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Abstract. The processes to derive the associated phase of an interference signal from the data of a series of recorded frames are performed, and we find that the sampling frequency being lower than the Nyquist sampling rate can also be applied to the full-field heterodyne interferometry. Two optimal sampling conditions for a commonly used CCD camera are proposed based on the relation between the heterodyne frequency and the contrast of the interference signal under the condition that the phase error is set to be 0.05 deg. [DOI: 10.1117/1.3567172]

Subject terms: CCD camera; Nyquist sampling rate; full-field heterodyne interferometry; three-parameter sine fitting algorithm.

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1 Introduction

In our previous work,[1] we proposed an optimal sampling condition for the full-field heterodyne interferometry[2,3] according to the sampling theorem.[4] Originally, the sampling theorem is suitable for a complicated multi-frequency signal and its sampling rate should not be lower than the Nyquist sampling rate to avoid aliasing. However, the signal to be processed in the heterodyne interferometry is a simple single frequency cosine signal whose spectrum includes three delta functions, which come from the dc term and the cosine term. Because its spectrum is so simple, the sampling frequency might have a loose condition.[5,6] The processes to derive the associated phase of an interference signal from the data of a series of recorded frames are performed anew; we find that the sampling frequency being lower than the Nyquist sampling rate can also be applied to the full-field heterodyne interferometry. Based on the relation between the heterodyne frequency and the contrast of the interference signal, two optimal sampling conditions for a commonly used CCD camera are proposed under the condition that the phase error is set to be 0.05 deg. Although it needs more sampled points to obtain the high resolution results, a current personal computer is qualified enough to perform the processes quickly.

2 Principle

The interference signal of a general heterodyne interferometer is

$$I(t) = I_0[1 + r\cos(2\pi ft + \phi_0)], \quad (1)$$

where f is the heterodyne frequency; I_0, r, and ϕ_0 are the average intensity, the contrast, and the phase of the interference signal, respectively. A camera with frame frequency f_s and frame exposure time a is used to sample the interference signal. In the recording time T, the camera records N frames which are numbered from 0 to $N-1$. To extend the sampling frequency to a wider range, a can be written as

$$a = \frac{m + a'}{f}, \quad (2)$$

where m is a non-negative integral number and a' is a proper fraction. Under the condition $m = 0$, the sampling frequency is higher than the Nyquist sampling rate as that in our previous work. The associated interference signal is sampled as shown in Fig. 1(a), in which $t_s = 1/f_s$ is the frame period. Consequently, the condition $m = 0$ will not be discussed in this paper. As to the condition $m > 0$, the sampling frequency is lower than the Nyquist sampling rate. Here, $(m + a')$ periods of the interference signal are integrated in one exposure time a, and the associated interference signal is sampled as shown in Fig. 1(b). Hence, the interference intensity measured at any pixel on the k'th frame becomes

$$I_{ck} = \frac{1}{(m + a')/f}\int_{kt_s}^{kt_s+(m+a')/f} I_0[1 + r\cos(2\pi ft + \phi_0)]dt$$
$$= I_0[1 + r'\cos(2\pi fkt_s + \psi)]. \quad (3)$$

The contrast becomes

$$r' = \frac{r\sin(\pi a')}{m\pi + \pi a'}, \quad (4)$$

and the phase ψ can be written as

$$\psi = \phi_0 + \pi a'. \quad (5)$$

Let I_{ck} be quantized in gray-level units, then it can be expressed as

$$I_d = \text{Round}\left(\frac{I_{ck}}{2I_0}\times 2^n\right). \quad (6)$$

where n is the number of gray-level and Round(x) is a mathematical operator to round the number x to an integer. Next, the three-parameter sine fitting algorithm[7–9] is used to process a series of data I_d and obtain an optimal fitted cosine wave curve. It can be represented by

$$I_f(t) = A_0\cos(2\pi ft) + B_0\sin(2\pi ft) + C_0$$
$$= \sqrt{A_0^2 + B_0^2}\cos(2\pi ft + \psi') + C_0, \quad (7)$$

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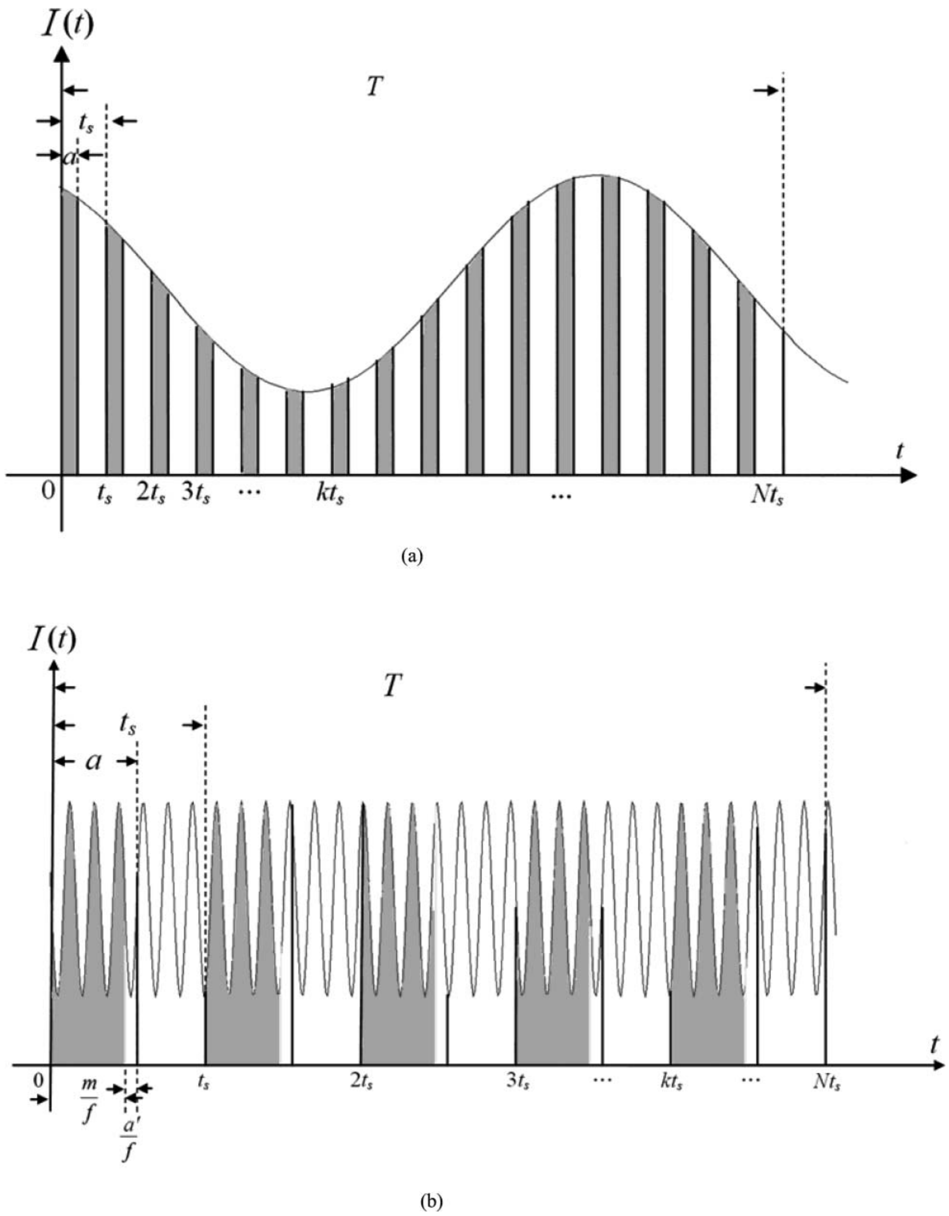

Fig. 1 The interference signals are sampled as (a) $a < 1/f$ and (b) $a > 1/f$.

where ψ' is the phase of the fitted wave; A_0, B_0, and C_0 are real numbers, and they can be obtained with the Fourier sine and cosine transforms under the condition $I_f(t)$ is specified. Thus, we have

$$A_0 = 2f \int_0^{1/f} I_f(t) \cos(2\pi f t) dt, \tag{8a}$$

$$B_0 = 2f \int_0^{1/f} I_f(t) \sin(2\pi f t) dt, \tag{8b}$$

and

$$\psi' = \tan^{-1}\left(-\frac{A_0}{B_0}\right). \tag{9}$$

According to Eq. (5), it can be seen that ψ' includes a phase drift term $\pi a'$. This term can be obtained under the experimental condition in which a' is specified. Consequently, the measured phase and its sampling error can be expressed as

$$\phi_s = \psi' - \pi a', \tag{10}$$

and

$$\Delta\phi = \phi_s - \phi_0, \tag{11}$$

respectively. If we apply the measurement processes repeatedly to every pixel, then its associated phase and sampling error can be obtained.

3 Numerical Calculations

From Eq. (4), we know that the contrast of the interference signal decreases after sampling. Although the low contrast interference signal can also be processed to obtain its associated phase, its resolution becomes worse because only the limited number of the gray-level is effective. To obtain the high contrast interference signal, the relation curves between r'/r and a' at different m can be calculated and depicted in

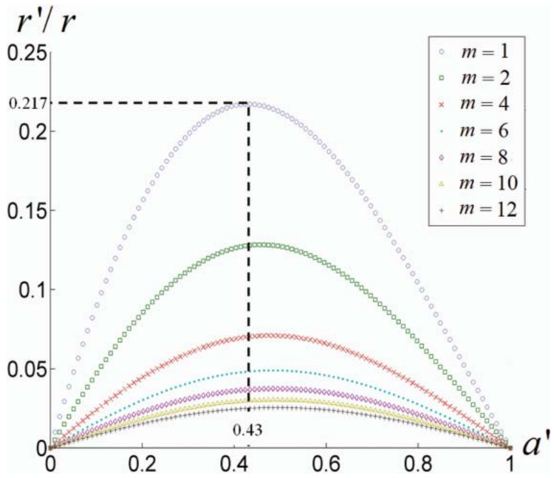

Fig. 2 The relation curves between r'/r and a' at different m.

Fig. 2. It can be seen that r'/r becomes maximum under the conditions $m = 1$ and $a' \cong 0.43$. Consequently, we have $f_s \cong 5f/14$. On the other hand, the phase error $\Delta\phi$ can be reduced by increasing the number N of sampled points. We define $u = f_s a$, which means the ratio of the frame exposure time to the frame period. The relation curves between $\Delta\phi$ and N for a commonly used CCD digital camera with $u = 0.5$ and $n = 8$ bit are calculated under the condition $f_s \cong 5f/14$ at $\phi_0 = 0$ deg, 20 deg, 40 deg, 60 deg, and 80 deg, respectively, and depicted in Fig. 3. It can be seen that $\Delta\phi$ is reduced to 0.05 deg when $N \geq 120$. Therefore, we can conclude that a commonly used CCD camera can be applied to the full-field heterodyne interferometry with good results under these two conditions

$$\begin{cases} f_s = \dfrac{5}{14} f; \\ N \geq 120. \end{cases} \tag{12}$$

4 Discussions

To avoid the affection of the environmental vibration on the interference signal, the heterodyne frequency should not be too low and it is better to be larger than 50 Hz.[10,11] For a commonly used CCD digital camera with $f_s = 30$ Hz, we have $f = 84$ Hz from Eq. (12). Consequently, the environmental vibration has almost a negligible effect on interference signals. On the other hand, we also derive the condition $f_s = 15f = 1260$ Hz as $f = 84$ Hz according to our previous work.[1] Hence, it needs an expensive high speed camera. According to our knowledge, its cost is over 10 times that of a commonly used CCD digital camera.[12] Besides, they need at least 15 and 120 frames in our previous work and in this case, respectively, to obtain the results with the same resolution. If each frame is recorded with 8 bit gray-levels and 800×600 pixels, then they need about 3 and 26 Mb memory capacities; 0.5 and 6 s processing times for one full-process operation by using a current personal computer with a 1 Tb hard disk and a 3 GHz CPU. So it is qualified enough to operate the increment of data without any additional cost in this case.

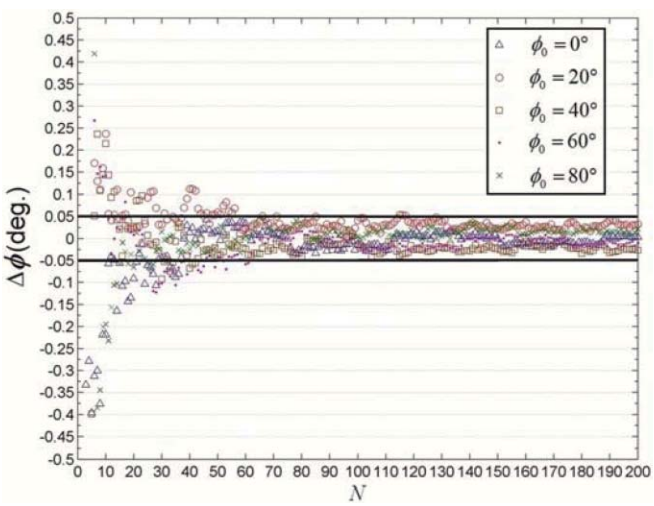

Fig. 3 The relation curves between $\Delta\phi$ and N at different ϕ_0.

5 Conclusion

The processes to derive the associated phase of an interference signal from the data of a series of recorded frames have been performed anew. We have found that the sampling frequency being lower than the Nyquist sampling rate can also be applied to the full-field heterodyne interferometry. Based on the relation between the heterodyne frequency and the contrast of the interference signal, two optimal sampling conditions written in Eq. (12) for a commonly used CCD camera with $u = 0.5$ and $n = 8$ bit have been proposed under the condition that the phase error is set to be 0.05 deg. Although it needs more sampled points to obtain the high resolution results, a current personal computer with a 1 Tb hard disk and a 3 GHz CPU is qualified enough to perform the processes quickly.

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