行政院國家科學委員會補助專題研究計畫成果報告

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計畫主持人：卓訓榮
共同主持人：

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中　華　民　國　八十九　年　十　月　三十一　日

行政院國家科學委員會專題研究計畫成果報告
兼具巨觀與微觀之動態車流理論之研究
A Dynamic Traffic Flow Model with Both Macroscopic and Microscopic Behavior

計畫編號：NSC 89-2211-E-009-019

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一、中文摘要

車流理論可構建或修正運輸供需模式關係；進行系統服務水準評估；檢驗運輸系統是否運作順暢。尤其發展智慧型運輸系統，需要即時交通資訊之預測。因此不論在交通資料之應用與預測上，車流理論均為一不可或缺的分析工具與處理程序。本研究擬構建以連續方程式(continuity equation)**為基礎的動態車流模式，波動方程式為** 1955 **年** Whitham**，**Lightwill**，**Richard **等人由流體現象引入交通領域。由於波動方程式可描述動態車流變化以及車隊追逐行為，期望藉此描述動態巨觀車流。然而，以往巨觀車流模式與微觀模式之間有加總偏差**(aggregated bias)**。因此，本研究承襲並擴充先前計畫之結果，將藉由統計分配描述總體行為並配合個體行為，構建出兩者相合的動態模式。**

關鍵詞：車流理論、偏微分方程、守恆律。

Abstract

By traffic flow theory, planners can construct models describing relationships between traffic systems and environment, designers can evaluate traffic systems, and operators can check if there is something wrong in the system. So traffic flow theory is an important research field of traffic and transportation. Especially, in the develop trend of ITS, the analysis of traffic flow can provide the application of raw traffic data and real time prediction of traffic situation. In this study, we follow and expand the research, which we have done before, and try to develop a general dynamic traffic flow model, which is based on continuity equation. Whitham, Lightwill, and Richard applied the wave equation to traffic flow theory in 1955 first. To avoid aggregated bias of microscopic models, we are going to develop a dynamic macroscopic traffic model that coincident with microscopic behavior herein. In addition, we are going to discuss the algorithm to solve the model and extend it to applications of real road situation.

Keywords: traffic flow theory、partial differential equation、 conservation law

二、緣由與目的

Traffic flow theory is a new science, which has addressed questions related to understanding traffic processes and to optimizing these processes through proper design and control. The former questions could be described as basic research and the latter as applied research, especially in the worldwide trend of intelligent transportation system (ITS), forecasting and controlling dynamic traffic phenomena is becoming more important. Dynamic traffic flow and dynamic traffic assignment are two powerful tools for ITS applications. Essentially, link travel time, which is provided by dynamic traffic models, is an important part of dynamic traffic assignment models. Therefore, formulating dynamic traffic phenomena, which describes traffic situations to adapt the requirement of dynamic traffic assignment models and ITS applications is a valuable research topic. This study proposes a macroscopic dynamic traffic flow model which is based on the LWR model (Lighthill and Whitham, 1955, Richards, 1956), car-following theory and the high-order kinetic model (Michalopoulos, et. al., 1980, Michalopoulos, and Pisharody, 1980, Michalopoulos, et. al., 1981, Michalopoulos, et. al., 1983). There are two main equations considered in the model, the first one describes conservation of vehicle numbers and the second one describes motion of vehicles (conservation of momentum). However, macroscopic models still can hardly analyze the influence of vehicular behavior. To improve the drawback, an interaction function is introduced to describe the interaction between vehicles, so as to construct a self-consistent model, which includes microscopic and macroscopic behaviors. Mostly, the LWR-like and higher-order models need some specific speed-density relationship as state equations. As a speed-density relationship is employed, an assumption is made. Sometimes the assumption of speed-density relation is not consistent with the whole model. The model proposed herein generalizes the speed-density relationship and makes the whole model self-consistent. After deriving the model, discussion and analysis are mentioned. In addition, the comparison with the LWR-like and higher-order models are made. Numerical experiences are left to further research.

三、結果與討論

Mostly, microscopic traffic flow models are based on car-following theory that is every driver who finds himself in a single-lane traffic situation is assumed o react mainly to a stimulus from his immediate environment according to the relationship

$$(\text{Reaction})_{t+T} = a\,(\text{Stimulus})_t \tag{1}$$

where a is a sensitivity coefficient and T a reaction time-lag, the combined effect of the sluggishness of the driver and his car. It is reasonable to consider as reaction the acceleration of the car, over which the brake and gas pedal. If one further assumes that the sensitivity, a, is constant, he obtains the "linear car-following model"

$$\frac{d^2 x_n(t+T)}{dt^2} = \lambda\left[\frac{dx_{n-1}(t)}{dt} - \frac{dx_n(t)}{dt}\right] \tag{2}$$

in which n denotes the position of a car in a line of cars and x_n the position of the nth car along a highway. Gazis, Herman and Rothery, (1961) checked the linear car-following model and discussed nonlinear car-following models. In studies of nonlinear car-following model, the main difference between them is the formulation of sensitivity. They combined Eq.(2) and

$$\lambda = \frac{a\left(x'(t+T)\right)_{n+1}^{m}}{\left[x_n(t) - x_{n+1}(t)\right]^{l}} \tag{3}$$

and obtained

$$x''_{n+1}(t+T) = \frac{a\left(x'(t+T)\right)_{n+1}^{m}\left[x'_n(t) - x'_{n+1}(t)\right]}{\left[x_n(t) - x_{n+1}(t)\right]^{l}}. \tag{4}$$

Comparing the car-following theory and the kinetic mechanism, the basic concepts of both are the same. Both of them are illustrated in the form: $F = ma$. In physics, field is a concept to describe uncontact force, like magnetic field, gravity field and so on. For the safety sake, vehicles on a road also adjust their velocity and headway so as to avoid collision. Thus, the interaction force between vehicles can also be treated as a traffic field.

$$F = eE \tag{5}$$

, where e is a scalar which denotes per vehicle equivalent and E is traffic field, which is a vector. From the discussion above, it can be found that traffic field (E) is dependant on headway. To simplify the complication of the problem, we assume E depends on headway only and satisfies the inverse-square law. Considering interaction between two vehicles, traffic field acting on vehicle 0 can be formulated as:

$$E_0 = \frac{e_1}{\varepsilon}\left(\frac{x_1 - x_0}{\left|x_1 - x_0\right|^3} i + \frac{y_1 - y_0}{M^2\left|y_1 - y_0\right|^3} j\right) \tag{6}$$

, where ε is the interacting parameter. For the convenience sake, we convert the space from $\Omega \to \tilde{\Omega}$, where $\tilde{x} = x, \tilde{y} = My$ and traffic field acting on vehicle 0 in $\tilde{\Omega}$ is denoted by

$$\tilde{E}_0 = \frac{e_1}{£}\frac{\tilde{\mathbf{X}}}{\left|\tilde{\mathbf{X}}\right|^3}. \tag{7}$$

where $\tilde{\mathbf{X}}$ denotes the tensor from vehicle 0 to vehicle 1. Under the assumption of superposition, in continuous space

$$\tilde{E} = \frac{e}{\varepsilon}\int_\Omega \frac{\tilde{k}(\tilde{x},\tilde{y})}{\left|\tilde{\mathbf{X}}\right|^2} d\Omega \tag{8}$$

Eq.(8) is assumed that vehicles on the road section are the same. Since $\tilde{E}$ is a conservative field, which means that $curl\tilde{E} = 0$ $\left(\nabla\times\tilde{E} = 0\right)$, there exists a potential function $\tilde{E} = -\nabla\varphi$. We have to know the magnitude of traffic field so as to determine how the vehicles distribute. Thus, we have

$$divE = -\Delta ¨ = \frac{e\left(\tilde{k} + \bar{K}\right)}{£}. \tag{9}$$

where $div\ E$ gives the magnitude of traffic field and $\bar{K}$ denotes the modified road condition such as lane width, grade and so on and gives in terms of density. From Eq. (9), we can found that density moves from high traffic potential to low traffic potential.

Notice that previous discussion is in $\tilde{\Omega}$ space, the continuity equation is represented as

$$\frac{\partial\tilde{k}}{\partial t} + \nabla\cdot\tilde{\mathbf{q}} = 0 \tag{10}$$

where $\tilde{k} = \frac{1}{M}k$ denotes density in $\tilde{\Omega}$, $\tilde{q}_x = \frac{1}{M}q_x$, $\tilde{q}_y = q_y$ denote flow density in $\tilde{\Omega}$.

Suppose there are m spatial interval, denote as $\Delta\tau_i$, and each interval has n_i particles, and each particle has energy ε_i

$$Max \quad W = \frac{N!}{\prod_i n_i!}, \tag{11}$$

$$s.t. \quad \sum_i n_i = n_1 + n_2 + \Lambda + n_m = N, \tag{12}$$

$$\sum_i n_i£_i = n_1£_1 + n_2£_2 + \Lambda + n_m£_m = £_{tol}, \tag{13}$$

where N is the number of total particle, and $£_{tol}$ is total energy. Both N and $£_{tol}$ are constants. From nonlinear programming, with KKT condition and introducing α, β as Lagrange multipliers, we have

$$n_i = e^{-\alpha-\beta\varepsilon_i}. \tag{14}$$

From the relation of F, E and φ, we can find $\varepsilon_i = e\varphi$, and transform the vehicle numbers into density, then Eq.(14) can represent as

$$\tilde{k} = \tilde{K}\exp\left(\left(¨ - ¤\right)/Z\right), \tag{15}$$

where $\tilde{K}$ is essential density, ¤ is reference potential and Z is the adjust coefficient. The last equation proposed herein is motion equation (or so called conservation of momentum), which is illustrated as:

$$\frac{\partial(\tilde{k}\tilde{\mathbf{u}})}{\partial t}+\nabla(\tilde{q}\tilde{\mathbf{u}})=e\tilde{k}\tilde{E}+scattering\quad term\cdot \quad (16)$$

It is assumed that the scattering is elastic scattering. Therefore Eq.(16) can be represented as

$$\tilde{k}\left[\frac{\partial\tilde{\mathbf{u}}}{\partial t}+\tilde{\mathbf{u}}\cdot(\nabla\tilde{\mathbf{u}})-¬\nabla\tilde{k}\cdot\nabla\tilde{\mathbf{u}}\right]=-\frac{\tilde{\mathbf{u}}}{2_m}+e\tilde{k}\tilde{E}\cdot \quad (17)$$

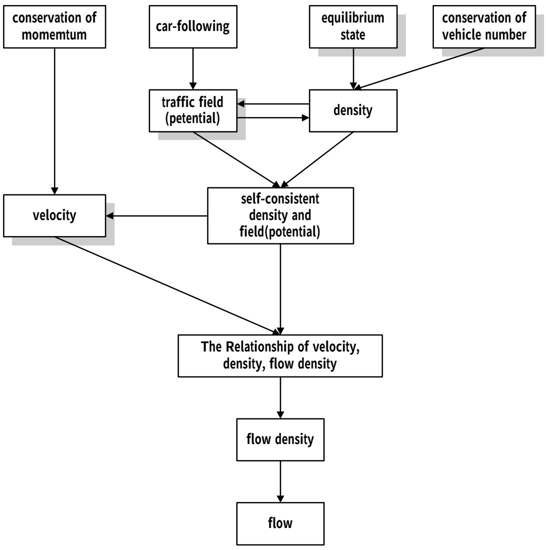
We discuss velocity under steady state and homogeneous condition and introduce mobility $« = \frac{e2_m}{\tilde{k}^*}$ herein, where $\tilde{k}^*$ is effective density and 2_m is relaxation time, such that Eq.(17) becomes

$$\tilde{\mathbf{u}}=\mu\tilde{E} \quad (18)$$

with the fundamental diagram and diffusion term:

$$\tilde{\mathbf{q}}=e\tilde{k}\tilde{\mathbf{u}}+e¬\nabla\tilde{k} \quad (19)$$

Eqs(9), (10), (15), (18) and (19) are the system equations of model we proposed. The modeling



structure are illustrated as figure 1.

Figure 1 Modeling structure of this study

After modeling, we are going to prove the existence and uniqueness of the solution under steady state condition. The steady state and spatial homogeneous model is illustrated as follows:

$$-\Delta ¨ = \exp(-¨)¸ + \overline{K}$$
$$\nabla(\exp(-¨)\nabla¸)=0 \quad (20)$$

where $¸(\mathbf{x})=e[\exp(¤)]/£$.

Then, we are going to develop a numerical method, which is called monotonic method. Firstly, we introduce the definitions and notation. We use the real scalar product, L_2 and uniform norms,

$$(f,g)=\int_{\tilde{\Omega}} f(\mathbf{x})g(\mathbf{x})d\mathbf{x},\quad \|f\|^2=(f,f),\quad \|f\|_0=\sup_{\mathbf{x}\in\tilde{\Omega}}|f(x)|, \quad (21)$$

and define the $W_2^{(1)}$ norm as

$$\|f\|_1^2 = ... |\nabla f|^2 dx \cdot \quad (22)$$

Let S denote the set of continuous functions in $W_2^{(1)}(\tilde{\Omega})$ satisfying

$$f(\mathbf{x})=0,\ \mathbf{x}\in\partial\tilde{\Omega}_1,\ \mathbf{n}\cdot\nabla f(\mathbf{x})=0,\ \mathbf{x}\in\partial\tilde{\Omega}_2 \quad (23)$$

for $f(\mathbf{x})\in S$, where $\mathbf{n}$ is the unit normal vector; then there exists a constant C_0 such that

$$\|f\|\le C_0\|f\|_1 \text{ for all } f\in S \quad (24)$$

It is assumed that the normalized vehicle density $\overline{K}(\mathbf{x})$ is uniformly bounded, and that the boundary data satisfy relations of the form

$$0<a\equiv\inf_{\mathbf{x}\in\partial\tilde{\Omega}_1}¸(\mathbf{x})\le b\equiv\sup_{\mathbf{x}\in\partial\tilde{\Omega}_2}¸(\mathbf{x}) \quad (25)$$

We define the constant C_1 by

$$C_1=\max\left(r,\sup_{x\in\partial\tilde{\Omega}_1}(¨(\mathbf{x}))\right), \quad (26)$$

where r are the solution of

$$-b\exp(r)=\inf_{\mathbf{x}\in\tilde{\Omega}}\overline{K}(\mathbf{x}) \quad (27)$$

From Eq.(20) and (25) and a standard maximum principle argument, it follows that any solution of (20) satisfies

$$a\le¸(\mathbf{x})\le b,\ \mathbf{x}\in\tilde{\Omega}\cup\partial\tilde{\Omega},\ \|¨\|_0=C_1. \quad (28)$$

The existence of the solution of (20) is illustrated as below. The derivation follows the concept of Mock (1972) and the proof is omitted herein.

Theorem 1 (existence). The system (20), with boundary conditions as described above, possesses a solution.

Theorem 1 can be proved by the Schauder fixed point theorem and the maximum principle. Since the solution of system (20) exists, we are going to develop a numerical method to solve it.

Let the function $¨(\mathbf{x})$ define a solution to the system (20), together with $¸(\mathbf{x})$, and the boundary data. We introduce the functions $¯_1(\mathbf{x})$, $¯_2(\mathbf{x})$ defined by

$$\Delta¯_1+\overline{K}(\mathbf{x})+a\exp(-C_1)=0,\ \mathbf{x}\in\tilde{\Omega},\ ¯_1-¨=0,\ \mathbf{x}\in\partial\tilde{\Omega}_1,$$
$$\Delta¯_2+\overline{K}(\mathbf{x})+a\exp(-C_1)=0,\ \mathbf{x}\in\tilde{\Omega},\ ¯_2-¨=0,\ \mathbf{x}\in\partial\tilde{\Omega}_1,$$
$$\mathbf{n}\cdot\nabla¯_1=\mathbf{n}\cdot\nabla¯_2=0,\ \mathbf{x}\in\partial\tilde{\Omega}_2 \quad (29)$$

The maximum principle and equations (26), (28), (29) imply

$$¯_1(\mathbf{x})\le¨(\mathbf{x})\le¯_2(\mathbf{x}),\ \mathbf{x}\in\tilde{\Omega}, \quad (30)$$
$$\Delta¯_1(\mathbf{x})\ge\Delta¨(\mathbf{x})\ge¯_2(\mathbf{x}),\ \mathbf{x}\in\tilde{\Omega}, \quad (31)$$

The sequence $\{¨_m\}$ is defined inductively by the following iteration scheme:

$$\nabla\exp(-¨_m)\nabla/_m=0,\ \mathbf{x}\in\tilde{\Omega},\ /_m-¸=0,\ \mathbf{x}\in\partial\tilde{\Omega}_1$$
$$\mathbf{n}\cdot\nabla/_m=0,\ \mathbf{x}\in\partial\tilde{\Omega}_2 \quad (32)$$
$$\Delta¤_m+\overline{K}(\mathbf{x})+/_m\exp(-¨_m)=0,\ \mathbf{x}\in\tilde{\Omega},$$
$$¤_m-¨=0,\ \mathbf{x}\in\partial\tilde{\Omega}_1,\ \mathbf{n}\cdot\nabla¤_m=0,\ \mathbf{x}\in\partial\tilde{\Omega}_2 \quad (33)$$
$$/_m(\mathbf{x})=(1-\sim)¨(\mathbf{x})+\sim¤(\mathbf{x}),\ \mathbf{x}\in\tilde{\Omega}\cup\partial\tilde{\Omega} \quad (34)$$
$$¨_{m+1}(\mathbf{x})=\min[¯_2(\mathbf{x}),\max[¯_1(\mathbf{x}),/_m(\mathbf{x})]],\ \mathbf{x}\in\tilde{\Omega}\cup\partial\tilde{\Omega} \quad (35)$$

Then, we show the Eqs(32), (33), (34) and (35) converges to unique solution.

Theorem 2 For all sufficiently small values of $\sim$,

$\log(b/a)$, the sequence $\{¨_m\}$ generated by successive applications of (32), (33), (34) and (35) converges geometrically in $W_2^{(1)}$ to the unique solution of (20).

Since, the existence and uniqueness of the solution of system (20) have been proven, we are going to show the error estimation and the condition for being well-posed. An immediate consequence of the lemma is the following error estimate for the computation scheme described above. Since $¨ - ¤_m \in S$, we obtain, integrating by parts and using (20), (32), (33) and (34),

$$\begin{aligned} \| ¨_m - ¨ \|_1^2 &= -(¨_m - ¨, \Delta ¨_m - \Delta ¨) \\ &= -(¨_m - ¨, \Delta ¨_m + \bar{K} + ¸ \exp(- ¨)) \\ &= -(¨_m - ¨, \Delta ¨_m + \bar{K} + ¦_m \exp(- ¨_m)) \\ &\quad -(¨_m - ¨, ¦_m \exp(- ¨_m) - ¸ \exp(¨)) \\ &\le -(¨_m - ¨, \Delta ¨_m + \bar{K} + ¦_m \exp(- ¨_m)) \\ &\le \| ¨_m - ¨ \| \| \Delta ¨_m + \bar{K} + ¦_m \exp(- ¨_m) \| \\ &\le C_0 \| ¨_m - ¨ \|_1 \| \Delta ¨_m + \bar{K} + ¦_m \exp(- ¨_m) \| \end{aligned} \quad (36)$$

here we have also used. Thus

$$\| ¨_m - ¨ \|_1 \le C_0 \| \Delta ¨_m + \bar{K} + ¦_m \exp(- ¨_m) \| \quad (37)$$

A consequence of (24) is the following condition for being well posed:

THEOREM 3. Let $¨, ¸$ and $\tilde{¨}, \hat{¸}$ be two solution of (20) corresponding to different smooth boundary data on $\partial\tilde{\Omega}_1$, and suppose that, for all $\mathbf{x} \in \partial\tilde{\Omega}_1$,

$$| ¨(\mathbf{x}) - \tilde{¨}(\mathbf{x}) |, \ | ¸(\mathbf{x}) - \hat{¸}(\mathbf{x}) | \le ¢ \quad (38)$$

then $\| ¨ - \tilde{¨} \|_1 \le C_5 ¢$,where C_5 depends on the boundary data, but exists for all smooth data.

四、計畫成果自評

This work is based on LWR model, which is macroscopic dynamic traffic flow model and car-following theory and connects both of them by equilibrium distribution (state equation). Thus, our modeling philosophy contains both macroscopic and microscopic behavior and also it is a flexible structure, which can be extended to higher-order dynamic traffic flow model for more complicate traffic behavior. In addition, the model presented herein is more general than the LWR model. In this study, we develop a numerical method and its error estimation under steady state. The result of this study can be concluded as follows:

1. A dynamic traffic flow model with both microscopic and macroscopic behavior is developed herein.
2. Under steady state, the existence and uniqueness are proved.
3. Numerical method for steady state is derived and the convergence (error estimation) is proved..

All objectives mentioned in the proposal of this project are finished in this study. After this project, are further researches we suggest are as follows:

1. The proof of existence and uniqueness of time-dependent solution.
2. Numerical method of time-dependent model.
3. Numerical analyses and experimental study.

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