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Critical Transmission Radius for Topology Control in Large-Scale Wireless Ad Hoc Networks

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中英文摘要

摘要：為了有效的運作無線隨意網路系統，通常我們會建立及維護一個虛擬的網路骨幹，這個虛擬骨幹只是網路拓樸的一部份。我們稱一個網路拓樸的子集為架構圖（spanner）如果任兩設備在此子集中的總傳輸代價（譬如：距離或傳輸能量等）可控制在只比原來完整網路中的代價多出有限的倍數。許多的幾何結構，如：歐拉最小生成樹（EMST）、相關鄰近圖（relative neighbor graph）、蓋伯圖（Gabriel graph）、Delauney 三角化圖（Delauney triangulation）及姚氏圖（Yao's graph）等皆廣泛用於建構架構圖。與建構及維護虛擬骨幹相關的學問則稱為拓樸控制。一個拓樸控制的演算法如果只需收集及處理局部的資訊則稱為區域化演算法。

在很多的應用中，隨意設備是由電池提供電力，而且一般來說在，更換電池或為電池充電是不切實際且不可行的。因此提高能源的使用效率可增長無線設備及系統的使用壽命，其中一個重要的因子即是傳輸耗能的多寡。在這個計畫中，我們給定了最小的傳輸半徑使其所引致的網路拓樸可讓我們設計只需直接相鄰結點訊息的區域化分散式演算法來建構多種的幾何結構。這最小的傳輸半徑我們稱為臨界傳輸半徑。在本文中，我們將介紹本計畫在網路連通性、感測網路的覆蓋性及蓋伯圖的最長邊等主題上所得的研究結果。

關鍵詞：無線隨意及感測網路、拓樸控制、孤立點、覆蓋性、蓋伯圖。

ABSTRACT: To efficiently operate wireless ad hoc networks, subsets of network topology, called virtual backbones, will be constructed and maintained. A spanner is a subset of the network topology in which the minimal total cost, e.g. distance or energy consumption, to transmit packets between any pair of nodes is only a constant fact larger than in the original network topology. Hence spanners are good candidates of virtual backbones. Geometric structures, including Euclidean minimal spanning trees (EMST), relative neighbor graphs (RNG), Gabriel graphs (GG), Delauney triangulations (DT), Yao's graphs (YG), etc., are widely used ingredients to construct spanners. The related topics about how to construct and maintain virtual backbones are called topology control. A topology control algorithm is localized if each node only needs to collect information from few hops neighbors.

In many applications, ad hoc devices are powered by batteries. Usually, it is impractical or even not possible to exchange or charge the batteries. To elongate the lifetime of wireless devices and systems, energy efficiency is one of the most important issues. A major approach is to communicate with partners using the smallest transmission power. In this work, we gave the minimal transmission radius that allows us to construct the geometric structures by localized and distributed algorithms using only 1-hop neighbor information. The minimal transmission radius is called a critical transmission radius. In the report, we are going to brief our results on connectivity, coverage, and the longest Gabriel edge.

Keywords: Wireless ad hoc and sensor networks, topology control, isolated nodes, coverage, Gabriel graphs.

I. INTRODUCTION

A wireless ad hoc network is a collection of radio devices located in a geographic region. Each node is equipped with an omnidirectional antenna and has limited transmission power. A communication session is established either through a single-hop radio transmission if the communication parties are close enough, or through relaying by intermediate devices otherwise. Because of no need for a fixed infrastructure, wireless ad hoc networks can be flexibly deployed at low cost for varying missions such as decision making in the battlefield, emergency disaster relief and environmental monitoring. In most applications, the ad hoc wireless devices are deployed in a large volume. The sheer large number of devices deployed coupled with the potential harsh environment often hinders or completely eliminates the possibility of strategic device placement, and consequently, random deployment is often the only viable option. In some other applications, the ad hoc wireless devices may be continuously in motion. For all these applications, it is natural to represent the ad hoc devices by a finite random point process over the (finite) deployment region. Correspondingly, the wireless ad hoc network is represented by a random geometric graph.

In wireless ad hoc networks, due to constrain of hardware, each node is associated with a maximal transmission radius (range) which is corresponding to the maximal transmission power of the device. Two nodes in the network can communicate directly if they are within each other’s transmission range. Therefore, the induced network topology is a disk graph in which two nodes have an edge between them if the distance between them is not larger than their transmission ranges. Furthermore, if all devices have the same transmission radius r and let V denote the set of nodes, then the network topology is exactly the r-disk graph over V, denoted by $G_r(V)$.

The required cost and resource for maintaining a whole network topology is expensive. In the general case, a subset of the topology is enough to operate the network. Therefore, in order to efficiently operate such a network, only a subset of network topology, called a virtual backbone, will be constructed and maintained. The related topics about how to construct and maintain virtual backbones are called topology control. A spanner is a subset of the network topology in which the minimal total cost to transmit packets between any pair of nodes, e.g. distance or energy consumption is only a constant fact larger than the minimal total cost in the original network topology. Hence spanners are good candidates of virtual backbones. Geometric structures, including Euclidean minimal spanning trees (EMST), relative neighbor graphs (RNG), Gabriel graphs (GG), Delauney triangulations (DT), Yao's graphs (YG), etc., are widely used ingredients to construct spanners [1, 2, 3].

In addition, wireless ad hoc devices are powered by batteries in many applications. Usually, it is impractical or even not possible to exchange or charge the batteries. For such applications, energy efficiency is one of the most important issues. By increasing the efficiency of energy, we can elongate the lifetime of wireless devices and systems. For the sake of power saving, it is interesting to find the minimal transmission radius such that the induced network topology has some desired good properties, e.g. connectivity of communication networks, coverage of wireless sense systems, and the possibility of constructing geometric structures by 1-hop information.

II. OBJECTIVES

A. The Number of Isolated Nodes

For a randomly-deployed wireless ad hoc network, the uncertainty of network connectivity is the first problem we need to face. A network can't function well if the induced network topology is not connected. The critical transmission radius for connectivity is the smallest transmission radius r such that the induced r-disk graph is connected. It was known that if an r-disk graph over a random point set is with no isolated node, it is connected with high probability. Thus, to investigate the connectivity of wireless ad hoc networks, we are interested in the number of isolated nodes. In a realistic system, nodes may become inactive, and links may become down. These inactive nodes and down links cannot take part in routing/relaying and thus may affect the connectivity. However, this aspect was not discussed in the most related research works. In our works, we gave the asymptotic probability distribution of the number of isolated nodes in networks with unreliable nodes and/or links.

B. Coverage by Randomly Deployed Wireless Sensor Networks

One of the main applications of wireless sensor networks is to provide proper coverage of their deployment regions. A wireless sensor network k-covers its deployment region if every point in its deployment region is within the coverage ranges of at least k sensors. In a randomly-deployed wireless sensor networks, the coverage is a random variable. Assume that the sensors are deployed as either a Poisson point process or a uniform point process in a square or disk region. We studied how the probability of the k-coverage changes with the sensing radius or the number of sensors.

C. The Longest Edge of Gabriel Graphs

In wireless ad hoc networks, without fixed infrastructures, virtual backbones are constructed and maintained to efficiently operate such networks. The Gabriel graph (GG) is one of widely used geometric structures for topology control in wireless ad hoc networks. In the GG, two nodes have an edge between them if and only if there is no other node on the disk using the segment of these two nodes as its diameter. Assume all nodes are with the same maximal transmission radius r. To construct the GG only by 1-hop neighbor information, the transmission radius r should be large enough such that the GG is a subgraph of the r-disk graph. Thus, the transmission radius should be not less than the length of the longest edge of the GG. On the other hand, for each node, if it can gather the information of nodes that are not farther than its farthest neighbor in the GG, it can decide all GG edges incident to it. Therefore, the length of the longest edge of the GG is the minimal transmission radius such that the GG can be constructed by using only 1-hop neighbor information and is called the critical transmission radius for the GG.

III. RELATED WORKS

The theory of random graphs was initiated by Erdős and Rényi (1969) [4], in which each pair of vertices is joined by an edge independently and uniformly at some probability. Since then hundreds of papers have been published in various areas of random graph theory. The monograph of Bollobás (2001) [5] and the treatise of Janson et al. (2000) [6] are excellent studies about random graphs. A broad survey of the probabilistic analysis of algorithms can be found in the articles by McDiarmid and Frieze (1997) [7] and Frieze and Reed (1998) [8].

The classic random graphs, however, is not suited to accurately represent networks of short-range radio nodes due to the presence of local correlation among radio links. This motivated Gilbert (1961) [9] to propose an alternative model. Gilbert's model assumes that all devices, represented by an infinite random point process over the entire plane, have the same maximum transmission radius r and two devices are joined by an edge if and only if their distance is at most r. For the modeling of wireless ad hoc networks which consist of finite radio nodes in a bounded geographic region, a bounded (or finite) variant has been used by Gupta and Kumar (1998) [10] and others. In this variant, by proper scaling, the random point process representing the ad hoc devices is typically assumed to be a uniform point process or a Poisson point process with density n over a disk or a square with unit area, denoted by $\mathcal{X}_n$ and $\mathcal{P}_n$ respectively, and the wireless ad hoc network is exactly the r-disk graph over $\mathcal{X}_n$ or $\mathcal{P}_n$, denoted by $G_r(\mathcal{X}_n)$ and $G_r(\mathcal{P}_n)$, respectively.

Before, in contract to the prosperity in random graphs, random geometric graphs have received only sporadic theoretic studies. In recent years, more and more researchers have been working on this field. The first well studied problem in random geometric graphs is about the connectivity of the random geometric graph. For any constant ξ, Dette and Henze (1989) [11] showed that

$$\lim_{n\to\infty}\Pr\left(\text{no isolated nodes in } G_{\sqrt{\frac{\ln n+\xi}{n\pi}}}\left(\mathcal{X}_n\right)\right)=\exp\left(-e^{-\xi}\right).$$

Penrose (1997) [12] established that if a random geometric graph of $G_r\left(\mathcal{X}_n\right)$ has no isolated nodes, it is almost surely connected. In other words,

$$\lim_{n\to\infty}\Pr\left(G_{\sqrt{\frac{\ln n+\xi}{n\pi}}}\left(\mathcal{X}_n\right)\text{is connected}\right)=\exp\left(-e^{-\xi}\right).$$

The same results also stand for $G_{\frac{\sqrt{\ln n+\xi}}{n\pi}}\left(\mathcal{P}_n\right)$. Shortly after, Penrose (1999) [13] extended this result and proved that for any $k\geq 2$, if the minimal node degree is not less than k, the random geometric graph is asymptotically almost surely k-connected. Later, Wan and Yi (2004) [14] derived the asymptotical probability distribution of the critical transmission radius for k-connected.

In addition to k-connectivity, k-coverage is another interesting topology problem in wireless sensor networks. The probabilistic studies of k-coverage by a random point process have been conducted for $k=1$ in [15] and arbitrary integer-valued constant k in [16] but with certain limitations. Both studies assume Poisson point processes on a square and use the toroidal metric, rather than the Euclidean metric which is more relevant to the applications. This renders their results hardly applicable to wireless sensor networks.

Recently, Kozma et al. (2004) [17] proved that the maximal length of an edge in the DT of a uniform n-point process in a unit disk is $O\left(\sqrt[3]{\ln n/n}\right)$.

IV. MAIN RESULTS

Let $X_1, X_2, \cdots$ be independent and uniformly distributed random points on a bounded region A in the plane. Given a positive integer n, the point process $\{X_1, X_2, \cdots, X_n\}$ is referred to as the uniform n-point process on A, and is denoted by $\mathcal{X}_n(A)$. Given a positive number λ, let $\mathrm{Po}(\lambda)$ be a Poisson random variable with parameter λ, independent of $\{X_1, X_2, \cdots\}$. Then the point process $\{X_1, X_2, \cdots, X_{\mathrm{Po}(\lambda)}\}$ is referred to as the Poisson point process with mean λ on A, and is denoted by $\mathcal{P}_\lambda(A)$.

A. The Number of Isolated Nodes

Let Ω be a unit-area square or disk. Assume that the wireless ad hoc network consists of n which is modeled by $\mathcal{X}_n(\Omega)$.

First, we consider wireless ad hoc networks in which nodes are not all active. The inactive or unavailable nodes may be caused by, for example, internal breakdown or being in the sleeping state. In such networks, a node is said isolated if it is not adjacent to active nodes. Assume nodes are active (or available) independently with probability p for some constant $0 < p \le 1$. We have the following theorem. [18]

Theorem 1. Suppose that all nodes have a maximum transmission radius $r_n = \sqrt{\frac{\ln n + \xi}{\pi p n}}$ for some constant ξ. Then the total number of isolated nodes is asymptotically Poisson with mean $e^{-\xi}$, and the total number of isolated active nodes is also asymptotically Poisson with mean $pe^{-\xi}$.

Next, we consider wireless ad hoc networks with unreliable nodes and links. Assume nodes are active independently with probability $0 < p_1 \le 1$, and links are up independently with probability $0 < p_2 \le 1$. A node is said to be isolated if it doesn't have an up link to an active node. We have the following theorem. [21]

Theorem 2. Suppose that $\lim_{n\to\infty} p_1 p_2 \ln n = \infty$ and nodes have the same maximum transmission radius $r_n = \sqrt{\frac{\ln n + \xi}{\pi p_1 p_2 n}}$ for some constant ξ. Then the total number of isolated nodes is asymptotically Poisson with mean $e^{-\xi}$, and the total number of isolated active nodes is also asymptotically Poisson with mean $p_1 e^{-\xi}$.

The work can be extended for secure wireless networks which apply m-composite key predistribution schemes. In the m-composite key predistribution scheme, the key pool contains K

distinct keys which are randomly chosen from the key space, and a key ring is composed of k distinct keys drawn from the key pool. Before deployed, each node randomly loads k distinct keys drawn from the key pool, which is called a key ring, into its memory. After deployed, two nodes within each other's transmission range have a secure link if their key rings have at least m common keys. A node is said to be isolated if it doesn't have a secure link. Let $q_i = \binom{k}{i}\binom{K-k}{k-i}\Big/\binom{K}{k}$ that is the probability of the event that two key rings have exactly i common keys, and $p = 1-(q_0 + q_1 + \cdots + q_{m-1})$ that is the probability of the event that two nodes (or key rings) have at least m common keys. We have the following theorem about the total number of isolated nodes in the secure wireless network. [21]

Theorem 3. In m-composite key predistribution schemes, let p be the probability that two neighbor nodes have a secure link. If $\lim_{n\to\infty} p \ln n = \infty$ and nodes have the same maximum transmission radius $r_n = \sqrt{\frac{\ln n + \xi}{\pi p n}}$ for some constant ξ, then the total number of isolated nodes is asymptotically Poisson with mean $e^{-\xi}$.

B. Coverage by Randomly Deployed Sensors

Here we study how the probability of the k-coverage changes with the sensing radius or the number of sensors. Let k be a fixed nonnegative integer, and Ω be the unit-area square or disk centered at the origin o. For any real number t, use $t\Omega$ to denote the set $\{tx : x \in \Omega\}$, i.e., the square or disk of area t^2 centered at the origin. Let $C_{n,r}$ (respectively, $C'_{n,r}$) denote the event that Ω is $(k+1)$-covered by the (open or closed) disks of radius r centered at the points in $\mathcal{P}_n(\Omega)$ (respectively, $\mathcal{X}_n(\Omega)$). Let $K_{s,n}$ (respectively, $K'_{s,n}$) denote the event that $\sqrt{s}\Omega$ is $(k+1)$-covered by the unit-area (closed or open) disks centered at the points in $\mathcal{P}_n(\sqrt{s}\Omega)$ (respectively, $\mathcal{X}_n(\sqrt{s}\Omega)$. Then, we would like to study the asymptotics of $\Pr[C_{n,r}]$ and $\Pr[C'_{n,r}]$ as n approaches infinity, and the asymptotics of $\Pr[K_{s,n}]$ and $\Pr[K'_{s,n}]$ as s approaches infinity.

To simplify the presentation of our results, we introduce some notation. Let η denote the peripheral of Ω, which is equal to 4 (respectively, $2\sqrt{\pi}$ if Ω is a square (respectively, disk). For any $\xi \in \mathrm{R}$, let

$$\alpha(\xi)=\begin{cases}\dfrac{\left(\sqrt{\pi}\eta+e^{-\frac{\xi}{2}}\right)^2}{16\left(2\sqrt{\pi}\eta+e^{-\frac{\xi}{2}}\right)}e^{-\frac{\xi}{2}}, \text{if } k=0;\\ \dfrac{\sqrt{\pi}\eta}{2^{k+6}(k+2)!}e^{-\frac{\xi}{2}}, \text{if } k\geq 1.\end{cases} \quad \text{and} \quad \beta(\xi)=\begin{cases}4e^{-\xi}+2\left(\sqrt{\pi}+\dfrac{1}{\sqrt{\pi}}\right)\eta e^{-\frac{\xi}{2}}, \text{if } k=0;\\ \dfrac{\sqrt{\pi}+\frac{1}{\sqrt{\pi}}}{2^{k-1}k!}\eta e^{-\frac{\xi}{2}}, \text{if } k\geq 1.\end{cases}$$

Our main results are summarized in the following two theorems. [19]

Theorem 4. Let $r_n=\sqrt{\dfrac{\ln n+(2k+1)\ln\ln n+\xi}{\pi pn}}$.

If $\lim_{n\to\infty}\xi_n=\xi$ for some $\xi\in\mathrm{R}$, then

$$1-\beta(\xi)\leq\lim_{n\to\infty}\Pr\left[C_{n,r_n}\right]\leq\frac{1}{1+\alpha(\xi)}, \text{ and } 1-\beta(\xi)\leq\lim_{n\to\infty}\Pr\left[C'_{n,r_n}\right]\leq\frac{1}{1+\alpha(\xi)}.$$

If $\lim_{n\to\infty}\xi_n=\infty$, then $\lim_{n\to\infty}\Pr\left[C_{n,r_n}\right]=\lim_{n\to\infty}\Pr\left[C'_{n,r_n}\right]=1$.

If $\lim_{n\to\infty}\xi_n=-\infty$, then $\lim_{n\to\infty}\Pr\left[C_{n,r_n}\right]=\lim_{n\to\infty}\Pr\left[C'_{n,r_n}\right]=0$.

Theorem 5. Let $\mu(s)=\ln s+2(k+1)\ln\ln s+\xi(s)$.

If $\lim_{s\to\infty}\xi(s)=\xi$ for some $\xi\in\mathrm{R}$, then

$$1-\beta(\xi)\leq\lim_{s\to\infty}\Pr\left[K_{s,\mu(s)s}\right]\leq\frac{1}{1+\alpha(\xi)}, \text{ and } 1-\beta(\xi)\leq\lim_{s\to\infty}\Pr\left[K'_{s,\mu(s)s}\right]\leq\frac{1}{1+\alpha(\xi)}.$$

If $\lim_{s\to\infty}\xi(s)=\infty$, then $\lim_{s\to\infty}\Pr\left[K_{s,\mu(s)s}\right]=\lim_{s\to\infty}\Pr\left[K'_{s,\mu(s)s}\right]=1$.

If $\lim_{s\to\infty}\xi(s)=-\infty$, then $\lim_{s\to\infty}\Pr\left[K_{s,\mu(s)s}\right]=\lim_{s\to\infty}\Pr\left[K'_{s,\mu(s)s}\right]=0$.

C. The Longest Edge of Gabriel Graphs

Let $\mathcal{P}_n$ denote a Poisson point process with density n over a unit-area disk. Assume that the wireless ad hoc network consists of n nodes modeled by $\mathcal{P}_n$, and all nodes have the same maximal transmission radius r_n. We use $G(\mathcal{P}_n)$ to denote the Gabriel graph over $\mathcal{P}_n$. For simplicity, the edges of GGs are call Gabriel edges. If G is a geometric graph, we use $\lambda(G)$ to denote the maximal length of an edge of G and $N(G,l)$ to denote the number of edges of G whose length is at least l. Our first result about the GG is the next theorem. [20]

Theorem 6. For any constant $\varepsilon>0$, we have

$$\lim_{n\to\infty}\Pr\left[(1-\varepsilon)2\sqrt{\frac{\ln n}{\pi n}}\leq\lambda(G(\mathcal{P}_n))\leq(1+\varepsilon)2\sqrt{\frac{\ln n}{\pi n}}\right]=1.$$

According to the theorem, if each node sets its maximal transmission radius to $r_n = \beta\sqrt{\frac{\ln n}{\pi n}}$ for some constant β, then the r_n-disk graph over $\mathcal{P}_n$ a.a.s. contains the GG if $\beta > 2$, and on the contrary, the r_n-disk graph a.a.r. contains the GG if $\beta < 2$. Therefore, $\beta = 2$ is the threshold for constructing the GG by 1-hop information. Now, we assume $r_n = \beta\sqrt{\frac{\ln n + \xi}{\pi n}}$ for some constant ξ. For a given ξ, we call the edge whose length is not less than r_n is a long edge. The next theorem gives us the asymptotic expectation of the number of long Gabriel edges.

Theorem 7. For the expectation of the number of long Gabriel edges, we have

$$\lim_{n\to\infty} \mathrm{E}\left[N\left(G(\mathcal{P}_n), 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right) \right] = 2e^{-\xi}.$$

Since $\Pr[X = 0] = 1 - \Pr[X \geq 1] \geq 1 - \mathrm{E}[X]$ for any non-negative integer value RV X,

$$\begin{aligned}\Pr\left[\lambda(G(\mathcal{P}_n)) < 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right] &= \Pr\left[N\left(G(\mathcal{P}_n), 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right) = 0 \right] \\ &\geq 1 - \mathrm{E}\left[N\left(G(\mathcal{P}_n), 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right) = 0 \right] \\ &\sim 1 - 2e^{-\xi}.\end{aligned}$$

Therefore,

$$\lim_{\xi\to\infty}\lim_{n\to\infty} \Pr\left[\lambda(G(\mathcal{P}_n)) < 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right] = 1 - 2e^{-\xi},$$

and $\xi \to \infty$ is an a.a.s. sufficient condition for $\lambda(G(\mathcal{P}_n)) < 2\sqrt{\frac{\ln n + \xi}{\pi n}}$. In the next theorem, we give the asymptotic probability distribution of the number of long Gabriel edges, and that implies the asymptotic probability distribution of the length of the longest edge.

Theorem 8. For any constant ξ, the total number of Gabriel edges whose lengths are at least $2\sqrt{\frac{\ln n + \xi}{\pi n}}$ is asymptotically Poisson with mean $2e^{-\xi}$.

Since

$$\Pr\left[\lambda(G(\mathcal{P}_n)) < 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right] = \Pr\left[N\left(G(\mathcal{P}_n), 2\sqrt{\frac{\ln n + \xi}{\pi n}} \right) = 0 \right],$$

according to the theorem, we have

$$\lim_{n\to\infty} \Pr\left[\lambda(G(\mathcal{P}_n)) < 2\sqrt{\frac{\ln n+\xi}{\pi n}}\right] = \exp\left(-2e^{-\xi}\right).$$

V. CONCLUSIONS (PERFORMANCE EVALUATION)

Supported by the NSC under the grant No. NSC94-2218-E-009-030, during the past year, we have published two journal papers in the IEEE Transactions on Communications [18] and IEEE Transactions on Information Theory [19]. In addition, we have one journal paper to appear in the IEEE Transactions on Parallel and Distributed Systems [20], and one conference paper to be presented in the IEEE GLOBECOM 2006 [21]. Overall speaking, we have finished most proposed works.

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