	0.29	0.31
A9	0.7	0.48	0.27	0.31	0.45	0.47	0.36	0.2	0.38	0	0.19	0.26	0.37	0.81	0.34	0.8	0.19	0.47	0.46	0.22	0.37
A10	0.52	0.3	0.35	0.15	0.55	0.41	0.23	0.23	0.26	0.19	0	0.34	0.41	0.68	0.31	0.85	0.19	0.35	0.41	0.17	0.43
A11	0.44	0.26	0.48	0.36	0.22	0.21	0.19	0.29	0.39	0.26	0.34	0	0.4	0.55	0.56	0.57	0.35	0.43	0.29	0.4	0.21
A12	0.8	0.58	0.59	0.47	0.44	0.59	0.49	0.34	0.53	0.37	0.41	0.4	0	0.83	0.66	0.79	0.43	0.51	0.57	0.42	0.48
A13	0.52	0.52	0.99	0.61	0.52	0.34	0.47	0.8	0.43	0.81	0.68	0.55	0.83	0	0.62	0.34	0.66	0.44	0.44	0.61	0.53
A14	0.65	0.51	0.42	0.26	0.67	0.47	0.39	0.5	0.25	0.34	0.31	0.56	0.66	0.62	0	0.92	0.27	0.38	0.53	0.25	0.55
A15	0.86	0.81	1.03	0.91	0.57	0.49	0.74	0.86	0.74	0.8	0.85	0.57	0.79	0.34	0.92	0	0.68	0.54	0.44	0.78	0.43
A16	0.7	0.49	0.41	0.32	0.48	0.33	0.36	0.31	0.25	0.19	0.19	0.35	0.43	0.66	0.27	0.68	0	0.28	0.28	0.21	0.28
A17	0.65	0.43	0.7	0.37	0.57	0.31	0.3	0.53	0.2	0.47	0.35	0.43	0.51	0.44	0.38	0.54	0.28	0	0.19	0.35	0.22
A18	0.62	0.4	0.68	0.47	0.43	0.2	0.3	0.51	0.29	0.46	0.41	0.29	0.57	0.44	0.53	0.44	0.28	0.19	0	0.46	0.09
A19	0.65	0.43	0.4	0.26	0.55	0.41	0.3	0.34	0.29	0.22	0.17	0.4	0.42	0.61	0.25	0.78	0.21	0.35	0.46	0	0.48
A20	0.65	0.43	0.6	0.49	0.35	0.23	0.31	0.43	0.31	0.37	0.43	0.21	0.48	0.53	0.55	0.43	0.28	0.22	0.09	0.48	0

The dissimilarity values represent the degree dissimilarity between any two DMUs. If the value is 1, then the DMUS are totally different. If the value is 0, then the two DMUs are exactly the same, so the coordinates of these two DMUs will be the same as well. The school name has been replaced by variables due to the size of the dissimilarity matrix. A0 represents the Ideal Solution, A1 represents UPenn, A2 represents Harvard, and so on. Figure 3.2 is the projection of these points on a 3D ball by using the coordinates in Table 3.5.

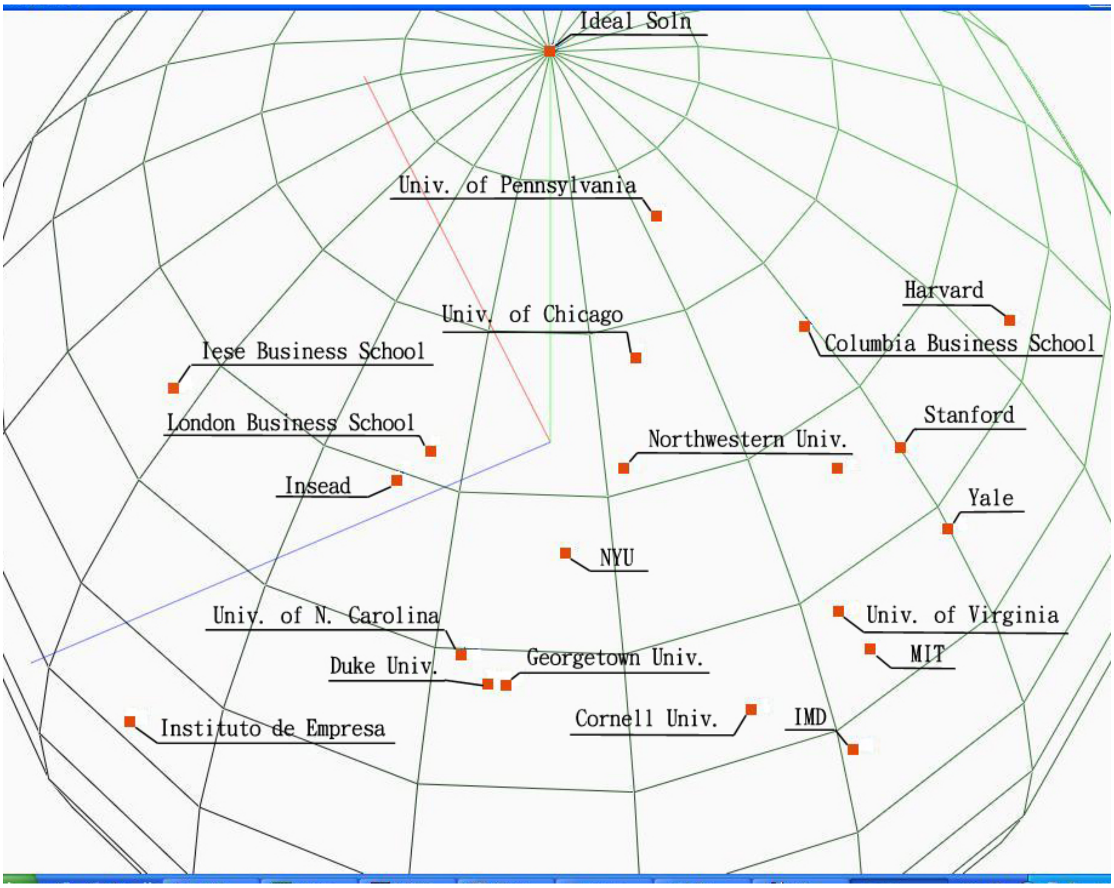

Figure 0.2 3D ball with DMUs projected on the surface

Notice that the North Pole is the ideal point. The points with higher altitudes are points with higher rankings. Universities that are closer to the equator are the ones with lower ranking and scores. Figure 3.2 clearly shows that Instituto de Empresa has the lowest ranking and IMD has the second lowest ranking, where University of Pennsylvania still has the best score.

4. Conclusion

People have being ranking DMU to show their importance and priorities since long ago. There are many ways to rank and each method has their strengths and weaknesses. From this study, we have proposed a method to help Decision Makers rank DMUs with out the needs to specify weightings for each criteria, which often is the most controversy and difficult in the whole ranking process. Using the techniques from Linear Programming, this model can produce a set of weightings for DMUs based on the absolute dominances relationships and preferences relationships, given by the Decision Makers. The 3D Ball representation not only has given Decision Makers the views they can not have by only looking at the table, but also allows them to categorize the DMUs and change the groupings for DMUs.

This model has focused on the mathematical models. There are still many issues can be studied in this area. Following are some suggestions for future works:

- Efficiency and validity in data collection and criteria selection.
- Although this model provides the function of changing groupings for DMUs, the clustering function can be improved. Certain clustering technique could be applied and help the groupings to be more accurate.
- The mathematical model can be modified to produce a more profound model, which can reduce the computation time and returns globally optimized solution.

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八、計畫成果自評

1、本研究已發展出亞洲大學與商學院評比系統及決策球，目前正撰寫文章投稿國際期刊中。

2、本研究已協助與訓練下列研究生完成論文寫作。

(i) 陳建治，「亞洲商學院之排序與分群（Ranking and Grouping of Asian Business Schools）」，碩士論文，國立交通大學資訊管理研究所，2004.

(ii) 柯彤昀，「IMD 國家競爭力排名方法之改進與視覺化模型（Ranking and Displaying Models for IMD World Competitiveness）」，碩士論文，國立交通大學資訊管理研究所，2004.

(iii) 楊秉中，「全球商學院之排序與分群（Ranking and Grouping on World Business Schools）」，

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