行政院國家科學委員會專題研究計畫　成果報告

多處理機的診斷錯誤之研究(I)

計畫類別：個別型計畫
計畫編號：NSC94-2213-E-009-138-
執行期間：94 年 08 月 01 日至 95 年 07 月 31 日
執行單位：國立交通大學資訊科學學系(所)

計畫主持人：譚建民

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本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

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多處理機的診斷錯誤之研究(I)
Conditional Diagnosability Measures for Large Multiprocessor Systems

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主持人：譚建民 國立交通大學資訊工程學系 教授

一、中英文摘要

在此計劃中，我們研究連結網路的錯誤診斷問題。診斷能力在連結網路的可靠度研究方面扮演了一個相當重要的角色，很多典型的錯誤診斷問題已經在很多文獻中被廣泛的討論，且很多著名網路的診斷能力也已經被研究出來。

文獻中提出了幾種不同的診斷模式，PMC model，由Preparata、Metze and Chien 三人最早提出。在此模式下，兩個相連接的點可以互相偵測是否為壞點。之後Maeng and Malek提出一種comparison model 稱為MM-model。他們對錯誤診斷的基本構想是由一個點向相鄰的兩個點，送出信號，然後由回收的訊號，比較並判斷是否有錯誤。為了要收集到最多的資料以供錯誤診斷，在MM*-model 下，規定任一個點都對其所有相鄰的兩個點作偵測及比較。

在此計畫裡，我們對PMC模式下的錯誤診斷能力提出了一個新的測量方式，稱作”條件式診斷能力”，因為對許多已知的網路結構來說，傳統的錯誤診斷能力經常受限於最小的鄰接點數(minimum degree)，但對網路中任一個處理器而言，他所有的鄰接處理器均是錯誤的機率不高，因此條件式診斷的作法為，在測量一個網路的診斷能力時，我們考慮以下情況，在網路中的任何一個點，其所有鄰居並沒有全都壞掉，也就是至少要有一個好鄰居，在此條件下，一個n維的超立方體結構結構網路其診斷能力將上升至4(n-2)+1，這比傳統診斷能力上升了約四倍。除此之外，我們也提出了一些有用的充份條件，利用這些充份條件可以用來判斷一個網路是否為”t-可診斷系統”， 另外我們也介紹”強 t-可診斷系統”的觀念，並證明一些為t-可診斷系統的網路其也是強t-可診斷系統。

關鍵詞：錯誤診斷、PMC-診斷模式、比較式診斷模式、MM*-診斷模式、診斷能力、條件式診斷能力

英文摘要

In this project, we study the diagnosis problem of interconnection network. Diagnosability has played an important role in the reliability of an interconnection network. The classical problem of fault diagnosis is discussed widely and the diagnosability of many well-known networks has been explored.

For the purpose of diagnosing a given system, several different models have been

proposed in literature. The PMC-model, introduced by Preparata, Metze, and Chien, is the first model for system level diagnosis. In this model, it is assumed that a vertex can test the faulty or fault-free status of an adjacent vertex. The comparison model, called MM model, introduced by Maeng and Malek, is considered to be another practical approach for system level diagnosis. In this approach, a self-diagnosable system is often represented by a multigraph M(V,C), where V is the vertex set and C is the labeled edge set. Let $(u, v)_w$ be a labeled edge. If (u, v) is an edge labeled by w, then $(u, v)_w$ is said to belong to C, which implies that the vertex u and v are being compared by w. The same pair of vertices may be compared by different comparators, so M is a multigraph. For $(u, v)_w \in C$, we use $r((u, v)_w)$ to denote the result of comparing vertices u and v by w such that $r((u, v)_w)=0$ if the outputs of u and v agree and $r((u, v)_w)=1$ if the outputs disagree. Obviously, if $r((u, v)_w)=0$ and w is fault-free, then both u and v are fault-free. If $r((u, v)_w)=1$, then at least one of the three vertices u, v and w must be faulty. If the comparator w is faulty, then the result of comparison is unreliable.

In classical diagnosability measures, it has generally been assumed that any subset of vertices can potentially fail at the same time. As a consequence, the diagnosability of an interconnection network is upper bounded by its minimum degree. In this project, we introduce a new measure of diagnosability, called conditional diagnosability, by restricting that any faulty set cannot contain all the neighbors of any vertex in the graph under the PMC model. Based on this requirement, the conditional diagnosability of the n-dimensional hypercube is shown to be 4(n-2)+1, which is about four times as large as the classical diagnosability. Besides, we propose some useful conditions for verifying if a system is t-diagnosable and introduce a new concept, called a strongly t-diagnosable system, under the PMC model. Applying these concepts and conditions, we investigate some t-diagnosable networks which are also strongly t-diagnosable.

Keywords: fault diagnosis, PMC model, Comparison model, MM* model, Diagnosability, Conditional Diagnosability.

二、計劃緣由及目的

High-performance signal processing architectures have become quite common with continuing advances in semiconductor technology. These architectures are used in several real-time applications and in high-performance large multiprocessor systems. However, the complexity of these systems can adversely affect the reliability. Therefore, the testing and diagnosis of these systems become an important aspect of system design.

The hypercube structure [24] is a well-known interconnection model for multiprocessor systems. Fault-tolerant computing for the hypercube structure has been of interest to many researchers. A hypercube of dimension n, denoted by Q_n, is an undirected graph consisting of 2^n vertices and $n2^{n-1}$ edges. The hypercube Q_1 is a complete graph K_2 with two vertices {0, 1}. For n > 1, Q_n is constructed from two copies of Q_{n-1} by adding a perfect matching between them. Each vertex u of Q_n can be distinctly labeled by a binary n-bit string, $u_{n-1}u_{n-2}\ldots u_1u_0$. There is an edge between two vertices if and only if their binary labels differ in exactly one bit position.

There are several variations of the hypercube, for example, the Crossed cube [6], the Twisted cube [13], and the Mobius cube [3]. For each of these cubes, an n-dimensional cube can be constructed from two copies of (n-1)-dimensional subcubes by adding a perfect matching between the two subcubes. The main difference is that each of these cubes has various perfect matching between its subcubes. An n-dimensional cube has 1) 2^n vertices, 2) connectivity n, and 3) each vertex has the same degree n (the two terms connectivity and degree will be defined subsequently). We define the cube family to include all such cubes which are constructed recursively by joining two subcubes with a perfect matching. For n=0, 1, and 2, an n-dimensional cube is a single vertex, an edge, and a cycle of length four, respectively.

In this project, we use the widely adopted PMC model [23] as the fault diagnosis model. In [11], Hakimi and Amin proved that a multiprocessor system is t-diagnosable if it is t-connected with at least 2t+1 vertices. Besides, they gave a necessary and sufficient condition for verifying if a system is t-diagnosable under the PMC model. In this project, we also propose a new necessary and sufficient condition which will be useful from the graph theoretical point of view.

Reviewing the previous papers [1], [2], [9], [10], [11], [14], [15], [24], the Hypercube Q_n, the Crossed cube CQ_n, the Mobius cube MQ_n, and the Twisted cube TQ_n, all have diagnosability n under the PMC model. Moreover, we observe that they are almost (n+1)-diagnosable except for the case where all the neighbors of some vertex are faulty simultaneously. Closely related to this observation, we introduce the concept of a strongly t-diagnosable system and propose some conditions to assure which networks are strongly t-diagnosable.

The connectivity of a system is an important measure of fault tolerance. It is well-known that, for a system G, the connectivity of G is less than or equal to its minimum degree (this term will be defined subsequently). For example, the hypercube Q_n has connectivity n and this value n is equal to its minimum degree n. However, a scalable hypercube multiprocessor system can consist of thousands of processors. Under this complicated environment, more processors are likely to fail. To explore a more proper measure of fault tolerance, the conditional connectivity has been investigated in several research works [7], [12], [17], [22], [25].

Under the classical PMC diagnosis model, only processors with direct connections are allowed to test one another. Given a system, if all the adjacent neighbors of a processor v are faulty simultaneously, it is not possible to determine whether processor v is fault-free or faulty. Hence, for most practical systems that are sparsely connected, only a small number of faulty processors can be recognized with the classical diagnosis model. So, it is an interesting problem to explore some measures for better reflecting fault patterns in a real system than the existing ones. For example, Das et al. [5] investigated fault diagnosis with local constraints.

In this project, we propose a new measure of diagnosability, called conditional diagnosability, and study the conditional diagnosability of the hypercube. In classical measures of system-level diagnosability for multiprocessor systems, it has generally been assumed that any subset of processors can potentially fail at the same time. As a consequence, the diagnosability of a system is upper bounded by its minimum degree. We then consider these measures by restricting that, for each processor v in the network, all the processors which are directly connected to v do not fail at the same time. Under this condition, we show that the conditional diagnosability of Q_n is 4(n-2)+1, which is about four times larger than that of the classical diagnosability of Q_n.

三、研究方法與成果

這些年來我們實驗室的研究領域集中在連結網路(interconnection network)上，而

對於連結網路上的錯誤診斷問題我們做了相當深入的研究。我們發現在連結網路中的一些觀念如: connectivity、conditional connectivity、cube family都與錯誤診斷的研究息息相關。所以藉由這些關念，我們定義出一些錯誤診斷問題的新名詞，如：conditional diagnosability、strongly t-diagnosable system、local diagnosability 及 strong local diagnosability property 等等。對於這些新的觀念我們已有一些研究成果，這些成果有些已經投稿並被接受，有些還在撰寫論文繼續努力中。我們實驗室每週都會定時做一些研討，研討工作如下：

1. 收集文獻

   我們藉由圖書館、各期刊網站及國內外研討會等，來收集我們所需要的相關文獻。

2. 探討文獻及發現問題

   由計畫中的成員分工合作，將收集到的相關文獻作一個初步的分析與探討後，每週固定時間報告文獻中的內容，並且從文獻中發現更進一步的新的研究問題，由計畫主持人帶領成員對這些新主題做完整的研究。

3. 解決問題、程式撰寫、及定理證明

   對於解決問題的方式，我們由老師帶領博士班學生，博士班學生帶領碩士班學生來共同研究，在解決問題的過程中，我們時常需要撰寫程式來輔助定理的證明。經由程式不斷的累積，我們實驗室已經自行發展出相當多的例子測試的軟體程式。最後將研究之成果整理好撰寫成一篇完整的論文。

4. 成果發表

   這些年來，我們實驗室已有多篇論文被刊登在國際知名期刊。而本計畫結果已刊登在2005年IEEE Transactions on Computers期刊：

   P. L. Lai, J.M. Tan, C. P. Chang, and L. H. Hsu, ”Conditional Diagnosability Measures for Large Multiprocessor Systems,” IEEE Trans. on Computers, vol. 54, no. 2, pp.165-175 Feb. 2005.

四、結論與討論

本次研究計畫在所有成員的努力和主持人的帶領下，每週固定時間將所收集的資料做一番的報告與討論，並且分析和比較文獻中所提到的各種方法的優點及缺點，在大家多方面的不斷討論下，使我們對一些連結網路的診斷能力特性有了更清楚的了解，對於這些特性我們也提出了一些問題並且也提出了解決方式。

近幾年來，連結網路的診斷能力相關問題一直是我們實驗室重要的研究主題之一，所以為我們累積了相當多的經驗，每個參與的計畫人員都能對診斷能力的特性有充分的了解，也因此為本計劃的執行奠定了良好的基礎，且順利完成預定的研究進度。希望我們所研究的這些問題能提升連結網路診斷能力相關問題這方面的領域。而我們在研究過程中所得的經驗和知識，希望能對一些尚未解決的問題有所幫助，也希望這些經驗和知識在日後做更深入的研究時有所幫助。

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