行政院國家科學委員會專題研究計畫成果報告

最佳容錯漢米爾頓及容錯漢米爾頓連通性圖的建構機制

Construction schemes of optimal fault-tolerant hamiltonian and hamiltonian connected graphs

計畫編號：NSC 90-2213-E-009-149

執行期限：90 年 8 月 1 日至 91 年 7 月 31 日

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一、中文摘要

本計劃是延續前數年實驗室之研究成果，其初始動機是觀察到超立方體(hypercube) Q_n，是在文獻中已被研究很多及實際應用在許許多多處理器大型電腦上的一種連結網路(interconnection network)，日後也陸續發出超立方體 Q_n 的各種推廣及變形，如: 雙扭超方體 TQ_n，交叉超方體 CQ_n，梅氏超方體(Mobius cube)及超級超方體(Super cube) [1,2,4,5,13,24,37,39]。它們都是將超立方體的線路連接方式按某一種特殊規則為基準作一些改變。而使得這些推廣的變形超方體連接網路具有較好的特性。

在延續過去本實驗室的計劃我們針對這些變形的超方體，雙扭超方體，交叉超方體，梅氏超方體，在任何一個連結網路使用中都可能發生損壞(fault)之情況下，研究其漢米爾頓環路的容錯性(fault-tolerant hamiltonian cycle)，而漢米爾頓環路在應用上正是號誌環(Token Ring)，所以研究其漢米爾頓環路的容錯性正是代表了號誌環網路的容錯能力。我們將討論其節點與邊的容錯能力，其定義為總共最多可容許損壞多少節點及邊，而使得剩下的連接網路仍存有一個漢米爾頓環路即 $H_f(G)$=max{k |G-F remains hamiltonian for every FĊV(G)ĆE(G) with |FáK|}，稱為漢米爾頓容錯量(fault-tolerant hamiltonicity)。另外我們也討論到容錯漢米爾頓連通性如下：若一個連接網路任何兩節點之間都存在一個漢米爾頓路徑(hamiltonian path)，則稱之漢米爾頓連通。而一個連接網路的漢米爾頓連通容錯量，$H_f^k(G)$，定義為總共最多可容許壞掉多少節點及邊，而剩下的連接網路仍存有漢米爾頓連通性 $H_f^k(G)$ =max{k |G-F remains hamiltonian connected for every FĊV(G)ĆE(G) with |FáK|} [16,23,25,26,27,28,29, 33,34,35,38,40]。

我們曾經證明 twisted cube [21,22], crossed cube [17,18,19,20], mobius cube [16], super cube [30,31,32], 和 recursive circulant graph [36] 都具有最佳的漢米爾頓容錯量(fault-tolerant hamiltonicity)及最佳的漢米爾頓連通性 (fault-tolerant hamiltonian connectivity)。因此以漢米爾頓容錯性而言雙扭超方體及交叉超方體較傳統的超立方體是好了很多，在此計劃中我們已經對這方面再作深入的研究，並且提出一些具體的建構法則建構一連串 $H_f(G)=n-2$ and $H_f^k(G)=n-3$ 的 graphs，並將所得結果撰寫為論文並投稿至國外期刊發表。

關鍵詞：漢米爾頓環路，漢米爾頓連通，容錯，超方體，雙扭超方體，交叉超方體，連結網路，號誌環。

Abstract

Among all interconnection networks proposed in the literature, the hypercube Q_n is one of the most popular topology and is used in many multi-processor computers. There are many variations of the hypercube proposed in literature which are all derived by changing some connections of hypercube Q_n according to specific rules such as twisted cube, crossed cube, mobius cube and super cube. Recently, many topological properties of these variation cubes are studied [1,2,4,5,13,24,37,39]. All these results indicate that the performances of these variations are better than that of Q_n.

In this proposal, we consider the fault-tolerant hamiltonicity, $H_f(G)$, which is defined as that G-F remains hamiltonian for $F \subset V(G) \cup E(G)$ with $|F| \leqq k$ and k is maximum. Obviously, $H_f(G) \le \min\{H_v(G), H_e(G)\} \le$ ¢$(G) - 2$. We also consider the "fault-tolerant hamiltonian connectivity". A graph G is hamiltonian connected if there exists a hamiltonian path joining any two vertices of G. The fault-tolerant hamiltonian connectivity is defined to be the maximum integer k such that G-F remains hamiltonian connected for every $F \subset V(G) \cup E(G)$ with $|F| \leqq k$ if G is hamiltonian connected, and undefined if otherwise. Obviously, $H_f^k(G) \le$ ¢$(G) - 3$. A graph G is called k-fault-tolerant hamiltonian (k-fault-tolerant hamiltonian connected respectively) or simply it k-hamiltonian (k-hamiltonian connected respectively) if it remains hamiltonian (hamiltonian connected respectively), after removing at most k vertices and/or edges [16,23,25,26,27,28,29,33,34,35,38,40].

We have proved that n-dimensional twisted cube [21,22], crossed cube [17,18,19,20], mobius cube [16], super cube [30,31,32], and recursive circulant graph [36] are (n-2)-fault-tolerant hamiltonian and (n-3)-fault-tolerant hamiltonian connected. In this proposal, we investigate some construction schemes to construct k-regular interconnection networks with (k-2)-fault-tolerant hamiltonicity and (k-3)-fault-tolerant hamiltonian connectivity.

Keywords : hamiltonian cycle, hamiltonian connected, fault-tolerant, hypercube, twisted cube, crossed cube, interconnection network, token ring.

二、緣由與目的

Network topology is a crucial factor for interconnection networks since it determines the performance of a network. Many interconnection network topology have been proposed in literature for connecting hundreds of processing elements. The hypercube is the most popular one and there are many variations of hypercube proposed in literature [1,2,4,5,13,17,25,27], such as twisted cube, crossed cube, Mobius cube and super cube.

Since node faults and link faults may happen when a network is used, it is practically meaningful to consider faulty networks. The vertex fault-tolerant hamiltonicity and the edge fault-tolerant hamiltonicity, proposed by Hsieh, Chen, and

Ho [7], measures the performance of the hamiltonian property in the faulty networks. The vertex fault-tolerant hamiltonicity, $H_v(G)$, is defined to be the maximum integer k such that G-F remains hamiltonian for every $F \subset V(G)$ with |F| ≦ k if G is Hamiltonian, and undefined if otherwise. Obviously, $H_v(G) \le \delta(G) - 2$ where $\delta(G) = \min\{\deg(v) \mid v \in V(G)\}$. Similarly, the edge fault-tolerant hamiltonicity, H_e(G), is defined to be the maximum integer k such that G-F remains hamiltonian for every $F \subset E(G)$ with |F| ≦ k if G is hamiltonian, and undefined if otherwise. Again, it is obvious that $H_v(G) \le \delta(G) - 2$. Many related works have appeared in literature, for example[7, 8, 9, 11, 12] .

In this proposal, we consider a more general parameter. The fault-tolerant hamiltonicity, $H_f(G)$, is defined to be the maximum integer k such that G-F remains hamiltonian for every $F \subset V(G) \cup E(G)$ with $|F| \leqq k$ if G is hamiltonian, and undefined if otherwise. Obviously, $H_f(G) \le \min\{H_v(G), H_e(G)\} \le \delta(G) - 2$. For technical reason, we also introduce the term "fault- tolerant hamiltonian connectivity". A graph G is hamiltonian connected if there exists a hamiltonian path joining any two vertices of G. The fault-tolerant hamiltonian connectivity, is defined to be the maximum integer k such that G-F remains hamiltonian connected for every $F \subset V(G) \cup E(G)$ with $|F| \leqq k$ if G is hamiltonian connected, and undefined if otherwise. Obviously, $H_f^k(G) \le \delta(G) - 3$. A graph G is called k-fault-tolerant hamiltonian (k-fault-tolerant hamiltonian connected respectively) or simply it k-hamiltonian (k-hamiltonian connected respectively) if it remains hamiltonian (hamiltonian connected respectively), after removing at most k vertices and/or edges.

We have proposed some construction schemes to construct with flexibility infinitely many k-hamiltonian and k-hamiltonian connected graphs. Additionally, there are many popular interconnection networks which are k-hamiltonian and k-hamiltonian connected. Some of them, e.g., twisted cube, crossed cube, Mobius cube, and recursive circulant graph, can be constructed using our construction schemes. And therefore, they are in fact a subclass of our proposed family of graphs.

三、 結果與討論

本研究計劃最後的成果如下：

在這個計劃中，我們主要針對下面兩個主題來做研究:

一: k-正則圖之超漢米爾頓圖之建構方式.

一個 k-正則圖 G，如果去除掉至多 k-2 個點和/或線仍是漢米爾頓，並且如果去除掉至多 k-3 個點和/或線仍是漢米爾頓連通，則我們說圖 G 是超容錯漢米爾頓. 在這個計劃中，我們提供了一些建構方式來建構出超容錯漢米爾頓圖. 且此結果已發表:

Y-Chuang Chen, Chang-Hsiung Tsai, Lih-Hsing Hsu, and Jimmy J.M. Tan, Construction schemes of fault-tolerant Hamiltonian graphs, Proceedings of ISAS SCI 2001, 5 (2001), pp.183--187.

二: k-正則圖之超連通圖之建構方式.

令 G=(V,E)是一個 k-正則圖，並且連通數是κ，線連通數是λ. 如果κ=k，我們說 G 是最大連通；如果λ=k，我們說 G 是最大線連通. 此外，如果 (1)G 是一個完全圖；或 (2)G 是最大連通且對某些點 v∈V，每一個最小的使圖不連接的點集合都是{x|(v,x)∈E}，則

我們說圖 G 是超級連通. 如果 G 是最大線連通且對某些點 v∈V, 每一個最小的使圖不連接的點集合都是{(v,x)|(v,x)∈E}, 則我們說圖 G 是超級連通. 在這一次的計劃當中, 我們發表了三種可以建構超連通圖. 應用這些建構機制, 我們可以很容易的得知超立方體, 雙扭超方體, 交錯超方體, 和梅氏超方體的超連通性質. 並且此結果已被期刊 Applied Mathematics and Computations 所接受: Y.C. Chen, Jimmy J.M. Tan, L.H. Hsu, S.S. Kao, "Super-connectivity and Super-edge-connectivity for Some Interconnection Networks, Applied Mathematics and Computation, (2002).

四、計畫成果自評

在本計畫的支援下, 我們已經成功地找出 k-正則圖之超漢米爾頓圖之建構方式, 以及 k-正則圖之超連通圖之建構方式, 並且這些結果也都被國外的期刊與會議論文所接受。

接下來我們繼續再針對雙扭超方體, 交錯超方體, 和梅氏超方體的特性做進一步的研究, 我們希望在這些網路有錯的情形下, 它們仍然能夠保有泛圈圖 (Pancyclic) 的性質, 也就是說, 從長度為四的環路一直到漢彌爾頓環路都可以被嵌入到這幾個網路之中, 而目前我們也已經有了一些初步的結果。

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計畫類別：　個別型計畫　　□整合型計畫
計畫編號：NSC　90－2213－E－009－149－
執行期間：90 年　8 月　1 日至　91 年　7 月　31 日

計畫主持人：譚建民
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