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Adomian's decomposition method for electromagnetically induced transparency

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We developed the Adomian's decomposition method to work for the electromagnetically induced transparency (EIT) problem. The method is general and capable to solve the coupled nonlinear partial differential equations for a light pulse passing through a three-level Λ-type coherent medium. This EIT system is described by the coupled Maxwell-Schrödinger equations and optical Bloch equations. In the weak probe field case, the results agree with perturbation solutions and experimental data. In the stronger probe field case while perturbation may fail, our results reproduce experimental data well. With the techniques of spatial and time partitions, we extend the decomposition method that will be versatile for the investigation of the light pulse propagating through a coherent atomic medium.

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I. INTRODUCTION

Recent progress for the control of light pulse propagations through a coherent medium inspires interesting discoveries. Namely, the electromagnetically induced transparency (EIT) [1] and the frozen of light [2] are two of the amazing phenomena. The technique to manipulate properties of light pulses [3] can be used to create large populations of coherently driven atoms. Types of optoelectronic devices [4] may be invented by this kind of technology. The basic principles involved in the two problems are delicate quantum interferences. To our knowledge, except for the perturbational solutions, the general way of solutions to the system is still demanding. We aim in this paper to develop a general tractable method to solve the related partial differential equations for the system. So, some realistic experimental physical parameters are used and the results of the experiment and numerical calculations are compared.

The method we developed here is the Adomian's decomposition method (ADM). ADM solves nonlinear differential equations with decompositions; neither linearization nor perturbation is necessary for the nonlinear part. The method has been widely applied to various domains in science and engineering. Adomian himself treated many physical topics such as the Navier-Stokes equations, Burger's equation, the advection-diffusion equation, Korteweg–de Vries equation, nonlinear Schrödinger equation, etc. [5]. ADM is extremely powerful for nonlinear physical problems, but no general treatments for eigenvalue problems are found. We recently developed a general ADM method for either linear or nonlinear eigenvalue problems [6]. Some of the techniques developed in the previous paper are adopted here, also.

The ADM treatment of the quantum optical propagation problem is valuable to computational physics. It provides nonperturbative, semianalytical solutions. So the solution may be used to analyze the behaviors when the physical parameters change. This advantage is by no means beyond the capacity of usual numerical grids methods. Thus, the ADM method will enhance the understanding of EIT and related problems.

Consider a coherent medium consisting of Λ-type three-level atoms with two metastable lower states $|1\rangle$, $|2\rangle$, and an excited state $|3\rangle$. In a typical experiment [7], the medium contains one billion of ultracold ^{87}Rb atoms produced by a magneto-optical trap. The three states are chosen as hyperfine levels of $|5S_{1/2},F=1\rangle$, $|5S_{1/2},F=2\rangle$, and $|5P_{3/2},F'=2\rangle$. A weak probe pulse $E_p(x,t)=\varepsilon_p e^{-i\omega_p t}+\mathrm{c.c.}$ couples state $|1\rangle$ and the excited state $|3\rangle$, and a stronger control field $E_c(x,t)=\varepsilon_c e^{-i\omega_c t}+\mathrm{c.c.}$ couples state $|2\rangle$ and state $|3\rangle$, where the pulse envelopes are slowly varying. Due to the long wavelength condition, the spatial dependence $e^{\pm ikx}$ of electric fields is neglected, however ε_p and ε_c still depend on x.

The Maxwell-Schrödinger equations (MSE) with the Rabi frequencies $\Omega_p \equiv d_{13}\varepsilon_p/\hbar$ and $\Omega_c \equiv d_{23}\varepsilon_c/\hbar$ for the probe field and control field are

$$\frac{1}{c}\frac{\partial \Omega_p(x,t)}{\partial t}+\frac{\partial \Omega_p(x,t)}{\partial x}=i\eta\,\mathrm{Im}[\rho_{31}], \tag{1}$$

$$\frac{1}{c}\frac{\partial \Omega_c(x,t)}{\partial t}+\frac{\partial \Omega_c(x,t)}{\partial x}=i\eta\,\mathrm{Im}[\rho_{32}], \tag{2}$$

where d_{ij} is the dipole-matrix element between states i and j, $\eta=3\lambda^2\Gamma N/(8\pi)$, and Γ is the spontaneous decay rate of the excited state $|3\rangle$. The decay rates from $|3\rangle$ to $|1\rangle$ and to $|2\rangle$ are usually assumed equal [8]. N is the number density of the

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medium, and λ is the wavelength of the probe laser. We consider that the coupling frequency and the carrier frequency of the probe pulse are tuned resonantly. The density matrix elements ρ_{ij} are obtained from the optical Bloch equations (OBE) under the rotating-wave approximation [9,10]:

$$\frac{\partial\rho_{11}}{\partial t} = \frac{\Gamma}{2}\rho_{33} - \frac{i\Omega_p}{2}(\rho_{13}-\rho_{31}),$$

$$\frac{\partial\rho_{22}}{\partial t} = \frac{\Gamma}{2}\rho_{33} - \frac{i\Omega_c}{2}(\rho_{23}-\rho_{32}), \tag{3}$$

and

$$\frac{\partial\rho_{12}}{\partial t} = -\gamma\rho_{12} - \frac{i\Omega_c}{2}\rho_{13} + \frac{i\Omega_p}{2}\rho_{23},$$

$$\frac{\partial\rho_{23}}{\partial t} = -\frac{\Gamma}{2}\rho_{23} - \frac{i\Omega_p}{2}\rho_{21} - \frac{i\Omega_c}{2}(\rho_{22}-\rho_{33}),$$

$$\frac{\partial\rho_{31}}{\partial t} = -\frac{\Gamma}{2}\rho_{31} + \frac{i\Omega_c}{2}\rho_{21} + \frac{i\Omega_p}{2}(\rho_{11}-\rho_{33}), \tag{4}$$

where ρ_{ij} are functions of x, and t and are complex in general. In the OBE, γ is the relaxation rate of the ground-state coherence. It can affect the amplitude of the output probe pulse significantly. The γ is a practical physical parameter but was usually neglected in other calculations. For example, in Ref. [8] the authors calculate the problem nonperturbatively. Merely, they did not consider the relaxation rate of ground-state coherence. In Ref. [11], the authors treat the probe pulse as the perturbation, since they do not have the equations of populations. However, a constant population distribution is only valid for the perturbative calculation, i.e., the ground-state population of the probe field, $\rho_{11}\sim 1$. Our goal is to solve the combined MSE and OBE by ADM with practical experimental parameters that may lie beyond the perturbative regime.

The paper is organized as follows. In Sec. II we introduce the scheme to decompose the spatial variable. In Sec. III we describe the decomposition of the time variable. In Sec. IV the time domain partition method is used to overcome the trouble of divergence in the series expansion of t. In Sec. V, for a stronger probe pulse case, we use the spatial domain partition method to resolve the convergent problem of x and present the results. Section VII is devoted to the concluding remarks.

II. DECOMPOSITION OF THE POSITION VARIABLE

In the following, we denote the probability of the excited state as $\rho_e=\rho_{33}$, and the reduced probability of the ground state as $\rho_g=1-\rho_{11}$ for convenience. The conservation of probability requires $\rho_{22}=\rho_g-\rho_e$. We find from Eq. (4) that ρ_{12} is real and ρ_{23} and ρ_{31} are imaginary. We denote $\rho_{12}=\rho_\alpha$, and $\rho_{23}=i\rho_\beta$, $\rho_{31}=i\rho_\gamma$, where ρ_e, ρ_g, ρ_α, ρ_β, ρ_γ, Ω_c, and Ω_p all are real functions. Thus the coupled MSE and OBE become

$$\frac{\partial\rho_g}{\partial t} = -\frac{\Gamma}{2}\rho_e + \Omega_p\rho_\gamma,$$

$$\frac{\partial\rho_e}{\partial t} = -\Gamma\rho_e + \Omega_p\rho_\gamma - \Omega_c\rho_\beta,$$

$$\frac{\partial\rho_\alpha}{\partial t} = -\gamma\rho_\alpha - \frac{\Omega_c}{2}\rho_\gamma - \frac{\Omega_p}{2}\rho_\beta,$$

$$\frac{\partial\rho_\beta}{\partial t} = -\frac{\Gamma}{2}\rho_\beta - \frac{\Omega_p}{2}\rho_\alpha - \frac{\Omega_c}{2}(\rho_g - 2\rho_e),$$

$$\frac{\partial\rho_\gamma}{\partial t} = -\frac{\Gamma}{2}\rho_\gamma + \frac{\Omega_c}{2}\rho_\alpha + \frac{\Omega_p}{2}(1-\rho_g-\rho_e), \tag{5}$$

and

$$\frac{1}{c}\frac{\partial\Omega_p(x,t)}{\partial t} + \frac{\partial\Omega_p(x,t)}{\partial x} = -\eta\rho_\gamma, \tag{6}$$

$$\frac{1}{c}\frac{\partial\Omega_c(x,t)}{\partial t} + \frac{\partial\Omega_c(x,t)}{\partial x} = \eta\rho_\beta. \tag{7}$$

The ansatz of ADM is to expand the unknown solutions $\Omega_p(x,t)$, $\Omega_c(x,t)$, and $\rho_i(x,t)$ in infinite series:

$$\Omega_p(x,t) = \sum_{n=0}^{\infty}\Omega_p^n(t)x^n,$$

$$\Omega_c(x,t) = \sum_{n=0}^{\infty}\Omega_c^n(t)x^n,$$

$$\rho_i(x,t) = \sum_{n=0}^{\infty}\rho_i^n(t)x^n, \tag{8}$$

where the superscript n designates the order of decomposition. The initial conditions in this problem are

$$\Omega_p^0(t) = \lim_{t\to 0^+}\Omega_p(x=0,\ t>0),$$

$$\Omega_c^0(t) = \Omega_c(\text{const}),$$

$$\rho_i^0(t=0) = \rho_i(x=0,\ t=0) = 0. \tag{9}$$

Let L_x represent $\partial/\partial x$ and L_t represent $\partial/\partial t$. By the assumption of Eq. (8), we rewrite the OBE in the form

$$L_t\rho_g^0 = -\frac{\Gamma}{2}\rho_e^0 + \Omega_p^0\rho_\gamma^0,$$

$$L_t\rho_e^0 = -\Gamma\rho_e^0 + \Omega_p^0\rho_\gamma^0 - \Omega_c\rho_\beta^0,$$

$$L_t\rho_\alpha^0 = -\gamma\rho_\alpha^0 - \frac{\Omega_c}{2}\rho_\gamma^0 - \frac{\Omega_p^0}{2}\rho_\beta^0,$$

$$L_t\rho_\beta^0 = -\frac{\Gamma}{2}\rho_\beta^0 - \frac{\Omega_p^0}{2}\rho_\alpha^0 + \frac{\Omega_c}{2}(\rho_g^0 - 2\rho_e^0),$$

$$L_t\rho_\gamma^0 = -\frac{\Gamma}{2}\rho_\gamma^0 + \frac{\Omega_c}{2}\rho_\alpha^0 + \frac{\Omega_p^0}{2}(1-\rho_g^0-\rho_e^0). \tag{10}$$

By solving Eq. (10) with conditions of Eq. (9), we can get $\rho_i^0(t)$. If we write Eqs. (6) and (7) in the form

$$L_x\Omega_p = -\eta\rho_\gamma - \frac{1}{c}L_t\Omega_p, \tag{11}$$

$$L_x\Omega_c = \eta\rho_\beta - \frac{1}{c}L_t\Omega_c, \tag{12}$$

and operate with the inverse operators L_x^{-1}, we have

$$\Omega_p = -\eta L_x^{-1}\rho_\gamma - \frac{1}{c}L_x^{-1}L_t\Omega_p, \tag{13}$$

$$\Omega_c = \eta L_x^{-1}\rho_\beta - \frac{1}{c}L_x^{-1}L_t\Omega_c. \tag{14}$$

By the assumption of Eq. (8), we get the ordinary differential equation

$$\Omega_p^{n+1}(t) = -\frac{1}{n+1}\eta\rho_\gamma^n(t) - \frac{1}{(n+1)c}L_t\Omega_p^n(t), \tag{15}$$

$$\Omega_c^{n+1}(t) = \frac{1}{n+1}\eta\rho_\beta^n(t) - \frac{1}{(n+1)c}L_t\Omega_c^n(t), \tag{16}$$

where $n \geqslant 0$. Using $\Omega_p^0(t)$, $\Omega_c^0(t)$, $\rho_i^0(t)$ and Eqs. (15) and (16) we can get $\Omega_p^1(t)$ and $\Omega_c^1(t)$.

For the higher orders of ρ_i, by Eq. (8) and the OBE, we have

$$L_t\rho_g^n = -\frac{\Gamma}{2}\rho_e^n + N_x^n[\Omega_p\rho_\gamma],$$

$$L_t\rho_e^n = -\Gamma\rho_e^n + N_x^n[\Omega_p\rho_\gamma] - N_x^n[\Omega_c\rho_\beta],$$

$$L_t\rho_\alpha^n = -\gamma\rho_\alpha^n - \tfrac{1}{2}N_x^n[\Omega_c\rho_\gamma] - \tfrac{1}{2}N_x^n[\Omega_p\rho_\beta],$$

$$L_t\rho_\beta^n = -\frac{\Gamma}{2}\rho_\beta^n - \frac{1}{2}N_x^n[\Omega_p\rho_\alpha] - \tfrac{1}{2}N_x^n[\Omega_c\rho_g] - N_x^n[\Omega_c\rho_e],$$

$$L_t\rho_\gamma^n = -\frac{\Gamma}{2}\rho_\gamma^n + \frac{1}{2}N_x^n[\Omega_c\rho_\alpha] + \tfrac{1}{2}(\Omega_p^n - N_x^n[\Omega_p\rho_g] - N_x^n[\Omega_p\rho_e]), \tag{17}$$

where the nonlinear function $N_x^n[CD]$ is the nth order decomposition of function $C(x,t)$ times $D(x,t)$ with variable x; defined as

$$N_x^n[CD] \equiv \frac{1}{n!}\left[\left(\frac{\partial}{\partial x}\right)^n [C(x,t)\times D(x,t)]\right]_{x=0}. \tag{18}$$

For example, if $C(x,t)=\Sigma C^n(t)x^m$, and $D(x,t)=\Sigma D^n(t)x^m$, then $N_x^1[CD]=C^0(t)D^1(t)+C^1(t)D^0(t)$.

In summary, our iteration procedure is to first apply the initial condition $\Omega_i^0(t)$ to Eq. (10) and obtain $\rho_i^0(t)$. Then from Eqs. (15) and (16) we obtain $\Omega_i^1(t)$; the next order $\rho_i^1(t)$ is given through Eq. (17), and so on. Schematically: $\Omega_i^0(t)\to\rho_i^0(t)\to\Omega_i^1(t)\to\rho_i^1(t)\cdots\Omega_i^k(t)\to\rho_i^k(t)\to\Omega_i^{k+1}(t)\to\rho_i^{k+1}(t)\cdots$. We finally construct the functions $\Omega_i(x,t)$ and $\rho_i(x,t)$ by Eq. (8).

In the next section we show the method of solving differential equations with variable t by ADM.

III. DECOMPOSITION OF THE TIME VARIABLE

Expand the decomposition orders $\Omega_p^n(t), \Omega_c^n(t)$, and $\rho_i^n(t)$ in series:

$$\Omega_p^n(t) = \sum_{m=0}^{\infty}\Omega_p^{n,m}t^m,$$

$$\Omega_c^n(t) = \sum_{m=0}^{\infty}\Omega_c^{n,m}t^m,$$

$$\rho_i^n(t) = \sum_{m=0}^{\infty}\rho_i^{n,m}t^m, \tag{19}$$

and with the initial conditions

$$\Omega_p^0(t) = \sum_{m=0}^{\infty}\Omega_p^{0,m}t^m, \tag{20}$$

$$\Omega_c^0(t) = \Omega_c^{0,0} = \Omega_c \tag{21}$$

$$\rho_i^{0,m} = 0.$$

So the coefficients $\Omega_i^{0,m}$ are all known.

Now, operate the inverse operators L_t^{-1} on both sides of Eq. (10), we have

$$\rho_g^0 = \rho_g^{0,0} + L_t^{-1}\left\{-\frac{\Gamma}{2}\rho_e^0 + \Omega_p^0\rho_\gamma^0\right\},$$

$$\rho_e^0 = \rho_e^{0,0} + L_t^{-1}\{-\Gamma\rho_e^0 + \Omega_p^0\rho_\gamma^0 - \Omega_c\rho_\beta^0\},$$

$$\rho_\alpha^0 = \rho_\alpha^{0,0} + L_t^{-1}\left\{-\gamma\rho_\alpha^0 - \frac{\Omega_c}{2}\rho_\gamma^0 - \frac{\Omega_p^0}{2}\rho_\beta^0\right\},$$

$$\rho_\beta^0 = \rho_\beta^{0,0} + L_t^{-1}\left\{-\frac{\Gamma}{2}\rho_\beta^0 - \frac{\Omega_p^0}{2}\rho_\alpha^0 - \frac{\Omega_c}{2}(\rho_g^0 - 2\rho_e^0)\right\},$$

$$\rho_\gamma^0 = \rho_\gamma^{0,0} + L_t^{-1}\left\{-\frac{\Gamma}{2}\rho_\gamma^0 + \frac{\Omega_c}{2}\rho_\alpha^0 + \frac{\Omega_p^0}{2}(1-\rho_g^0-\rho_e^0)\right\}. \tag{22}$$

Substitute Eq. (19) into Eq. (22), we obtain

$$\rho_g^{0,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_e^{0,m} + N_t^m[\Omega_p^0\rho_\gamma^0]\right\},$$

$$\rho_e^{0,m+1} = \frac{1}{m+1}\{-\Gamma\rho_e^{0,m} + N_t^m[\Omega_p^0\rho_\gamma^0] - N_t^m[\Omega_c^0\rho_\beta^0]\},$$

$$\rho_\alpha^{0,m+1} = \frac{1}{m+1}\left\{-\gamma\rho_\alpha^{0,m} - \tfrac{1}{2}N_t^m[\Omega_c^0\rho_\gamma^0] - \tfrac{1}{2}N_t^m[\Omega_p^0\rho_\beta^0]\right\},$$

$$\rho_\beta^{0,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_\beta^{0,m} - \frac{1}{2}N_t^m[\Omega_p^0\rho_\alpha^0] + \frac{1}{2}N_t^m[\Omega_c^0\rho_g^0] - N_t^m[\Omega_c^0\rho_e^0]\right\},$$

$$\rho_\gamma^{0,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_\gamma^{0,m} + \frac{1}{2}N_t^m[\Omega_c^0\rho_\alpha^0] + \frac{\Omega_p^{0,m}}{2} - \frac{1}{2}N_t^m[\Omega_p^0\rho_g^0] - \frac{1}{2}N_t^m[\Omega_p^0\rho_e^0]\right\}, \tag{23}$$

where the nonlinear function $N_t^n[EF]$ is defined as the nth order decomposition of function $E(t)$ and by multiplying $F(t)$ for variable t,

$$N_t^n[EF] \equiv \frac{1}{n!}\left[\left(\frac{d}{dt}\right)^n [E(t)\times F(t)]\right]_{t=0}. \tag{24}$$

For instance, if $E(t)=\Sigma E^m t^m$ and $F(t)=\Sigma F^m t^m$, then $N_t^2[EF]=E^0F^2+E^1F^1+E^2F^0$. From the initial conditions and Eq. (23), the functions $\rho_i^0(t)$ are obtained.

Substitute Eq. (19) into Eqs. (15) and (16), we get

$$\sum_{m=0}\Omega_p^{n+1,m}t^m = -\frac{1}{n+1}\eta\sum_{m=0}(\rho_\gamma^{n,m}t^m) - \sum_{m=0}\left(\frac{m}{(n+1)c}\Omega_p^{n,m-1}t^{m-1}\right), \tag{25}$$

$$\sum_{m=0}\Omega_c^{n+1,m}t^m = \frac{1}{n+1}\eta\sum_{m=0}(\rho_\beta^{n,m}t^m) - \sum_{m=0}\left(\frac{m}{(n+1)c}\Omega_c^{n,m-1}t^{m-1}\right), \tag{26}$$

or

$$\Omega_p^{n+1,m} = -\frac{1}{n+1}\eta\rho_\gamma^{n,m} - \frac{m+1}{(n+1)c}\Omega_p^{n,m+1}, \tag{27}$$

$$\Omega_c^{n+1,m} = \frac{1}{n+1}\eta\rho_\beta^{n,m} - \frac{m+1}{(n+1)c}\Omega_c^{n,m+1}. \tag{28}$$

With the initial conditions $\rho_\gamma^{0,m}$, $\Omega_p^{0,m}$ and $\Omega_c^{0,m}$, the next order $\Omega_p^{1,k}$ and $\Omega_c^{1,k}$ for any k are derived from the above equation. Then $\Omega_p^1(t)$ and $\Omega_c^1(t)$ can be derived. Similar procedures, $\Omega_p^n(t)$ and $\Omega_c^n(t)$ for $n\geqslant 2$, can be found.

To find higher orders $\rho_i^m(t)$, we operate the inverse operators L_t^{-1} to both sides of Eq. (17), then

$$\rho_g^{n,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_e^{n,m} + N^{n,m}[\Omega_p\rho_\gamma]\right\},$$

$$\rho_e^{n,m+1} = \frac{1}{m+1}\{-\Gamma\rho_e^{n,m} + N^{n,m}[\Omega_p\rho_\gamma] - N^{n,m}[\Omega_c\rho_\beta]\},$$

$$\rho_\alpha^{n,m+1} = \frac{1}{m+1}\left\{-\gamma\rho_\alpha^{n,m} - \tfrac{1}{2}N^{n,m}[\Omega_c\rho_\gamma] - \tfrac{1}{2}N^{n,m}[\Omega_p\rho_\beta]\right\},$$

$$\rho_\beta^{n,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_\beta^{n,m} - \frac{1}{2}N^{n,m}[\Omega_p\rho_\alpha] + \frac{1}{2}N^{n,m}[\Omega_c\rho_g^{n,m}] - N^{n,m}[\Omega_c\rho_e^{n,m}]\right\},$$

$$\rho_\gamma^{n,m+1} = \frac{1}{m+1}\left\{-\frac{\Gamma}{2}\rho_\gamma^{n,m} + \frac{1}{2}N^{n,m}[\Omega_c\rho_\alpha] + \frac{\Omega_p^{n,m}}{2} - \frac{1}{2}N^{n,m}[\Omega_p\rho_g] - \frac{1}{2}N^{n,m}[\Omega_p\rho_e]\right\}, \tag{29}$$

where the nonlinear function $N^{n,m}[CD]$ is defined as the nth order in variable x and mth order in variable t of function $C(x,t)$; multiply $D(x,t)$,

$$N^{n,m}[CD] \equiv N_t^m[N_x^n[CD]] = \frac{1}{n!\,m!}\left[\left(\frac{\partial}{\partial x}\right)^n\left(\frac{\partial}{\partial t}\right)^m [C(x,t)D(x,t)]\right]_{x=0,t=0}. \tag{30}$$

For example, if $C(x,t)=\Sigma C^{n,m}x^n t^m$, and $D(x,t)=\Sigma D^{n,m}x^n t^m$, then $N^{1,2}[CD]=(C^{0,0}D^{1,2}+C^{0,1}D^{1,1}+C^{0,2}D^{1,0})+(C^{1,0}D^{0,2}+C^{1,1}D^{0,1}+C^{1,2}D^{0,0})$.

We now already have enough information to obtain all the functions. For a known $\Omega_p^k(t)$ and $\Omega_c^k(t)$, our procedure starts from the initial condition $\rho_i^{k,0}$, then proceeds schematically $\rho_i^{k,0}\to\rho_i^{k,1}\to\cdots\to\rho_i^{k,m},\ldots,$ to construct the functions $\rho_i^k(t)$. We use $\rho_\gamma^{k,m}$ and $\Omega_p^{k,m+1}$ to obtain $\Omega_p^{k+1,m}$, and use $\rho_\beta^{k,m}$ and $\Omega_c^{k,m+1}$ to obtain $\Omega_c^{k+1,m}$, to construct the higher decomposition function $\Omega_p^{k+1}(t)$ and $\Omega_c^{k+1}(t)$.

Combine the decomposition x and t method, all the functions $\Omega_i(x,t)$ and $\rho_i(x,t)$, and hence the solution to the coupled equations are obtained.

IV. THE TIME DOMAIN PARTITION METHOD

To justify the convergence of series expansion of t, we divide the time domain $[0,T]$ into a union of q partitions $[T_{m-1},T_m]$, $m=1,2,3,\ldots,q$. And the electric field function $\Omega_i^n(t)$ and density functions $\rho_i^n(t)$ in the mth partition are denoted as $\Omega_{i,m}^n(t)$ and $\rho_{i,m}^n(t)$ with $t\in[T_{m-1},T_m]$. The global solution is given by

$$\begin{aligned} \Omega_i^n(t) &= \sum_{m=1}^{q} \Omega_{i,m}^n(t)\chi_m(t), \\ & \qquad\qquad\qquad\qquad t \in [0,T], \\ \rho_i^n(t) &= \sum_{m=1}^{q} \rho_{i,m}^n(t)\chi_m(t), \end{aligned} \tag{31}$$

where

$$\chi_m(t) = \begin{cases} 1, & t \in [T_{m-1}, T_m], \\ 0, & t \neq [T_{m-1}, T_m], \end{cases} \tag{32}$$

and

$$[0,T] = \bigcup_{m=1}^{q} [T_{m-1}, T_m]. \tag{33}$$

We reset the connection conditions at $t=T_m$ as

$$\begin{aligned} \Omega_{i,m}^n(\Delta_m T) &= \Omega_{i,m+1}^n(0), \\ \rho_{i,m}^n(\Delta_m T) &= \rho_{i,m+1}^n(0), \end{aligned} \tag{34}$$

where $\Delta_m T = T_m - T_{m-1}$ is the length of the interval $[T_{m-1}, T_m]$. With this partition method, we overcome the problem of convergence in time domain. The method was developed in our previous treatment of ADM eigenvalue problems [6].

In the following, we consider a Gaussian temporal pulse passing through the coherent medium. The initial condition of the field is

$$\Omega_p^0(t>0) = \Omega_0 \exp\left[-\left(\frac{t-t_0}{\sqrt{2}\tau}\right)^2\right], \tag{35}$$

$$\Omega_c^0(t) = \Omega_c. \tag{36}$$

Our first example is a weaker field case so that we can compare the results with perturbation theory. We take $\Omega_0 = 0.01\Gamma, t_0 = 3\tau, \tau = 240/\Gamma, \gamma = 0.001\Gamma$, and $\eta = 9.06\Gamma/L$, where L is the length of the medium. Here $L = 780$ μm, and $\Gamma = 3.77 \times 10^7$/sec; $\Omega_c = 0.3\Gamma$ is the Rabi frequency of the coupling laser.

Notice that the reset initial condition for each interval is

$$\begin{aligned} \Omega_{p,m}^0(t) &= \Omega_p^0(T_{m-1}+t) \\ &= \Omega_0 e^{-[(T_{m-1}+t-t_0)/\sqrt{2}\tau]^2}, \quad t \in [0, \Delta_m T]. \end{aligned} \tag{37}$$

We choose $T = 6\tau$, and slice the time domain into $q = 24\,000$ parts. The choice of the number q is related to the size of time slice $\Delta_m T$. Assume the machine accuracy of a numerical computation is ϵ, then after propagation of q steps, the accumulative error is estimated to be $O(q \times \epsilon)$. Thus, the smaller value of q provides higher accuracy than larger value of q. On the other hand, smaller q means larger value of $\Delta_m T$. To reach the expected accuracy, the ADM expansion must go to higher order. For nonlinear PDE, higher order terms are complicated. The compromise is to choose a larger value of q and keep modest order in expansion so that proper accuracy can be obtained. If the accuracy criterion is set to $O(10^{-10})$, and $\epsilon = 10^{-16}$, we need $q \leqslant O(10^6)$.

In Fig. 1, we depict the results of $\Omega_p(x,t)$ in Eq. (8) and its associated decompositions up to the sixth order at the end

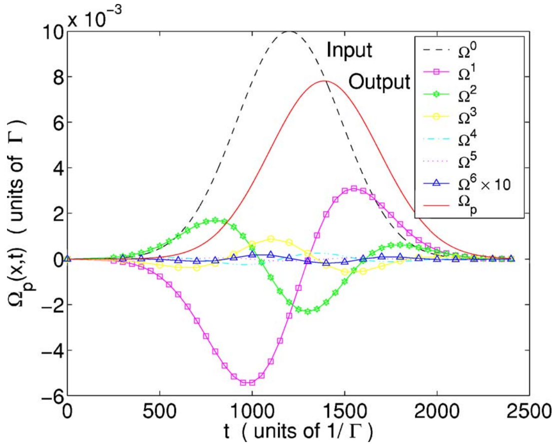

FIG. 1. (Color online) The probe field in Eq. (8). The curve Input denotes the probe pulse at the beginning end $x=0$, and the curve Output is at the end of medium $x=L$. Also shown are decomposition orders. The labels Ω^n are shorthand for Ω_p^n in Eq. (8). The parameters are $\Omega_0 = 0.01\Gamma$, $\Omega_c = 0.3\Gamma$, $\tau = 240/\Gamma$, $\gamma = 0.001\Gamma$, and $\eta = 9.06\Gamma/L$.

point of medium $x=L$. Note that Ω_p^0 is the input pulse at the beginning end $x=0$. The peak ratio of $\Omega_p(x=0,t)$ to $\Omega_p(x=L,t)$ agrees with the spatial variation from the perturbational theory and will be discussed later. We can also see that the magnitude decreases from order to order and the magnitude of the sixth order is negligibly small. Thus, the convergence in expansion orders is justified. Figure 2 shows the time evolutional results of states probabilities and the coherence at the end $x=L$. During the calculation, the spatial domain expansion order is 6 and the time domain expansion order is 4. The numerical error in time is less than 10^{-9}, and is less than 10^{-3} in spatial expansion.

A. The perturbation theory

In the previous calculation, the amplitude of control field Ω_c has the same order of magnitude of Γ and is 30 times of

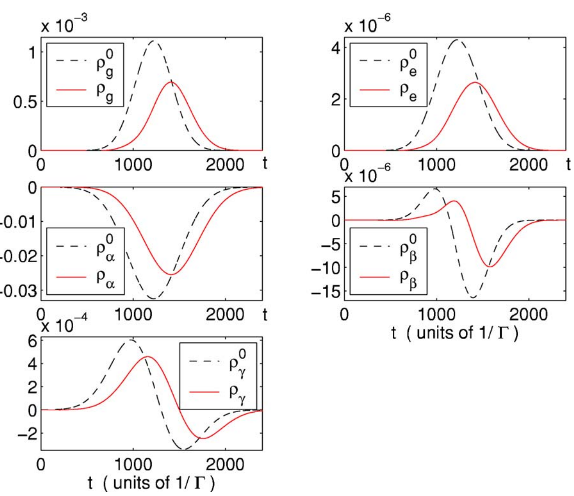

FIG. 2. (Color online) The density functions $\rho_i(t)$ as functions of time at the end of medium $x=L$. Also shown are $\rho_i^0(t)$ at the beginning end $x=0$. The parameters are the same as Fig. 1. The meanings of indices are described in the text.

the probe pulse Ω_p. We have $\Omega_p \ll \Omega_c, \Gamma$. Due to the optical pumping effect by the strong control field, all population is nearly in the $|1\rangle$ state and, hence, $\rho_g, \rho_e \ll 1$. The optical coherence is qualitatively proportional to the Rabi frequency of the driving field multiplied by the ground-state population. The population in the $|1\rangle$ state is driven by the very weak Ω_p and the negligible ρ_g is driven by the control field. It is reasonable that $\rho_\beta, \rho_\gamma \ll \rho_\alpha$ and the change rates of ρ_β and ρ_γ are much smaller than Ω_c and Γ. Under the above conditions, we neglect the term of $\Omega_p \rho_\beta / 2$ in the third equation of Eq. (5); $1-\rho_g-\rho_e \approx 1$ and the terms of $\partial\rho_\gamma/\partial t$ and $\Gamma\rho_\gamma/2$ are negligible in the fifth equation of Eq. (5). We then have

$$\frac{\partial \rho_\alpha}{\partial t} \simeq -\gamma\rho_\alpha - \frac{\Omega_c}{2}\rho_\gamma,$$

$$0 \simeq \frac{\Omega_c}{2}\rho_\alpha + \frac{\Omega_p}{2}. \tag{38}$$

By using the results of Eq. (38) and the condition that Ω_c is a constant, we obtain

$$\rho_\gamma \simeq \frac{2}{\Omega_c^2}\frac{\partial \Omega_p}{\partial t} + \frac{2\gamma}{\Omega_c^2}\Omega_p. \tag{39}$$

Plug this expression ρ_γ into MSE of Eq. (6), the traveling wave equation becomes

$$\left(\frac{1}{c} + \frac{2\eta}{\Omega_c^2}\right)\frac{\partial \Omega_p(x,t)}{\partial t} + \frac{\partial \Omega_p(x,t)}{\partial x} = -\frac{2\gamma\eta}{\Omega_c^2}\Omega_p, \tag{40}$$

so we find that the group velocity v_g satisfies

$$\frac{1}{v_g} = \frac{1}{c} + \frac{2\eta}{\Omega_c^2}. \tag{41}$$

The amplitude as a function of the position is

$$\Omega_p(x,t) = \Omega_p(t)\exp\left(-\frac{2\gamma\eta x}{\Omega_c^2}\right). \tag{42}$$

B. The comparison of ADM and perturbational results

If $2\eta/\Omega_c^2 \gg 1/c$, with the physical parameters described above, we then have

$$v_g \simeq \frac{\Omega_c^2}{2\eta} = 145.9 \text{ m/s}. \tag{43}$$

The traveling time of the pulse through the medium is L/v_g, in the dimensionless unit

$$r\Delta t = \frac{L}{v_g} = 201.5. \tag{44}$$

At $x=L$, the decay factor is

$$\exp(-\gamma\Delta t) = \exp(-0.202) = 0.817. \tag{45}$$

From the ADM data presented in Fig. 1, the decay in the peak heights of the probe pulse at the input and output position (curves labeled with Input and Output), agrees quite well with the perturbation formula of Eq. (45). Our ADM results also reproduce the experimental data as shown in Fig. 3 with parameters described in the captions of that figure.

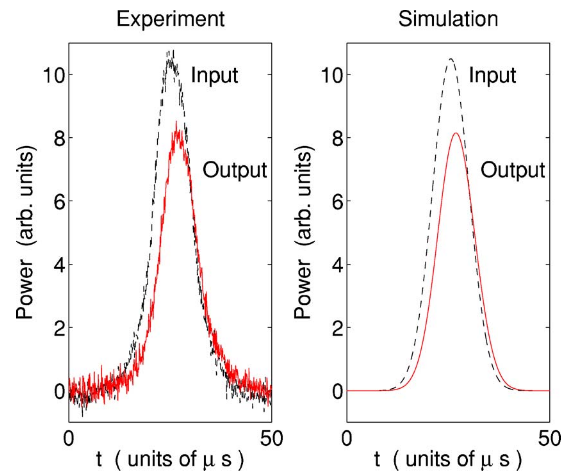

FIG. 3. (Color online) The comparison of the experimental data and numerical simulation. $\Omega_c=0.35\pm0.03\Gamma$, $\Omega_0=0.07\pm0.01\Gamma$, $\eta=3.6\pm0.2\Gamma/L$, $\gamma=(3\pm1)\times10^{-3}\Gamma$, and $\tau=240/\Gamma$ are determined experimentally [7]. $\Omega_c=0.35\Gamma$, $\Omega_0=0.07\Gamma$, $\eta=3.6\Gamma/L$, $\gamma=2\times10^{-3}\Gamma$, and $\tau=240/\Gamma$ are used in the calculation.

V. THE SPATIAL DOMAIN PARTITION METHOD

In the next simulation, we increase the light pulse amplitude but reduce the width of the pulse, so that the perturbation theory may not work. The direct use of the same ADM as described previously is divergent in spatial series. To guarantee the convergence of series expansion in x, we divide the domain $[0,X]$ into r partitions $[X_{m-1},X_m]$, $m=1,2,3,\ldots,r$, just like the method we developed for the nonlinear eigenvalue problem of the Gross-Pitaevskii equation [6]. So the field function $\Omega_i(x,t)$ and density functions $\rho_i(x,t)$ in the mth partition are denoted as $\Omega_{i,m}(x,t)$ and $\rho_{i,m}(x,t)$ with $t \in [T_{m-1},T_m]$. The global solutions are given by

$$\begin{aligned} \Omega_i(x,t) &= \sum_{m=1}^{q} \Omega_{i,m}(x,t)\chi_m(x), \\ \rho_i(x,t) &= \sum_{m=1}^{q} \rho_{i,m}(x,t)\chi_m(x), \end{aligned} \quad x \in [0,X], \tag{46}$$

where

$$\chi_m(x) = \begin{cases} 1, & x \in [X_{m-1},X_m], \\ 0, & x \neq [X_{m-1},X_m], \end{cases} \tag{47}$$

and

$$[0,X] = \bigcup_{m=1}^{r} [X_{m-1},X_m]. \tag{48}$$

The connection conditions at $x=X_m$ are

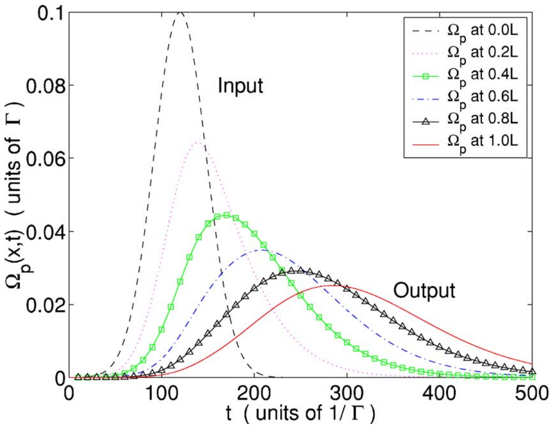

FIG. 4. (Color online) The pulse shape of fields passing through a medium of Λ-type three-level atoms at various places in the medium. Ω_p are shorthand for $\Omega_p(x,t)$ at labeled positions. The parameters are $\Omega_0=0.1\Gamma$, $\Omega_c=0.3\Gamma$, $\tau=28/\Gamma$, $\gamma=0.001\Gamma$, and $\eta=9.06\Gamma/L$.

$$\Omega_{i,m}(\Delta_m X,t) = \Omega_{i,m+1}(0,t),$$

$$\rho_{i,m}(\Delta_m X,t) = \rho_{i,m+1}(0,t), \tag{49}$$

where $\Delta_m X = X_m - X_{m-1}$ is the length of the interval $[X_{m-1}, X_m]$. With the spatial partition method, the convergence problem in the series expansion of the spatial domain is resolved. With the above condition for light pulse, we divide the space into ten partitions and solve the coupled OBE and MSE by ADM. We show in Fig. 4 the time evolutions of a Gaussian pulse passing through a medium of Λ-type three-level atoms at various positions in the medium. In the simulation, Ω_0 is 0.1Γ which is ten times larger than the case in Fig. 1, and with much shorter width $\tau=28/\Gamma$. Interestingly, we find that the stronger and narrower light pulse is now more diffused by the medium than the case in the previous section. Experiments show that the EIT frequency transmission window is narrower compared to the previous parameter set of a weaker field case. The narrower

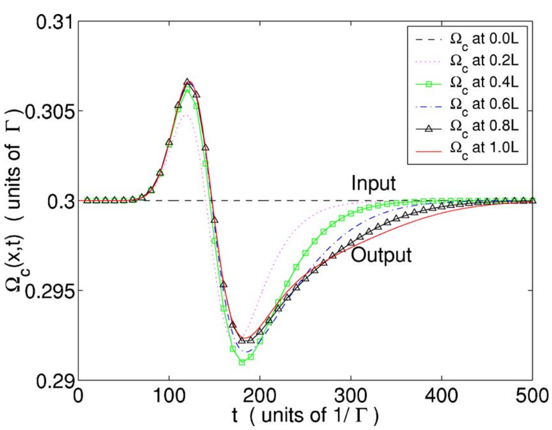

FIG. 5. (Color online) The time evolutions of the control field at several places in the medium of Λ-type three-level system. Ω_c are shorthand for $\Omega_c(x,t)$ at labeled positions. The parameters are the same as Fig. 3.

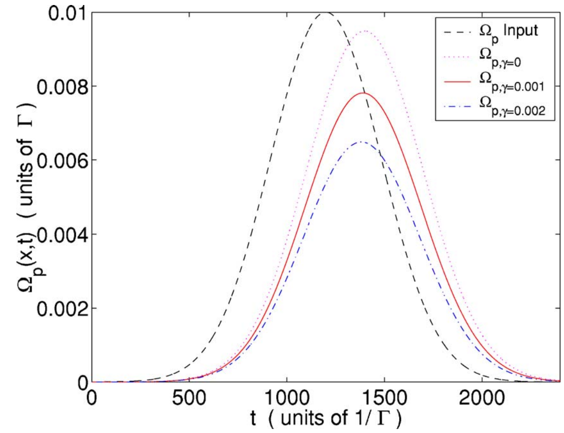

FIG. 6. (Color online) The input Ω_c $(x=0,t)$ and output probe pulses Ω_c $(x=L,t)$ at $\gamma=0$, 0.001, and 0.002Γ. The other parameters are equal to those of Fig. 1.

frequency band transmitted causes the pulse shape to be broader in the time domain. This phenomenon has not been explained by the perturbation calculation.

Figure 5 depicts the time evolutions of the control field at several places in the medium. At the beginning end $(x=0)$, the field is fixed at the level of 0.3. The limiting case with $\gamma=0$ was shown in [8]. Our results take γ into consideration; γ can affect the amplitude of output probe pulses significantly. The behavior of the output probe pulse in the medium is quite different from those calculated with $\gamma=0$. To exhibit the effect of γ, we show in Fig. 6 the results of $\gamma=0$, 0.001, and 0.002Γ and the other parameters are set equal to those of Fig. 1. The output probe pulse amplitude of the case with nonvanishing γ is significantly lower than the case with $\gamma=0$.

Figure 7 shows the comparisons between calculation and experimental results in real time. Significantly different from the case shown in Fig. 3, the output probe pulse is broadened

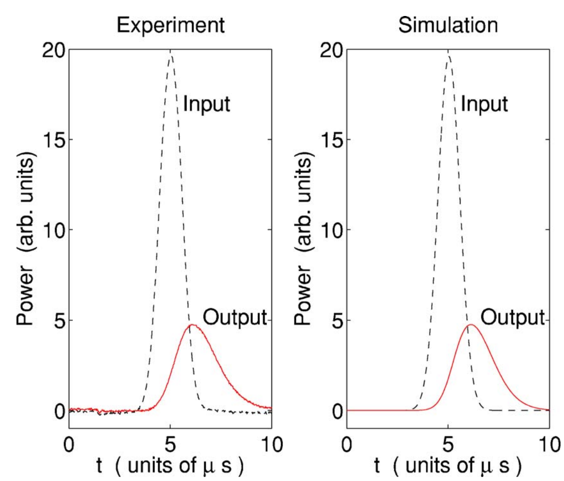

FIG. 7. (Color online) The comparison of the experimental data and numerical simulation. $\Omega_c=0.30\pm0.03\Gamma$, $\Omega_0=0.10\pm0.01\Gamma$, $\eta=3.6\pm0.2\Gamma/L$, $\gamma=(3\pm1)\times10^{-3}\Gamma$, and $\tau=28/\Gamma$ are determined experimentally [7]. The experimental data are taken through a 20-MHz low-pass filter. $\Omega_c=0.33\Gamma$, $\Omega_0=0.10\Gamma$, $\eta=3.6\Gamma/L$, $\gamma=2\times10^{-3}\Gamma$, and $\tau=28/\Gamma$ are used in the calculation.

and asymmetrical with a longer tail. The agreement between the experimental data and the theoretical prediction is satisfactory. The agreements in both weak and stronger field cases are excellent. This justifies that our ADM method works well for either perturbative or nonperturbative cases.

VI. CONCLUSION

We show the Adomian's decomposition method can provide semianalytical solutions for the problems of light pulse passing through a medium of Λ-type three-level atoms. Unlike other numerical grids methods, the ADM technique gives explicit forms of solution. The direct use of the power series expansion method (or modified decomposition method) may not be able to solve the partial differential equation straightforwardly. In that case, the spatial partition method is a recipe to provide convergent results.

In summary, we have developed in this paper a new and efficient algorithm to solve the coupled partial differential equations of MSE and OBE for the three-level EIT problem. All the computations reported here are carried out on a personal computer. The four levels are related to the problems; such as *photon switching by quantum interference* [12]. It is far more complicated than the three-level system but is interesting. The method of the solution is currently under development and will be reported in the future.

ACKNOWLEDGMENTS

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