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彈性期間定期盤存制最適採購政策之研究

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彈性期間定期盤存制最適採購政策之研究
(NSC94-2416-H-009-010)

（一）計畫中文摘要

本計畫研究含彈性期間定期盤存制之存貨模式。所謂彈性期間即期間之長度可依實際情況訂定。本計畫延續前一年度計畫(NSC93-2416-H-009-007)對訂購周期定期盤存制之研究。前一年度計畫使用較複雜之動態規劃模式，並假設固定訂購成本為可忽略。本計畫發展較精簡之動態規劃模式，且明確考慮固定訂購成本之存在。

本計畫發展缺貨後補與喪失銷貨存貨模式，並將以期末存貨計算存貨貯存與缺貨成本。本計畫顯示缺貨後補模式之最適採購政策為一(訂購點，訂購水準)存貨政策，此結果與定期盤存制之文獻一致。本計畫另顯示前置時間為零之喪失銷貨模式之最適採購政策亦為一(訂購點，訂購水準)存貨政策。

本計畫並研究前置時間為正之喪失銷貨模式之最適採購政策。本計畫對此模式將確實執行動態規劃程式，並提供最適操作參數。本計畫建議廠商若定期盤存制之期間可彈性調整，可採用本計畫所推導出之最適採購政策，以適度削減存貨，降低營運成本。
關鍵詞: 定期盤存制;最適採購政策;喪失銷貨

（二）計畫英文摘要

In this research, we consider inventory models for periodic-review systems with flexible periods. By flexible periods, we mean that their length can be tailored to the needs of an application. This research is an extension of a previous study on periodic-review systems with replenishment cycles (NSC93-2416-H-009-007). In that study, the dynamic programs formulated have two subscripts for the cost function. Moreover, it assumes that the fixed cost of ordering is negligible. In this research, we propose a more concise dynamic program and incorporate a positive fixed cost of ordering.

We will use dynamic programming to formulate both the backorder and lost-sales models, and propose to charge inventory holding and shortage costs based on the ending inventory of periods. We will show that an (s, S) policy is optimal for the infinite-horizon backorder model. This agrees with the periodic-review inventory literature. In addition, we extend this result to the lost-sales problem with zero lead-time.

Moreover, we consider the lost-sales problem with positive lead-time. Although we cannot derive any properties that can be used in the dynamic programming computation, we provide some interesting results. We advocate that firms use the proposed models and reduce their inventory-related costs.
Keywords: Periodic Review System, Optimal Ordering Policy, Lost-Sales

I. Introduction

Periodic-review inventory systems are commonly found in practice, especially if many different items are purchased from the same supplier and the coordination of ordering and transportation is important. In a recent survey [8, p. 69], material managers indicate the effectiveness of periodic-review systems for reducing inventory levels in a supply chain.

Although most studies on periodic-review inventory models have (implicitly) assumed that the review periods are as small as one day (see, e.g., Porteus [6] and references therein), periodic-review systems in practice often have the review periods (i.e., *replenishment cycles* or simply *cycles*) that are a few days or weeks long and regular orders are placed at a review epoch (see, e.g., Chiang [3] and Chiang and Gutierrez [4] for periodic systems where an emergency order can be placed at a review epoch or virtually at any time between two review epochs). For such periodic-review systems, it is appropriate to compute holding and shortage costs based on respectively, the average inventory of a replenishment cycle and the duration of shortage (assuming that demand not immediately filled is backlogged). Due to the difficulties involved in exact analysis, the approximate (heuristic) treatment of such systems is often used in textbooks (e.g., Hadley and Whitin [5, Sec. 5-2] and Silver et al. [7, Sec. 7.9.4]) to obtain easy-to-implement solutions. However, as Chiang [2] recently pointed out, there are many shortcomings with the approximate treatment. Chiang thus proposed a dynamic programming model in which a cycle consists of a number of small periods and holding and shortage costs will be computed based on the ending inventory of small periods (rather than only on the ending inventory of cycles). As periods can be chosen to be any time units (see [2] for a detailed discussion), the period length is tailored to the needs of an application.

In this research, we present an alternative (and concise) dynamic programming model. Moreover, we allow the possibility of a positive fixed ordering cost that is not considered in Chiang [2]. The optimal policy is of the familiar (s, S) type, i.e., if inventory drops to or below s at a review epoch, an order is placed to raise the inventory to a predetermined level S. As in the periodic-review inventory literature (see, e.g., Veinott and Wagner [9]), we extend this result to the lost-sales periodic problem with zero lead-time. Computation shows that the long-run average cost is rather insensitive to the choice of the period length. This indicates that a rough estimate of the period length is acceptable.

In addition, we show how the proposed model is modified to handle the backorder problem where shortage is charged on a per-unit basis irrespective of its duration (as in [5, Sec. 5-2]), which is also not considered in [2]. Finally, we investigate the lost-sales problem with positive lead-time. Although we are unable to derive any properties (as in [2]) that can be used in the dynamic programming computation, we provide some interesting computational results. We advocate that firms use the proposed models for obtaining optimal ordering policies.

II. Research Methodology

In this research, we use dynamic programming to formulate both the backorder and lost-sales models. The dynamic programming model in [2] uses two recursions to determine the cost function: an outer one for the number of cycles and an inner one for the number of periods remaining until the end of the planning horizon. In this research, we formulate a dynamic programming model that involves only a recursion on the number of cycles. The recursion on the number of periods can be avoided by using a refined intra-cycle time scale. This scale embeds the number of periods and defines clearly how inventory holding and shortage costs are computed.

We also use dynamic programming to show how the proposed model is modified to handle the backorder problem where shortage is charged on a per-unit basis irrespective of its duration, which is not studied in [2].

In addition, we incorporate a positive cost for placing orders (which is also not considered in [2]). It is shown that an (s, S) policy is optimal for the infinite horizon problem. We further extend this result to the lost-sales problem with zero lead-time, as in the periodic-review inventory literature.

III. Purpose of Research

We will show that a base-stock policy is optimal for the backorder model. This agrees with the periodic-review inventory literature. We also devise a simple expression for computing the optimal base-stock level. Moreover, we allow the possibility of a positive fixed ordering cost. The optimal policy is of the familiar (s, S) type due to the convexity of the one-cycle cost function.

For the lost-sales problem with positive lead-time, although we are unable to derive any properties (as in [2]) that can be used in the dynamic programming computation, we provide some interesting computational results. Computational results indicate that it does not take many cycles for operational parameters to converge. Hence, we advocate that firms use the proposed method (of computing holding and shortage costs) and implement optimal ordering policies.

IV. The model

This research evolves into a paper titled “Optimal ordering policies for periodic-review systems with a refined intra-cycle time scale,” which has been accepted by European Journal of Operational Research [1]. Please see [1] for details of the model.

V. Conclusions and Suggestions

This research presents an alternative dynamic programming model for periodic-review inventory systems with a refined intra-cycle time scale. It also incorporates a positive fixed cost of ordering. The optimal policy is of the familiar (s, S) type because of the convexity of the one-cycle cost function. As in the periodic-review inventory literature, we extend this result to the lost-sales periodic problem with zero lead-time. Computational results show that the long-run average cost is rather insensitive to the choice of the period length. This indicates that a rough estimate of the period length is acceptable.

Moreover, this research contributes to the periodic-review literature by showing that the backorder problem where shortage is charged per unit irrespective of its duration can be easily handled in the proposed model. It is found that Hadley and Whitin’s approximate model usually yields the optimal order-up-to level for problems with a low holding-to-shortage cost ratio. However, if this ratio is high, Hadley and Whitin’s model may yield a lower level and increase the long-run average cost by more than 2%.

Finally, this research investigates the lost-sales periodic problem with positive lead-time. Although we are unable to develop any properties that can be used in the dynamic programming computation, we provide some interesting results. Further research on the lost-sales problem with more than one outstanding order allowed and/or stochastic supply lead-time is possible, though a dynamic programming model seems complex or difficult to formulate.

References

[1] C. Chiang, Optimal ordering policies for periodic-review systems with a refined intra-cycle time scale, forthcoming in European Journal of Operational Research.

[2] C. Chiang, Optimal ordering policies for periodic-review systems with replenishment cycles, European Journal of Operational Research 170 (2006) 44-56.

[3] C. Chiang, Optimal replenishment for a periodic review inventory system with two supply modes, European Journal of Operational Research 149 (2003) 229-244.

[4] C. Chiang, G.J. Gutierrez, Optimal control policies for a periodic review inventory system with emergency orders, Naval Research Logistics 45 (1998) 187-204.

[5] G. Hadley, T.M. Whitin, Analysis of Inventory Systems, Prentice-Hall, Englewood Cliffs, NJ, 1963.

[6] E.L. Porteus, Stochastic inventory theory, in: D.P. Heyman, M.J. Sobel (Eds.), Handbooks in OR & MS, Vol. 2, Elsevier Science Publishers, Amsterdam, 1990, Ch. 12.

[7] E.A. Silver, D.F. Pyke, R. Peterson, Inventory Management and Production Planning and Scheduling, John Wiley & Sons, New York, 1998.

[8] D. Simchi-Levi, P. Kaminsky, E. Simchi-Levi, Designing and Managing the Supply Chain, McGraw-Hill, New York, 2003.

[9] A.F. Veinott Jr., H.M. Wagner, Computing optimal (*s*, *S*) inventory policies, Management Science 11 (1965) 525-552.

計畫成果自評

In this research, we consider inventory models for periodic-review systems with a refined intra-cycle time scale. This research presents a more concise dynamic programming model than the previous research (NSC93-2416-H-009-007). The proposed model involves only a recursion on the number of cycles to determine the cost function and embeds the recursion on the number of periods on a refined time scale. This is expected to make a good contribution to the inventory literature.