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行政院國家科學委員會專題研究計畫成果報告
利用無母數方法架構整合績效評估
Integrating Ratio Analysis under Non-Parametric Framework

計畫編號：NSC 94－2213－E－009－078
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主 持 人：陳文智 國立交通大學工業工程與管理學系

一、中英文摘要

如何正確且有效的衡量評估系統組織的績效一直都是工業工程和作業研究領域最基本的議題，各種學術或實際應用的問題，如製程的監測與管制、系統的改善、規劃或最佳化、以及方案的選擇，都牽涉到績效評估。過去數十年來，產學界已發展了許多績效評估的方法，各個方法都有其優缺點、假設和研究社群。直到今日，各方法仍是各自獨立的典範(paradigm)且互相競爭，少有研究試著整合各個方法，特別是做理論性的整合上。本研究計劃的主要目的是試著提供一完整的理論架構來整合不同的績效評估方法，特別是專注在傳統比率分析(ratio analysis)和無母數(non-parametric)方法間。如果本研究計劃得以完成，傳統的比率分析將可以整合到無母數方法的架構下，如此將使我們可以更清楚瞭解不同績效評估的方法差異的定量(quantitative)性質，進而得以發展出可靠、有效的績效評估方法。
關鍵詞：績效評量；比率分析；資料包絡面分析

Abstract

A fundamental problem of industrial engineering and operations research is to measure and assess the performance of an organization and/or a system. Many applications from process control and monitoring, system improvement to design alternative selection are all rooted on performance assessment. There are many different methods for this goal with their *pros* and *cons*, and assumptions and research communities. Today, each methodology is an isolated paradigm and competes with each other, however there is little research, especially not theoretically, on integrate these methods. The objective of the proposed research is to develop a theoretical framework which integrates different performance metrics, especially focusing on traditional ratio analysis and DEA. If concluded successfully, traditional ratio analysis can be analyzed under the non-parametric framework. Therefore, better understanding of different approaches will be achieved and theory-based, cost-effective, and practical performance assessment methods will be developed.
Keywords：Performance Assessment, Ratio Analysis, DEA

二、緣由與目的

Organizations utilize resources (inputs) to produce value-generating goods or services (outputs). Design, planning and control problems, especially in industrial engineering and operations research, are rooted in making this input-output conversion process better. As a result, a fundamental problem of industrial engineering and operations research is to measure and assess the process to determine overall resource efficiency. There are many different methods for performance assessment with their *pros* and *cons*, assumptions and research communities. However, till so far, each methodology is an isolated paradigm and competes with each other; adoption of another method becomes a difficult paradigm shift. There is little common context among these methods. One noticeable example is that despite the appeal of data envelopment analysis (DEA) and a history of 25 years of theoretical and applied research, most practitioners still utilize output-input ratio analysis for performance assessment. Therefore, the ultimate objective of the proposed research is to develop a theoretical framework which integrates different performance metrics. In particular, this one year project is trying to develop a theory to reconcile different performance metrics, especially focusing on traditional ratio analysis and DEA.

A *ratio productivity*, or so-called *single-factor productivity* or, simply, *output-input ratio*, is the ratio of only one output to only one input, e.g. order fulfillment per labor hour, or customers served per dollar of cost (Agrell and Wikner, 1996; Bhargava et al., 1994; Brinkerhoff and Dressler, 1990; Fabricant, 1983, Kendrick, 1977; Lyons, 1995). The associated term for relative comparison is *ratio efficiency* (RE), which is the ratio of a ratio productivity value to some reference value, such as the corresponding "best in class" ratio productivity based on this specific productivity definition.

The main problem for ratio productivity or ratio efficiency is a lack of inclusiveness since each ratio only catches a piece of the whole picture. To resolve single-factor measures' weakness, a set of ratios can be sued to represent all aspects related to the DMU. For example, in financial the analysis, we are looking at many different index (ROI, ROA, etc) for different managerial aspects. The main disadvantage is that the ratio measures in this family usually are not consistent with each other (Bhargava et al., 1994; Lyons, 1995; Martin and Roman, 2001). Though listing a set of measures may cover all pertinent aspects, the lack of convergence makes evaluating overall performance difficult.

Non-parametric approaches consider multiple inputs and outputs simultaneously, but, in contrast to TFP, DEA requires neither *a priori* weights nor a prespecified functional form. One of the well-known approach, DEA, is in this category. DEA, introduced by Charnes *et al.* in 1978, is a mathematical programming based approach that evaluates the efficiencies of an organization or, in general, a *decision making unit* (DMU) relative to a set of comparable organizations. However, instead of calculating the productivity for a particular DMU, DEA constructs an empirical production possibility set (EPPS) and an efficient frontier, and provides a single efficiency score for that DMU by comparing to a "virtual producer" on the frontier. Since it was introduced, there have been over 3000 publications concerning DEA including more than 1200 journal papers and 171 dissertations by the end of year 2001 (Tavares 2002).

While DEA has been studied for the last two decades and used in a large number of special studies in specific industries, it is NOT used as a routine performance assessment tool in practice. To the contrast, <u>ratio analysis is still the most widely-used</u> performance assessment approach today in practice. Surprisingly, these two approaches with the same purposes are disconnected. The proposed research is to bridge the disconnection among different performance assessments. With the theoretical connection built, we are able to have better understanding of performance assessment and resolve the addressed problems.

There is limited literature on the relationship between DEA and ratio productivity or efficiency measurements. Most of the published work on this issue presents empirical comparisons of DEA and ratio productivity metrics. Some authors use the same data set to check (1) the consistency between these two approaches, and (2) the consistency with the overall economic performance, or the goal of the organizations. For instance, Schefczyk (1993) compares ratio productivity and DEA in a study of warehouse performance. Lyons (1995) compares the ratio measures, total factor productivity (TFP) and DEA using a data set for urban transit. Thanassoulis *et al.* (1996) study the district health authorities (DHA) in the UK. Yeh (1996) studies bank performance in Taiwan. Worthington (1998) compares the financial performance of thirty Australian gold producers based on accounting-based ratios and DEA. Feroz *et. al.* (2003) use data from oil and gas industry as a case study. These studies all conclude the inconsistency between DEA and ratio analysis.

Besides using productivity as the performance indicator, productivity and efficiency could be used to identify potential improvement opportunities. Only Thanassoulis *et al.* (1996) compare ratio measures and DEA for this purpose. They state that identifying a "best in class" as the benchmark by a specific ratio productivity metric and setting this benchmark as the improvement target is not suitable. They demonstrate this point by a numerical example that gives an unrealistic improvement target.

Another group of authors tries to integrate ratio measures and DEA to create a better performance assessment tool. Chen and Ali (2002) prove that top-ranked performance by ratio productivity analysis is on the DEA frontier, and suggest that this property can help to select some efficient DMUs without solving the DEA model.

Most earlier work provides an empirical comparison between ratio measures and DEA and/or evaluates the consistency between the metrics. Some authors discuss the inconsistency between the two approaches. Some simple extreme cases in ratio productivity and DEA are proposed as a complementary tool to reduce DEA computation effort. There is, however, no prior study that explains the relationship between DEA and the conventional ratio productivity in detail and forms a theoretical basis for connecting and integrating these two approaches.

三、結果與討論

System-Based Efficiency

Consider an input set *I* and an output set *J*. Let $\mathbf{x} \in \Re_+^{|I|}$ be the input vector and $\mathbf{y} \in \Re_+^{|J|}$ be the output vector. The *production possibility set*, *T*, is defined as {(**x**, **y**) : **y** can be produced by **x**}. The output set associated with **x** is defined as $P(\mathbf{x}) \equiv \{\mathbf{y} : (\mathbf{x}, \mathbf{y}) \in T\}$. In the same way, given **y**, the corresponding input set *L*(**y**) is defined as $L(\mathbf{y}) \equiv \{\mathbf{x} : (\mathbf{x}, \mathbf{y}) \in T\}$. The input Debreu-Farrell *technical efficiency* for a particular (**x**,**y**) is defined as $\min\{\alpha : \alpha\mathbf{x} \in L(\mathbf{y})\}$ (Debreu, 1951; Farrell, 1957).

In practice, *T* is unknown. However, given a set of DMU observations *S* with input-output vectors $\{(\mathbf{x}_1,\mathbf{y}_1\},(\mathbf{x}_2,\mathbf{y}_2),\ldots,(\mathbf{x}_{|S|},\mathbf{y}_{|S|})\}$, the *empirical* production possibility set, as described by Ray and Mukherjee (1996), is "an inner approximation to the true production possibility set" and "is the free disposal convex hull of the observed points". Further assuming *constant returns to scale* (CRS), the empirical production possibility set can then be expressed as a set of linear inequalities in |*S*| nonnegative variables and denote as:

$$T_c^S \equiv \left\{(\mathbf{x},\mathbf{y}) : \sum_{r \in S} \mathbf{x}_r \lambda_r \le \mathbf{x}; \sum_{r \in S} \mathbf{y}_r \lambda_r \le \mathbf{y}; \lambda_r \ge 0, r \in S\right\}.$$

Given input $\tilde{\mathbf{x}}$, the empirical output set is denoted as $P_c^S(\tilde{\mathbf{x}})$, and the empirical input set for given $\tilde{\mathbf{y}}$ is $L_c^S(\tilde{\mathbf{y}})$, and they both can be specified in the same way.

Combing concepts of technical efficiency and the empirical input set, the first DEA model is the CRS model introduced by Charnes, Cooper and Rhodes in 1978 (also named the CCR model). For $(\mathbf{x}_k, \mathbf{y}_k) \in S$, we have the input-oriented model

$$\theta_c^*(k) = \min_{\theta,\lambda}\left\{\theta : \sum_{r \in S} \mathbf{x}_r \lambda_r \le \theta\mathbf{x}_k ; \sum_{r \in S} \mathbf{y}_r \lambda_r \le \mathbf{y}_k ; \lambda_r \ge 0, r \in S\right\} \qquad \text{(CRS-I)}$$

DEA provides system-based efficiency measurement since it considers all inputs and outputs of the system simultaneously, and hereafter we refer the system-based efficiency as $\theta_c^*(k)$. In fact, $\theta_c^*(k)$ can be decomposed into many components with nice interpretations.

For example, Banker et al. (1984) show that $\theta_c^*(k)$ actually is the product of *pure* technical efficiency *TE*(*k*) and *scale efficiency* (*SE*(*k*)), which measures how far DMU *k* is from the *most productive scale size*. This leads to $\theta_c^*(k) = TE(k) \times SE(k)$ that reveals system structure, namely, the technical efficiency and scale economics. Other possible decompositions of $\theta_c^*(k)$, e.g. productivity changes (Fare et al., 1994, 1997), congestion or weight restriction, are related to system-based structural issues and not out of the scope of this study.

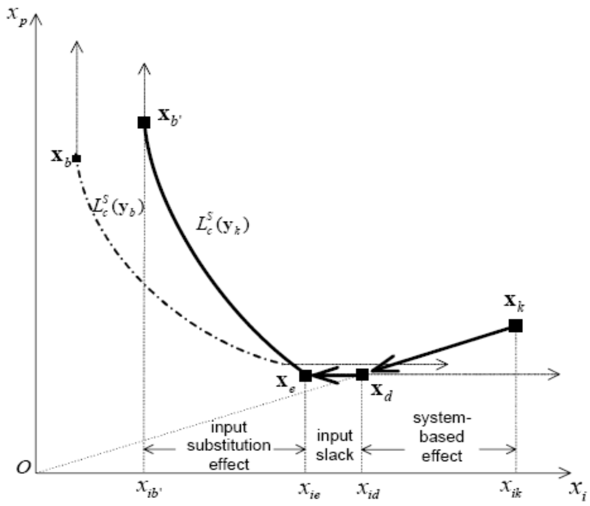

Figure 1: Two-input (input *i* and *p*) multiple-output illustration of the decomposition

Decomposition of Ratio Efficiency

The RE is the efficiency measure obtained from conventional ratio analysis. Consider the output-input ratio metric in which input $i\,(x_i)$ and output $j\,(y_j)$ are of interest; the RE for any DMU $k \in S$ is defined as $RE\,(i,j,k) \equiv \frac{y_{jr}}{x_{ik}} \Big/ \max_{r \in S}\left(\frac{y_{jr}}{x_{ik}}\right)$. The corresponding DMU with best $\frac{y_{jr}}{x_{ik}}$ denoted as $b \equiv \arg\max_{r \in S}\left(\frac{y_{jr}}{x_{ir}}\right)$ so that $RE\,(i,j,k) = \frac{y_{jk}}{x_{ik}} \Big/ \frac{y_{jb}}{x_{ib}}$. The theoretical relationship between RE and DEA can be built by decomposing RE under the non-parametric frontier analysis framework. In this report, we only facilitate the idea of decomposition by Figures 1 and 2, detail definitions and proofs have been done but cannot be addressed because of the length of the report.

Figure 1 is a 2-input (x_i and x_p) illustration for general multi-output cases and Figure 2 is a 2-output (y_j and y_q) illustration; in both, $\frac{y_j}{x_i}$ is chosen as the ratio metric of interest. For any DMU $k \in S$ with ($\mathbf{x}_k$,$\mathbf{y}_k$), *RE* (*i,j,k*) suggests to reduce x_i or increase y_j to achieve the best ratio value $\frac{y_{jb}}{x_{ib}}$. As shown in Figure 1, without changing outputs, $\mathbf{y}_k$, DMU *k* can reduce its input *i* from x_{ik} to $x_{ib'}$, which is the minimum possible of x_i to produce $\mathbf{y}_k$. This reduction generally contains three parts:

First, from x_{ik} to x_{id} is system-based since the reduction is possible while keeping the input mix the same. In addition, other inputs are also reduced by $100 \times \frac{x_{id}}{x_{ik}}$ %. The second part, from x_{id} to x_{ie}, is due to the excess of x_i over the amount necessary to produce $\mathbf{y}_k$ and $x_{id} - x_{ie}$ is called the slack of input *i*. The third part of the reduction, from x_{ie} to $x_{ib'}$, requires to increase at least one another input, e.g., x_p in Figure 1. That is using other inputs as the substitute so that x_i can be reduced while keeping $\mathbf{y}_k$ the same, and thus, this part is due to the input substitution.

Similar to the idea measuring scale and technical inefficiency (Banker et al., 1984), quantitative metrics are defined to measure the magnitude of the three parts composing the total reduction of x from x_{ie} to $x_{ib'}$. Apparently, $\mathbf{x}_d = \frac{x_{id}}{x_{ik}} \mathbf{x}_k$ in Figure 1, and, thus, $\frac{x_{id}}{x_{ik}}$ is identical to system-based $\theta_c^*(k)$ measuring the maximum proportionate input reduction, i.e., input-output mix is the same, to produce $\mathbf{y}_k$. $ISlk(i,k) = \frac{x_{ie}}{x_{id}}$ is the *input slack factor* of input i for DMU k and measures the "pure" slack effect of input i. We have $ISlk(i,k) \le 1$, and $ISlk(i,k) = 1$ indicates that there is no slack for input i; the smaller the $ISlk(i,k)$ is, the more serious is the impact of input i slack. $ISub(i,k) = \frac{x_{ib'}}{x_{ie}}$ is the *input substitution factor* measuring the substitution effect of input i for DMU k. It is clear that $ISub(i,k) \le 1$, and $ISub(i,k) = 1$ indicates that there is no means to substitute input i by other inputs. However, from $ISub(i,k)$, we cannot determine how and by how much other inputs change to enable the decrease of input i; this measure only addresses the change of input i. Consequently, $RE(i,j,k)$ can be decomposed and expressed as:

$$RE(i,j,k) \equiv \frac{\frac{y_{jk}}{x_{ik}}}{\frac{y_{jb}}{x_{ib}}} = \frac{x_{id}}{x_{ik}} \times \frac{x_{ie}}{x_{id}} \times \frac{x_{ib'}}{x_{ie}} \times \frac{\frac{y_{jk}}{x_{ib'}}}{\frac{y_{jb}}{x_{ib}}} = \theta_c^*(k) \times ISlk(i,k) \times ISub(i,k) \times \frac{\frac{y_{jk}}{x_{ib'}}}{\frac{y_{jb}}{x_{ib}}} \qquad (1)$$

In Figure 1, although $x_{ib'}$ is the minimum amount of input i to produce $\mathbf{y}_k$, it is possible that $\frac{y_{jk}}{x_{ib'}} < \frac{y_{jb}}{x_{ib}}$ after the input i reduction, and we can fill the gap $\frac{y_{jb}}{x_{ib}} - \frac{y_{jk}}{x_{ib'}}$ of by increasing output j.

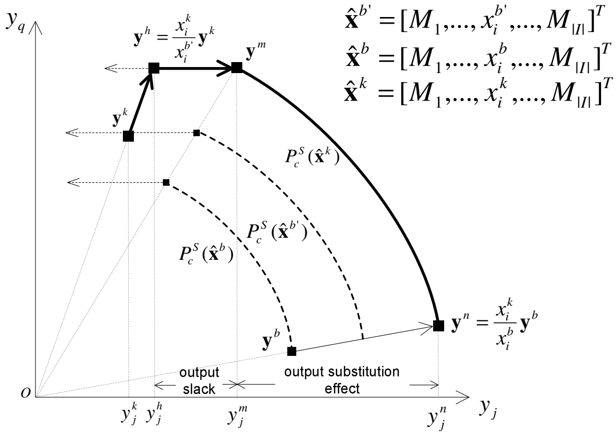

Figure 2: Two-output (output j and q) multiple-input illustration of the decomposition

Figure 2 continuing from Figure 1 is the second phase output-oriented analysis. First, $x_{ib'}$ is the result of the substitution effect and we do not know what inputs will substitute for input i or by how much. In addition, we are concern only with input i, xi is the critical resource and the only input constraint. Hence, we can represent $\mathbf{x}_{b'}$ by $\hat{x}_{b'} = [M_1, ..., x_{ib'}, ..., M_{|I|}]^T$, where M is a sufficiently large number, and the computation results remain the same. As a result, in the output analysis we use $\hat{x}_k = [M_1, ..., x_{ik}, ..., M_{|I|}]^T$, $\hat{x}_b = [M_1, ..., x_{ib}, ..., M_{|I|}]^T$ and $\hat{x}_{b'} = [M_1, ..., x_{ib'}, ..., M_{|I|}]^T$ to represent the available resources.

Second, we will show that $\frac{y_{jk}}{x_{ib'}}$ and $\frac{y_{jb}}{x_{ib}}$, the last term of (1), are equivalent to $\frac{y_{jh}}{x_{ik}}$ and $\frac{y_{jn}}{x_{ik}}$ in Figure 2, respectively. As the result of input i reduction, $\mathbf{y}_k$ can be produced by $\hat{x}_{b'}$

efficiently. Because of CRS, reducing inputs to $100\times\frac{x_{ib'}}{x_{ik}}$ % is same as increasing outputs to $\frac{x_{ik}}{x_{ib'}}$; $\mathbf{y}_h=\frac{x_{ik}}{x_{ib'}}\mathbf{y}_k$ denoted in Figure 2, thus, is produced by $\hat{x}_k$ efficiently. With the same argument, we have $\mathbf{y}_n=\frac{x_{ik}}{x_{ib}}\mathbf{y}_b$ produced by $\hat{x}_k$ efficiently as well. By scaling $\mathbf{y}_k$ and $\mathbf{y}_b$ as shown in Figure 2, they are both in the output set using resource $\hat{x}_k$, in which all resources but *xi* are unconstrained, system-based efficiently.

Figure 2 shows that to fill the gap between $\frac{y_{jk}}{x_{ik}}$ to $\frac{y_{jb}}{x_{ib}}$, one can reduce x_i from x_{ik} to x_{ib} (if possible) while keeping y_{ib}, or increase y_j from y_{jk} to y_{jn} with x_{ik} fixed. The part y_{jk} to y_{jh} representing the result of the first stage, and to fill the remaining gap y_{jh} to y_{jn} includes two parts:

The first part, y_{jh} to y_{jm} is the slack of output *j*, which is the extra amount of y_j can be produced by x_{ik} with the others resources always available. The part of from y_{jm} to y_{jn} is the maximum extra amount of output *j* can be; however, it requires sacrificing at least one another outputs, which can substitute with y_j, to transform to y_j. Thus this part is due to the substitution effect of outputs.

Similar to Figure 1, the input-oriented stage, metrics measuring the magnitude of the two parts can be defined. Corresponding to y_{jh} and y_{jm}, $OSlk(i,j,k)=\frac{y_{jm}}{y_{ih}}$ is the *output slack factor*, which measures the "pure" slack effect of output *j*. $OSub(i,j,k)=\frac{y_{in}}{y_{jm}}$ is the *output substitution factor*. Clearly, we have $OSlk(i,j,k)\geq 1$ and $OSub(i,j,k)\geq 1$. $OSlk(i,j,k)=1$ and $OSub(i,j,k)=1$ indicate that there is no slack of y_j and it is impossible to transform other outputs to y_j, respectively.

Therefore, the last term of the right hand size of (1) can be rewritten as

$$\frac{\frac{y_{jk}}{x_{ib'}}}{\frac{y_{jb}}{x_{ib}}}=\frac{\frac{y_{jh}}{x_{ik}}}{\frac{y_{jn}}{x_{ik}}}=\frac{y_{jh}}{y_{jm}}\times\frac{y_{jm}}{y_{jn}}=\frac{1}{OSlk(i,j,k)}\times\frac{1}{OSub(i,j,k)}. \qquad (2)$$

Combining (1) and (2), the complete decomposition of *RE*(*i,j,k*) is:

$$RE(i,j,k)=\theta_c^*(k)\times ISlk(i,k)\times OSub(i,k)\div OSlk(i,j,k)\div OSub(i,j,k). \qquad (3)$$

Ratio efficiency can be decomposed as five factors including input/output slacks, input/output substitution and system-based efficiency, which can be further decomposed in a system-based context as shown in many earlier literature.

Implementation

The conceptual decomposition has been implemented by Matlab. The information retrieved from the decomposition can be implemented as diagnosis tool for ratio analysis. Figure 3 is an example, in which ratio using output A and input B is of interests. The gap of this particular ratio to the best in class is 82%, i.e., this particular DMU is only 18% as good as the best in class. The information of the decomposition provides further insights on how to fill the performance gap. Some parts of the gap are resulted from long term strategic aspect such as product mix (40%) or resource allocation (17%). Some parts are due to short term operational reasons. It is important to understand the causes of the gap before setting targets for improvements.

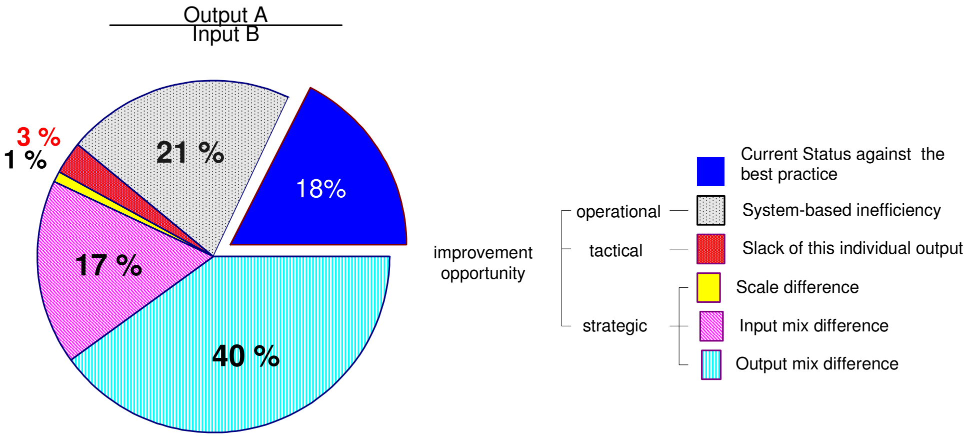

Figure 3: Gap analysis

四、計畫成果自評

The research results of this project are well fit with the original proposal as follows.

First, a comprehensive literature review on performance evaluation, productivity and efficiency analysis are studies. Different approaches are organized and summarized under the non-parametric activity analysis framework. Second, the theoretical relationship between widely used ratio analysis and DEA is proposed. Ratio efficiency, from ratio analysis comparing to the best in class, can be decomposed into five factors including CCR efficiency obtained from DEA. Third, the theory is implemented by Matlab. Some initial applications are proposed, and the applications of the theory are the future direction to extend the developed theory.

This research is suitable for publishing, and the principle investigator is working on the manuscript for publishing. Students involved in the project have the chance to explored theoretical knowledge and implementation of both performance assessment and computation experience. They are working on extensions of the decompositions and also the implementations of the theory. The initial implementation of the developed theory shows promising implications of performance evaluation, especially for diagnosing the gap of ratio analysis. Further studies on the issues are in progress and master theses on the topics will be finished next spring semester.

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