行政院國家科學委員會補助專題研究計畫 ■ 成果報告 □期中進度報告

BOT計畫放棄權與營收特許設計複合權評價之研究[1]

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計畫主持人：黃玉霖　副教授
共同主持人：
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一、摘要

公共建設具有提升生活品質、促進社會發展、厚植產業實力的作用。近年來，政府積極以 BOT 模式（Build-Operate-Transfer）推動民間參與公共建設，冀能引進民間的資金與經營效率，解決政府財政不足的窘局。然 BOT 計畫風險高且有其公共性質，為達社會公利與民間利益之平衡，常有重要特許設計安排，如履約保證金機制、強制收買、最低營收保證等。因此，本研究特應用實質選擇權相關理論，在最低營收保證政策下，評估 BOT 計畫於籌辦始點的各項價值，包括：投資淨現值、投資選擇權、放棄選擇權，及最低營收保證政策價值，並以台灣高速鐵路計畫財務資料進行案例模擬，期能提供 BOT 計畫特許設計之參考。研究結果顯示：第一，當計畫擁有放棄選擇權時，計畫整體價值將提高；第二，政府若另提供最低營收保證時，計畫的總價值將會增加，但由於其與放棄選擇權同屬消除營收下方風險之選擇權，故增加之價值小於兩者分別評價之和。

關鍵詞：BOT、實質選擇權、投資選擇權、放棄選擇權、最低營收保證

Abstract

This research applies real-option theories for the valuation of the option to abandon and minimum revenue guarantee (MRG) in Build-Operate-Transfer (BOT) infrastructure projects. The valuation of the option to abandon in the pre-construction phase is straightforward by the real-option approach, but the valuation of MRG is more complicated. In this research, MRG is constructed as a series of European style put options and formulated by a single option pricing model when used independently from the option to abandon. When combined with the option to abandon under the same BOT package, MRG is formulated by a compound option framework. Taiwan High-Speed Rail Project is used as a numerical case to apply the derived option price formulas. The results show both MRG and the option to abandon can create substantial financial values, but increasing the MRG level will decrease the value of the option to abandon under the compound option formulation.

Keywords: Build-Operate-Transfer, real option, option to abandon, minimum revenue guarantee

二、緣由與目的

BOT 為一種特許模式（Concession model）。為達促進民間參與公共建設的目的，又兼顧公共建設所擁有之公共財（Public goods）性質，將公共建設的推動與運作切割為興建（Build）、營運（Operate）、移轉（Transfer）三個階段，授予特許公司興建權與營運權；就財務面而言，原本由政府公部門編列預算興建公共建設的方式，轉化為利用民間資源從事公共建設興建與營運。

BOT 計畫的財務效益是能否吸引民間參與的重要關鍵，傳統現金流量折現法（Discounted cash flow method）所計算出的財務指標，如淨現值、內部報酬率、回收期間與自償率等，是決定是否參與投資的重要參考，然而此種評估方式忽略了投資計畫的幾樣特徵（Dixit & Pindyck, 1995）：

1. 環境是不確定的，實際的營收未必如同預期，不確定性愈大，表示營收的變動性愈大，此時，若有投資或營運決策上的彈性將能增加計畫整體評價。
2. 有些計畫的投資是可以延遲的（Deferrable），也就是說投資人可以等待更多的訊息揭露之後，再決定是否進行投資。

特許公司在建造時點之前，政府通常會給予一段籌辦時間，在籌辦期屆滿時，特許公司可視當時營收資訊，選擇是否建造公共

建設，此即特許公司於籌辦期初所擁有的投資選擇權（Option to invest），若籌辦期滿所預測的營收狀況不佳，特許公司擁有放棄選擇權（Option to abandon）選擇放棄投資建造；而於計畫營運期中，最低營收保證之特許設計能有效提升民間參與誘因，其相當於特許公司在籌辦期初相當於擁有以各營運期營收為標的之賣權（Put options），連同前述之放棄選擇權，整個計畫評估架構便成為一個複式選擇權（Compound option）的模式。

本研究之目的即是運用選擇權相關理論構建 BOT 計畫籌辦期期初統合價值評估模式，並計算放棄選擇權與最低營收保證對於計畫價值之影響。

三、結果與討論

（一）模式假設

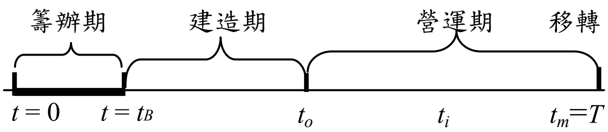
本研究假設計畫營運期營收遵循以下隨機過程：

$$\frac{dR}{R} = \alpha\ dt + \sigma\ dz = (\mu - \delta_R)\ dt + \sigma\ dz \qquad (1)$$

其中R為計畫營運期各期營運收入，為一變數，其預期成長率遵循Generalized Wiener process；α為單位時間內各期營運收入預期成長率，為一常數；σ為單位時間內營收預期成長率波動性(或預期成長率標準差)，為一常數；μ表示計畫折現率（或必要報酬率）；δ_R等於（μ-α），為營收成長率不足計畫折現率部分，即營收短少率（Shortfall rate），其概念類似股票選擇權中的股利支付率(Dividend payout rate)；dz為Generalized Wiener process隨機增量，其增量期望值E (dz)為 0，變異數V(dz)為dt。

（二）單一選擇權評估模式建立

本研究對於計畫各時點的定義如下：



圖一　BOT 計畫各時點示意圖

假設特許公司所擁有的投資選擇權為F，如同股票選擇權為股票之衍生性商品一樣，可視為由計畫營運收入所衍生而來。然計畫籌辦期滿t_B投入總成本I^{t_B}之性質與股票選擇權的履約價格有折現率差異，雖然前面已假設計畫投入總成本固定，在此仍將其視為變數I(但不為隨機變數)，以於模式中從事折現率之調整；故計畫籌辦期初投資選擇權F價值應為計畫營運收入R、投入總成本I，以及時間t的函數。其中投入總成本I之全微分式為：

$$dI = \varepsilon\ dt\ = (\mu - \delta_I)\ dt \qquad (2)$$

與式（1）性質不同的，因為I不為隨機變數，式（2）並沒有隨機增量項目dz；ε表示計畫投入總成本之預期成長率，由於已假設投入總成本I^{t_B}固定不變，因此ε等於0，另δ_I表示相應計畫折現率的短少率。依據Ito lemma，可得計畫籌辦期初投資選擇權評估模式，其中r表示無風險利率（Riskless rate）。

$$\frac{1}{2}\frac{\partial^2 F}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F}{\partial R} + (r-\delta_I)I\frac{\partial F}{\partial I} + \frac{\partial F}{\partial t} - rF = 0$$

$$F(R, I, t_B) = \left[\sum_{i=1}^{m} R_{t_1} e^{\alpha(t_i - t_1)} e^{-\mu(t_i - t_B)} - I_{t_B}\right] H(v)$$
$$= \left[P^{t_B} - I^{t_B}\right]\ H(P^{t_B} - I^{t_B})$$

$$H(v) = \{\ 1 \mid v \geq 0\ ,\ 0 \mid v < 0\ \} \qquad (3)$$

參考 Hwang & Jou（1994）之選擇權方程式之求解步驟，可得 BOT 計畫籌辦期初投資選擇權 F 評價公式為：

$$F(R, I, 0) = P^0 N(k_1) - I^0 N(k_2) \qquad (4)$$

$$k_1 = \frac{\ln\left(P^0/I^0\right) + \left(\sigma^2/2\right)\ t_B}{\sigma\sqrt{t_B}}$$

$$k_2 = \frac{\ln\left(P^0/I^0\right) - \left(\sigma^2/2\right)\ t_B}{\sigma\sqrt{t_B}} = k_1 - \sigma\sqrt{t_B}$$

P^0表示總營運收入籌辦期初現值，而I^0為總投入成本籌辦期初現值；$N(.)$為標準常態分佈累積機率函數。

上式除表示 BOT 計畫籌辦期初投資選擇權價值，其與計畫投資淨現值之差，即代

表在籌辦期屆滿特許公司可以選擇放棄投資的彈性價值，我們定之為 BOT 計畫籌辦期放棄投資選擇權 f，其於籌辦期初的評價為：

$$f = F - (P_0 - I_0) = I_0 N(-k_2) - P_0 N(-k_1) \quad (5)$$

放棄投資選擇權f的公式解與一存續期間為t_B歐式賣權相仿，其於選擇權的意涵乃是：當特許公司於計畫籌辦期屆滿時，若總營運收入現值 P^{t_B} 不及總投入成本現值 I^{t_B}，則特許公司將執行放棄選擇權f。

（三）最低營收保證之單一選擇權分析

營運期間「最低營收保證」是政府財務協助措施，其目的是考慮特許公司於營運期間因為營運收入的不確定性，而提供的營收補貼政策。以選擇權角度觀之，可將政府所提供最低營收保證模擬視為特許公司於營運期各期所擁有的「歐式賣權」，其標的物乃為營運期各期之營收現金流量。

假設當營運第n期營收R_{tn}低於M_{tn}時，政府將給予M_{tn}-R_{tn}的營收補貼，假設特許公司確定於籌辦期屆滿投資建造公共建設的情況下，其相當於政府授予特許公司一個存續期間為t_n的歐式賣權，評價模式如下：

$$\frac{1}{2}\frac{\partial^2 Q_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial Q_{t_n}}{\partial R} + (r-\delta_M)M\frac{\partial Q_{t_n}}{\partial M} + \frac{\partial Q_{t_n}}{\partial t} - rQ_{t_n} = 0$$

$$Q_{t_n}(R, M, t_n) = (M_{t_n} - R_{t_n})H(M_{t_n} - R_{t_n}) \quad (6)$$

則第 n 期最低營收保證賣權價值為：

$$Q_{t_n}(R, M, 0) = M_{t_n}^0 N(-d_2) - R_{t_n}^0 N(-d_1) \quad (7)$$

$$d_1 = \frac{\ln\left(R_{t_n}^0 / M_{t_n}^0\right) + \left(\sigma^2/2\right) t_n}{\sigma\sqrt{t_n}}$$

$$d_2 = \frac{\ln\left(R_{t_n}^0 / M_{t_n}^0\right) - \left(\sigma^2/2\right) t_n}{\sigma\sqrt{t_n}} = d_1 - \sigma\sqrt{t_n}$$

經由上式，在特許公司確定會於計畫籌辦期屆滿時投資建造的情況下，我們可以計算出在籌辦始點營運各期最低營收保證賣權價值 Q_{ti}（i = 1~ m），加總可得政府提供最低營收保證之政策價值$Q = \sum_{i=1}^{m} Q_i$。

（四）最低營收保證之複式選擇權分析

然而特許公司此時所在時點是計畫籌辦始點，因此必須執行投資選擇權後，才能享有最低營收保證之賣權價值，所以在籌辦始點，營運期的最低營收保證本質上是屬於複式選擇權。

特許公司於籌辦期屆滿時可視計畫之總營運收入 P^{t_B} 與各期最低營收保證賣權報償情況而決定是否投入總成本 I^{t_B}。將以上情況看成特許公司是在籌辦期屆滿投入總成本 I^{t_B} 以獲得m期的營運收入與m期最低營收保證，因此我們可以將投入總成本 I^{t_B} 分攤為m項，其值分別為 $D_{t_i}^{t_B}$（i = 1 ~m），以獲得各期「現金流量M_{ti}」與「執行價格M_{ti}、標的物為各期營收R_{ti}的歐式買權A_{ti}」；如針對營運第n期，若特許公司於籌辦期屆滿決定投資，則將投入總成本 I^{t_B} 之攤銷 $D_{t_n}^{t_B}$，以獲得營運第n期之「最低營收保證之現金流量M_{tn}」與「執行價格M_{tn}、標的物為第n期營收R_{tn}的歐式買權A_{tn}」。

假設第n期最低營收保證之投資選擇權價值為F_{tn}，可以得到微分方程式：

$$\frac{1}{2}\frac{\partial^2 F_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F_{t_n}}{\partial R} + (r-\delta_D)D\frac{\partial F_{t_n}}{\partial D} + \frac{\partial F_{t_n}}{\partial t} - rF_{t_n} = 0$$

$$F_{t_n}(R, D, t_B) = \left[A_{t_n} - (D_{t_n}^{t_B} - M_{t_n}e^{-(t_n - t_B)})\right] H\left[A_{t_n} - (D_{t_n}^{t_B} - M_{t_n}e^{-(t_n - t_B)})\right]$$

$$\frac{1}{2}\frac{\partial^2 A_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial A_{t_n}}{\partial R} + (r-\delta_M)M\frac{\partial A_{t_n}}{\partial M} + \frac{\partial A_{t_n}}{\partial t} - rA_{t_n} = 0$$

$$A_{t_n}(R, M, t_n) = (R_{t_n} - M_{t_n})H(R_{t_n} - M_{t_n}) \quad (8)$$

以上是針對營運第n期，所建構出在最低營收保證下，特許公司於計畫籌辦始點投資選擇權之聯立偏微分方程式，參考Geske（1979）之複式選擇權模式，可得在最低營收保證政策下，計畫營運第n期之籌辦始點投資選擇權F_{tn}等於：

$$F_{t_n}(R,D,0)=R_{t_n}^{0}B(a_{1_{t_n}},b_{1_{t_n}};\sqrt{t_B/t_n})-M_{t_n}^{0}B(a_{2_{t_n}},b_{2_{t_n}};\sqrt{t_B/t_n})-(D_{t_n}^{0}-M_{t_n}^{0})N(a_{2_{t_n}}) \quad (9)$$

$$a_{1_{t_n}}=\frac{\ln\left(R_{t_n}^{0}/R^{*0}_{t_n}\right)+\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}} \qquad a_{2_{t_n}}=\frac{\ln\left(R_{t_n}^{0}/R^{*0}_{t_n}\right)-\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}=a_{1_{t_n}}-\sigma\sqrt{t_B}$$

$$b_{1_{t_n}}=\frac{\ln\left(R_{t_n}^{0}/M_{t_n}^{0}\right)+\left(\sigma^2/2\right)t_n}{\sigma\sqrt{t_n}} \qquad b_{2_{t_n}}=\frac{\ln\left(R_{t_n}^{0}/M_{t_n}^{0}\right)-\left(\sigma^2/2\right)t_n}{\sigma\sqrt{t_n}}=b_{1_{t_n}}-\sigma\sqrt{t_n}$$

其中，$D_{t_n}^{0}$ 為計畫籌辦始點總投入成本之攤銷 $D_{t_i}^{t_B}$ 之現值；$R^{*0}_{t_n}$：為計畫籌辦期屆滿第 n 期臨界營收現值 $R^{*t_B}_{t_n}$ 之籌辦始點現值；$B(.)$：為雙變數常態分配累積分佈函數，$\sqrt{t_B/t_n}$ 為雙變數常態分配累積分佈函數相關係數。

各期臨界營收現值 $R^{*t_B}_{t_n}$ 與攤銷成本 $D_{t_i}^{t_B}$ 之攤銷方程式如下所示：

$$R^{*t_B}_{t_1}N(k_{1_{t_1}})-M_{t_1}^{t_B}N(k_{2_{t_1}})-(D_{t_1}^{t_B}-M_{t_1}^{t_B})=0$$
$$R^{*t_B}_{t_1}e^{(-\mu+\alpha)(t_2-t_1)}N(k_{1_{t_2}})-M_{t_2}^{t_B}N(k_{2_{t_2}})-(D_{t_2}^{t_B}-M_{t_2}^{t_B})=0$$
$$\vdots$$
$$R^{*t_B}_{t_i}e^{(-\mu+\alpha)(t_i-t_1)}N(k_{1_{t_i}})-M_{t_i}^{t_B}N(k_{2_{t_i}})-(D_{t_i}^{t_B}-M_{t_i}^{t_B})=0$$
$$\vdots$$
$$R^{*t_B}_{t_m}e^{(-\mu+\alpha)(t_m-t_1)}N(k_{1_{t_m}})-M_{t_m}^{t_B}N(k_{2_{t_m}})-(D_{t_m}^{t_B}-M_{t_m}^{t_B})=0$$
$$\sum_{i=1}^{m}D_{t_i}^{t_B}=I^{t_B} \quad (10)$$

加總各營運期之投資選擇權 $F_{t_i}, i=1\sim m$，可得在營收保證政策下計畫籌辦期初之投資選擇權，即$F_M=\sum_{i=1}^{m}F_{t_i}$。

另外，最低營收保證價值與放棄選擇權價值計算如下：

最低營收保證價值（有放棄選擇權下）：

$$Q_f=\sum_{i=1}^{m}\left[F_{t_i}(R,D,0)\big|_{M_{t_i}}-F_{t_i}(R,D,0)\big|_{M_{t_i}=0}\right]=F_M-F \quad (11)$$

放棄投資選擇權（在營收保證下）價值：

$$f_M=\sum_{i=1}^{m}F_{t_i}-\left[(P^0-I^0)+\sum_{i=1}^{m}Q_{t_i}\right] \quad (12)$$

（五）案例模擬

本案例分析，將以複式選擇權評估模式（式 9），使用台灣高速鐵路之財務預測資料進行模擬，最後並說明計畫投資選擇權、放棄選擇權及最低營收保證各價值之關係。

1.相關資料與假設條件

特許期(年)	籌辦期	1
	建造期	5
	營運期	30
投資成本 (百萬元)	籌辦成本 I^0:	3,400
	建造成本 B^0:	242,422
	營運成本 C^0:	101,429
估計參數	預期成長率 α	0.06
	成長率標準差 σ	0.3
	折現率 μ	12%
各期營收保證值 M_{t_i} i=1~30	各期營運成本 C_{t_i}	

2.模擬結果

(1) 計畫之 NPV 為 139,508 百萬元，此乃以傳統現金流量折現法計算而得。
(2) 在沒有最低營收保證下，計畫投資選擇權 F 為 146,944 百萬元，放棄選擇權 f 為 7,436 百萬元，放棄選擇權對於計畫整體價值有顯著提升。
(3) 在確定投資建造下，最低營收保證價值 Q 等於 7,716 百萬元，亦提升計畫整體價值。
(4) 當放棄選擇權與最低營收保證兩類型選擇權同時存在時，計畫投資選擇權價值 F_M等於 152,897 百萬元，最低營收保證價值Q_f 等於 5,953 百萬元，放棄選擇權價值f_M等於 5,673 百萬元。

(5) 當最低營收保證與放棄選擇權同時存在時，計畫的總價值將會增加，但由於兩者同屬消除營收下方風險（Down-side risk）之選擇權，故增加之價值小於兩者分別評價之和(Trigeorgis, 1993)，其下降之價值可稱為「聯合價值」（Joint value），其值等於 1,763 百萬元。可以從兩個角度來觀看聯合價值的意涵，首先，最低營收保證之價值必須等計畫確定投資以後才會實現，因此在籌辦期初無法完全獲得最低營收保證的價值 *Q*；第二，最低營收保證會提高放棄選擇權的標的資產（Underlying asset）的價值，進而使放棄選擇權的價值降低，意即較不會選擇放棄投資建造。

(6) 最低營收保證與放棄選擇權同時存在時，將增加 13,389 百萬元的選擇權價值，其值等於（*Q*＋*f*－*Joint value*）。

3. 最低營收保證水準敏感度分析

最低營收保證水準	NPV	F_M	Q_f	f_M	*Joint value*
0%	139,508	146,944 (= *F*)	0	7,436 (= *f*)	0
50%	139,508	148,267	1,323	6,992	444
100%	139,508	152,897	5,953	5,673	1,763
150%	139,508	161,233	14,290	4,132	3,304
250%	139,508	187,708	40,715	883	6,602
350%	139,508	227,037	80,093	10	7,426

上表顯示，最低營收保證的水準愈高，計畫的價值也愈高，然而，以本案為例，當最低營收保證為原來的 350%時，計畫的放棄選擇權價值f_M已經相當微小，意言當政府提供愈高水準的營收保證時，代表政府本身願意承擔愈高的營收下方風險，因此，特許公司愈不會執行放棄選擇權，而選擇投資建造。

四、計畫成果自評

本研究首先以單一選擇權評估模式，初步建立放棄選擇權與最低營收保證之評估架構，後續針對放棄選擇權與最低營收保證建構出複式選擇權價值評價模型，評估計畫之統合價值，選擇權評估架構之導入可彌補傳統現金流量折現法未能反應計畫決策彈性價值之缺點。

整體而言，以實質選擇權評估 BOT 計畫相關特許設計是可行，且是預期可以持續開展延伸，尤其 BOT 計畫具有高度的不確定性，相關特許設計的配套，關乎政府民間公私部門權利義務分配之公平合理與否，也因此，若能針對相關特許設計進行量化評估，將有助於權利義務之安排。

本研究雖以複式選擇權的方式統合評估放棄選擇權與最低營收保證同時存在情況下之計畫價值，然而，投資人在各階段其實均有投資、營運、擴充、退出與放棄的決策彈性，實際情況相形複雜許多，未來若能併同納入考量，將更能反映 BOT 計畫實切價值，在議訂特許合約時，俾利相關特許設計之安排。

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五、附錄

Valuation of the Option to Abandon and Minimum Revenue Guarantee in BOT Infrastructure Projects

Yu-Lin Huang* and Shih-Pei Chou

Department of Civil Engineering, National Chiao Tung University, 1001 Ta-Hsueh Road, Hsin-Chu 300, Taiwan

Abstract

The real option approach is used to value the option to abandon and minimum revenue guarantee (MRG) in Build-Operate-Transfer infrastructure projects. The option to abandon is formulated under an investment option held by the concessionaire at contract signing and to expire before construction commencement. MRG is formulated as a series of European style put options in a single option pricing model. When combined with the option to abandon in the pre-construction phase, MRG is reconstructed as a series of European style call options to develop a compound option pricing formula. Taiwan High-Speed Rail Project is chosen as a numerical case to apply the formulas. The results show both MRG and the option to abandon can create values. When MRG and the option to abandon are combined, they will counteract each other, and their values will thus be reduced. Increasing the MRG level will decrease the value of the option to abandon, and, at a certain MRG level, the option to abandon will be rendered worthless.

Keywords: BOT, infrastructure, real option, option to abandon, minimum revenue guarantee.

* For correspondence: huangyulin2000@hotmail.com

Introduction

Build-Operate-Transfer (BOT) infrastructure projects differ significantly from more traditional construction projects in how projects are implemented during the pre-construction phase. For traditional projects, the owner is responsible for project planning, environmental assessment, property acquisition, and project funding. For BOT projects, the concessionaire is usually required to undertake project development tasks. The concessionaire faces substantial project development risks. Even if the concessionaire can successfully accomplish these tasks, BOT projects often become financially not viable due to changes in external investment conditions. As a result, BOT projects are more likely to fail in the pre-construction phase than in other project phases. To reduce the concessionaire's investment risks, BOT concession contracts often grant the concessionaire an option to abandon during the pre-construction stage (Huang, 1995).

Likewise, BOT infrastructure projects usually face substantial revenue risks during the operation phase. To avoid the downside risks of revenues, the concessionaire often negotiates with the government to provide a minimum revenue guarantee (MRG). Under an MRG, the government is obligated to cover the shortfalls between a pre-specified level of MRG and operating revenues realized by the concessionaire. MRG can increase the concessionaire's willingness to invest. It can also enhance the credit worthiness of BOT projects facing high revenue risks, since the guaranteed cash inflows can provide a minimum level of debt coverage.

The presence of the abandonment option will increase the concessionaire's flexibility in investment decisions and thus increase project value. The valuation of the option to abandon can be done by real-option theories (Trigeorgis and Mason, 1989; Dixit, 1989; Trigeorgis, 1993). The use of real-option theories in BOT project evaluation is not a new idea. Ho and Liu (2002), for example, developed a real-option pricing model to value government debt guarantees in BOT projects. For traditional construction projects, the real-option approach has also been used. For example, David, Diane and John (2002) show that the real-option approach can be applied in traditional project planning; Tien (2002) analyzed time-to-build options in sequential construction; and Michael and Charles (2004) developed a model to evaluate strategic project deferments.

The presence of MRG can also increase project value, but the valuation of MRG is still an open issue. Paddock, Siegel and Smith (1988) had developed a real option approach to value offshore petroleum leases as staged-options, which also face substantial project development risks. But the obligation to pay under an MRG is very different from that of leasing. While the lessee usually promises to make a series of fixed payments to the lessor, the undertaker of an MRG pays only when project revenues fall below a pre-specified level of MRG. The undertaker will not pay at all when the realized operating revenues are higher than the MRG level.

This paper studies the valuation of the MRG and the option to abandon in the pre-construction stage. The option to abandon is formulated under an investment option held by the concessionaire, and the expiration date of the investment option is at the targeted construction commencement date. The MRG is constructed as a series of European style put options under a single option model. When the MRG is combined with the option to abandon, they will counteract each other, and the valuation becomes more complicated. The MRG is re-constructed as a series of European style call options to form a compound option.

Taiwan High Speed Rail Project is used as a numerical case to apply the derived option pricing formulas. The results show both the MRG and the option to abandon can create substantial values under the single option settings. In the compound option model, they counteract with each other, and their values are reduced. An increase of the MRG level will decrease the value of the option to abandon, and when the MRG level is high enough, the option to abandon will become worthless.

In the following sections, this paper begins with formulating a single option pricing model for the option to abandon in the pre-construction phase. MRG is then constructed as a series of European style put options to derive a single option pricing formula. A compound option model is further developed to combine the MRG with the abandonment option under the same BOT package. Taiwan High Speed Rail Project is used as a numerical case to apply the derived formulas, and then observations and policy implications are drawn accordingly.

Valuating the option to abandon during the pre-construction stage

Figure 1 shows the life-cycle of a typical BOT project after concession tendering. It begins with a pre-construction phase at time $t = 0$, when the concession contract is signed, and ends with project transfer at $t = T$, when the concession period expires. The concession contract often provides the concessionaire options to abandon during the lifecycle (Huang, 1995). We only focus on the pre-construction phase, since BOT projects are more likely to fail in this phase.

The concessionaire may decide to walk away at any time during the pre-construction phase, but we assume the final decision can only be made at the targeted construction commencement date, or $t = t_B$. We also assumed that once the concessionaire decides to invest, the project cannot be abandoned thereafter. Once the concessionaire decides to invest, the project will thus be completed on time and become operational at $t = t_O$. It will be operated for m years, from t_1 to t_m, and then transferred at the end of the concession, or $t = t_m = T$.

Based on these assumptions, the option to abandon is formulated under an investment option held by the concessionaire's at $t = 0$ and to expire at $t = t_B$. Let R denote the project's operating revenues. Suppose R is a random variable, and follows a generalized Wiener process:

$$R = \left\{ R_t \mid t \in [0, T] \qquad \frac{dR}{R} = \alpha \ dt + \sigma \ dz = (\mu - \delta_R) \ dt + \sigma \ dz \right\} \qquad (1)$$

Here, α is the growth rate of R. It has a variance σ^2 during a very short interval dt. δ_R is the shortfall rate between the project's discounted rate μ and the growth rate of R. The shortfall rate resembles the dividend payout rate of a stock option. The rate is the opportunity cost for keeping an option alive rather than exercising it. dz is the incremental part of the Wiener process. The expected value of dz is zero, and the variance of dz is equal to dt.

Further let I denote the project's total costs, and suppose I include both capital investment cost and operation cost. Suppose I is not stochastic in nature, and the growth rate of I is ε during a short interval dt. Then,

$$I = \left\{ I_t \mid t \in [0, t_B] \qquad \frac{dI}{I} = \varepsilon \ dt = (\mu - \delta_I) \ dt \right\} \qquad (2)$$

Here, the shortfall rate of I is denoted as δ_I. The rate is the opportunity cost avoided by holding the option to abandon until its expiration date t_B. If the amount of I is deterministic, then ε is zero, and $\delta_I = \mu$.

Let F denote the value of the concessionaire's option to invest. F is a function of state variables R, I, and time t, or $F = \{F(R,I,t) | \ t \in [0, t_B]\}$. By contingent claim analysis (Black and Scholes, 1973), F should satisfy:

$$\frac{1}{2}\frac{\partial^2 F}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F}{\partial R} + (r-\delta_I)I\frac{\partial F}{\partial I} + \frac{\partial F}{\partial t} - rF = 0 \tag{3}$$

The boundary condition of this partial differential equation (PDE) is:

$$\begin{aligned} F(R,I,t_B) &= \left[\sum_{i=1}^{m}(R_{t_1} e^{\alpha(t_i - t_1)} - C_{t_i})e^{-\mu(t_i - t_B)} - B_{t_B}\right] H(v) \\ &= \left[P_{t_B} - I_{t_B}\right] \ H(P_{t_B} - I_{t_B}) \end{aligned}$$

Here, $H(v) = \{\ 1 \mid v \ge 0\ \ , 0 \mid v < 0\ \}$. In this formulation, R_{t_1} is the revenue of the 1st operating period, r is the risk-free interest rate, and I^{t_B} is the present value of I at t_B. I^{t_B} includes both capital investment cost B^{t_B} and operation cost C^{t_B}, and P^{t_B} is the present value of total operating revenues at t_B. The boundary condition implies that the concessionaire will exercise the option to invest at t_B with strike price I^{t_B} if and only if $P^{t_B} > I^{t_B}$; otherwise, the project will be abandoned.

Following McDonald and Siegel (1986), let Z and $W(Z,t)$ be the transformation variables:

$$Z = \frac{R}{I} \qquad\qquad W(Z,t) = \frac{F}{I}$$

Accordingly, Equation (3) is simplified:

$$\frac{1}{2}\frac{\partial^2 W}{\partial Z^2}\sigma^2 Z^2 + (\delta_I - \delta_R) Z \frac{\partial W}{\partial Z} + \frac{\partial W}{\partial t} - \delta_I W = 0 \tag{4}$$

subject to

$$W(Z, t_B) = \left[\Phi(T) Z - 1\right] H(\Phi(T) Z - 1)$$

This PDE is similar to that of the original Black-Scholes model. Using the solution methods by Kutner (1988) and Hwang and Jou (1994), the price formula of the option to invest can be derived:

$$F(R, I, 0) = P^0 N(k_1) - I^0 N(k_2) \tag{5}$$

where

$$k_1 = \frac{\ln\left(P^0 / I^0\right) + \left(\sigma^2 / 2\right) t_B}{\sigma \sqrt{t_B}}$$

$$k_2 = \frac{\ln\left(P^0 / I^0\right) - \left(\sigma^2 / 2\right) t_B}{\sigma \sqrt{t_B}} = k_1 - \sigma \sqrt{t_B}$$

Let f denote the value of the option to abandon. The value is the difference between the value of the investment option and the project's NPV at $t = 0$:

$$f = F - (P^0 - I^0) \tag{6}$$

Substituting (5) into (6) gives the following price formula for f:

$$\begin{aligned} f &= P^0 \left[N(k_1) - 1\right] - I^0 \left[N(k_2) - 1\right] \\ &= I^0 N(-k_2) - P^0 N(-k_1) \end{aligned} \tag{7}$$

This solution is similar to that of a European style put option. Here, P^0 is the present value of operating revenues, I^0 is the present value of the total investment costs at $t = 0$, and $N(\cdot)$ is a cumulative normal distribution function.

Valuating Minimum Revenue Guarantee

In a single option setting, the MRG can be constructed and valued as a series of European style put options in the operation phase. Let M_{t_i} define the MRG level specified in the concession contract, i = 1 ~ m. The downside risk of operating revenues is limited to M_{t_i}, and the project's n[th] period payoff function is shown in Figure 2. To find the pricing formula for the MRG, we decompose the project's payoff function into two parts, namely an n[th] period operating revenue and an MRG cash inflow; see Figure 3 and Figure 4 respectively. In Figure 4, the payoff from the MRG is specified as ($M_{t_n} - R_{t_n}$), which defines the shortfall below M_{t_n} undertaken by the government.

Let Q_{t_n} denote the value of the n[th] MRG, R for the projected operating revenues, and δ_M for the shortfall rate with the same property as δ_I. Assume R is stochastic in nature as specified in the previous section. Q_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 Q_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial Q_{t_n}}{\partial R} + (r-\delta_M)M\frac{\partial Q_{t_n}}{\partial M} + \frac{\partial Q_{t_n}}{\partial t} - rQ_{t_n} = 0 \tag{8}$$

subject to

$$Q_{t_n}(R, M, t_n) = (M_{t_n} - R_{t_n})H(M_{t_n} - R_{t_n})$$

Solving by the same solution method gives the following price formula:

$$Q_{t_n}(R, M, 0) = M_{t_n}^0 N(-d_2) - R_{t_n}^0 N(-d_1) \tag{9}$$

where

$$d_1 = \frac{\ln\left(R^0_{t_n} \middle/ M^0_{t_n}\right) + \left(\sigma^2 \middle/ 2\right) t_n}{\sigma\sqrt{t_n}} \qquad d_2 = \frac{\ln\left(R^0_{t_n} \middle/ M^0_{t_n}\right) - \left(\sigma^2 \middle/ 2\right) t_n}{\sigma\sqrt{t_n}} = d_1 - \sigma\sqrt{t_n}$$

Here, $R^0_{t_n}$ is the present value of R_{t_n}, and $M^0_{t_n}$ the present value of M_{t_n}. Both of them are discounted by μ at $t = 0$. Let Q denote the total value of the MRG. Q is found by an aggregation of Q_{t_i}, $i = 1 \sim m$, or $Q = \sum_{i=1}^{m} Q_i$.

Combining MRG with the Option to Abandon

When the MRG and the option to abandon are combined under the same BOT package, they will interact with each other. Suppose the holding of the MRG option is dependent on the concessionaire's decision to invest at t_B. The concessionaire's option to invest at $t = 0$ can be formulated as an option on option, or a compound option, under which the exercise payoff of the option to invest is related to the MRG level.

To derive the compound option pricing formula, first reconstruct Figure 2 as two different payoff functions: a constant cash flow denoted as M_{t_n} (Figure 5), and a European style call option whose value is written as A_{t_n} (Figure 6). The strike price of the call is M_{t_n}, the exercise payoff is (R_{t_n} - M_{t_n}), and the expiration date is at the end of t_n.

At $t = t_B$, the concessionaire decides whether or not to invest I^{t_B} for obtaining the combined value of M_{t_i} and A_{t_i}, i = 1 ~ m. Once the concessionaire decides to invest, the project cannot be abandoned thereafter. Thus, if the concessionaire does not abandon at $t = t_B$, s/he will obtain all M_{t_i} and A_{t_i}, i = 1 ~ m. Each call option A_{t_i} is independent, to be exercised only if the realized operating revenue at t_i is above the MRG level according to Figure 6.

Suppose F_M is the value of the option to invest at $t = 0$. F_M is the aggregated value of M_{t_i} and A_{t_i}. To find F_M, first amortize the present value of I at t_B, denoted as I^{t_B}, by m portions, and let D_{t_i} denote the amortized cost at t_i, i = 1 ~ m. Under this amortization schedule, the concessionaire decides at $t = t_B$ either to invest the present value of D_{t_i} for obtaining M_{t_i} and A_{t_i} or not. The calculation of D_{t_i} is presented in the next section.

Then, let's start with the n^th^ period of the concession contract. When the concessionaire decides to invest at $t = t_B$, the present value of the n^th^ amortized cost $D_{t_n}^{t_B}$ is invested for M_{t_n} and A_{t_n}. We treat the contingent MRG cash inflows as cost-reducing factors for option valuation, and the amount invested for the n^th^ period can thus be rewritten as $(D_{t_n}^{t_B} - M_{t_n} e^{-\mu(t_n - t_B)})$, with a corresponding exercise payoff A_{t_n}. Here, M_{t_n} is discounted by μ at t_B.

Let F_{t_n} denote the value of the option to invest for the n^th^ operating period. F_{t_n} is the function of state variables, R_{t_n}, D_{t_n}, and time t: $F_{t_n} = \left\{ F(R,D,t) \middle| \ t \in [0, t_B] \right\}$. Accordingly, the strike price of F_{t_n} is $(D_{t_n}^{t_B} - M_{t_n} e^{-\mu(t_n - t_B)})$. By contingent claims analysis, F_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 F_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F_{t_n}}{\partial R} + (r-\delta_D)D\frac{\partial F_{t_n}}{\partial D} + \frac{\partial F_{t_n}}{\partial t} - rF_{t_n} = 0 \qquad (10)$$

Here, D is assumed to have the same property as I, as stated above. If the amount of D is fixed, then the shortfall rate of D is equal to that of I, which in turn is equal to the discount rate. Since F_{t_n} resembles a European style call option, equation (10) must satisfy the following boundary condition:

$$\begin{aligned} F_{t_n}(R,D,t_B) &= \left[A_{t_n} - (D_{t_n}^{t_B} - M_{t_n} e^{-(t_n - t_B)})\right] H\left[A_{t_n} - (D_{t_n}^{t_B} - M_{t_n} e^{-(t_n - t_B)})\right] \\ &= \left[A_{t_n} - D_{t_n}^{t_B}(1-\rho)\right] H\left[A_{t_n} - D_{t_n}^{t_B}(1-\rho)\right] \end{aligned} \tag{11}$$

Here, $\rho = M_{t_n} e^{-\mu(t_n - t_B)} / D_{t_n}^{t_B}$, and $H(\cdot)$ is the same unit step function as defined before. This boundary condition implies that, if $F_{t_n} > 0$ at t_B, then the amount $D_{t_n}^{t_B}(1-\rho)$ is invested for keeping the European style call option A_{t_n} alive. A_{t_n} is a function of state variables R, M, and time t: $F_{t_n} = \left\{ A(R,M,t) \middle| \; t \in \left[0, t_n\right] \right\}$.

Further let $R_{t_B}^{*t_n}$ be the critical asset price, so that $F_{t_n}(R,D,t_B) = 0$. If $R_{t_B}^{t_n} > R_{t_B}^{*t_n}$, then the concessionaire will decide to invest at t_B. By contingent claims analysis, the value of A_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 A_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial A_{t_n}}{\partial R} + (r-\delta_M)M\frac{\partial A_{t_n}}{\partial M} + \frac{\partial A_{t_n}}{\partial t} - rA_{t_n} = 0 \tag{12}$$

subject to

$$A_{t_n}(R, M, t_n) = (R_{t_n} - M_{t_n})H(R_{t_n} - M_{t_n})$$

Here, since the MRG level is specified, M has the same property as I. This boundary condition implies that M_{t_n} will be invested for R_{t_n} if and only if $A_{t_n} > 0$ at t_n. Since $M_{t_n} e^{-\mu(t_n - t_B)} = \rho D_{t_n}^{t_B}$, by chain rule,

$$M\frac{\partial A_{t_n}}{\partial M} = M\frac{\partial A_{t_n}}{\partial D}\frac{\partial D}{\partial M} = \rho D e^{\mu(t_n - t_B)}\frac{\partial A_{t_n}}{\partial D}\frac{e^{-\mu(t_n - t_B)}}{\rho} = D\frac{\partial A_{tn}}{\partial D} \tag{13}$$

Substituting (13) into (12) gives:

$$\frac{1}{2}\frac{\partial^2 A_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial A_{t_n}}{\partial R} + (r-\delta_D)D\frac{\partial A_{t_n}}{\partial D} + \frac{\partial A_{t_n}}{\partial t} - rA_{t_n} = 0 \qquad (14)$$

subject to

$$A_{t_n}(R,D,t_n) = (R_{t_n} - D_{t_n}^{t_B}\rho e^{\mu(t_n-t_B)})H[R_{t_n} - D_{t_n}^{t_B}\rho e^{\mu(t_n-t_B)}]$$

Furthermore, let ξ, $X(\xi,t)$, and $Y(\xi,t)$ be the following transformation variables:

$$\xi = \frac{R}{D} \qquad X(\xi,t) = \frac{F_{t_n}}{D} \qquad Y(\xi,t) = \frac{A_{t_n}}{D}$$

Accordingly, equation (10) is simplified:

$$\frac{1}{2}\frac{\partial^2 X}{\partial \xi^2}\sigma^2 Z^2 + (\delta_D - \delta_R)\xi\frac{\partial X}{\partial \xi} + \frac{\partial X}{\partial t} - \delta_D X = 0 \qquad (15)$$

subject to

$$X(\xi, t_B) = [Y - (1-\rho)]H[Y - (1-\rho)]$$

and (14) simplified as:

$$\frac{1}{2}\frac{\partial^2 Y}{\partial \xi^2}\sigma^2 Z^2 + (\delta_D - \delta_R)\xi\frac{\partial Y}{\partial \xi} + \frac{\partial Y}{\partial t} - \delta_D Y = 0 \qquad (16)$$

subject to

$$Y(\xi, t_n) = [\xi - \rho e^{\mu(t_n - t_B)}]H[\xi - \rho e^{\mu(t_n - t_B)}]$$

Equations (15) and (16) are the joint PDE of the compound option. Following the solution method by Geske (1979) gives the price formula for F_{t_n} at t = 0 as a closed-form solution:

$$F_{t_n}(R, D, 0) = R_{t_n}^{0} B(a_{1_{t_n}}, b_{1_{t_n}}; \sqrt{t_B / t_n}) - M_{t_n}^{0} B(a_{2_{t_n}}, b_{2_{t_n}}; \sqrt{t_B / t_n}) -$$

$$(D_{t_n}^{0} - M_{t_n}^{0}) N(a_{2_{t_n}}) \qquad (17)$$

where

$$a_{1_{t_n}} = \frac{\ln\left(R_{t_n}^{0} \big/ R^{*0}_{t_n}\right) + \left(\sigma^2 \big/ 2\right) t_B}{\sigma \sqrt{t_B}} \qquad a_{2_{t_n}} = \frac{\ln\left(R_{t_n}^{0} \big/ R^{*0}_{t_n}\right) - \left(\sigma^2 \big/ 2\right) t_B}{\sigma \sqrt{t_B}} = a_{1_{t_n}} - \sigma \sqrt{t_B}$$

$$b_{1_{t_n}} = \frac{\ln\left(R_{t_n}^{0} \big/ M_{t_n}^{0}\right) + \left(\sigma^2 \big/ 2\right) t_n}{\sigma \sqrt{t_n}} \qquad b_{2_{t_n}} = \frac{\ln\left(R_{t_n}^{0} \big/ M_{t_n}^{0}\right) - \left(\sigma^2 \big/ 2\right) t_n}{\sigma \sqrt{t_n}} = b_{1_{t_n}} - \sigma \sqrt{t_n}$$

In this solution, $B(\cdot)$ is a two-dimensional cumulative bivariate normal distribution function with a correlation coefficient $\sqrt{t_B/t_n}$ for overlapping Brownian increments. $R^{*0}_{t_n}$ is the present value of $R^{*t_B}_{t_n}$ at $t = 0$. The calculation of the critical asset price $R^{*0}_{t_n}$ is related to the amortized cost $D^{0}_{t_n}$, as presented in the next section. Let F_{t_i} denote the value of the option to invest for the i^th period. F_M is found as an aggregation of F_{t_i}, $i = 1 \sim m$, or $F_M = \sum_{i=1}^{m} F_{t_i}$.

Now, we can derive price formulas for the MRG and the option to abandon. First let Q_f denote the value of the MRG at $t = 0$. Q_f is given by:

$$Q_f = \sum_{i=1}^{m} \left[F_{t_i}(R, D, 0) \mid_{M_{t_i}} - F_{t_i}(R, D, 0) \mid_{M_{t_i}=0} \right] = F_M - F \qquad (18)$$

Here, $F_{t_i}(R, D, 0)\,|_{M_{t_i}=0}$ is the value of the option to invest at t = 0 for the ith period underlying assets without the MRG. Note that, in the pricing formula (17), if $M_{t_n}=0$, $R^{*0}_{t_n}$ is equal to $D^{0}_{t_n}$, and $F_{t_i}(R, D, 0)\,|_{M_{t_i}=0}$ is given by:

$$F_{t_i}(R,D,0)\,|_{M_{t_i}=0} = R^{0}_{t_i}N(a_{1_{t_i}}) - D^{0}_{t_i}N(a_{2_{t_i}}) \qquad (19)$$

This solution is the similar to equation (5).

Then, let f_M denote the value of the option to abandon at t = 0. Since the holding of the MRG is dependent on the concessionaire's decision to invest at t_B, f_M is calculated by subtracting the project's NPV and the value of the MRG from the value of the compound option at t = 0; that is,

$$f_M = \sum_{i=1}^{m} F_{t_i} - \left[(P^0 - I^0) + \sum_{i=1}^{m} Q_{t_i} \right] \qquad (20)$$

Calculating the Amortized Cost $D^{t_B}_{t_n}$ and the Critical Asset Price $R^{*t_B}_{t_n}$

The foregoing compound option is formulated under the condition that the decomposed m investment options are all exercised together at t = t_B or never, and the MRG is constructed as a series of independent options, which will be exercised only if the realized operating revenues fall below the pre-specified MRG level. In other words, once the concessionaire decides to invest at t = t_B, the following boundary conditions should be satisfied:

$$
\begin{aligned}
&F_{t_1}(R,D,t_B) = A_{t_1} - (D_{t_1}^{t_B} - M_{t_1}^{t_B}) = 0 \\
&F_{t_2}(R,D,t_B) = A_{t_2} - (D_{t_2}^{t_B} - M_{t_2}^{t_B}) = 0 \\
&\vdots \\
&F_{t_i}(R,D,t_B) = A_{t_i} - (D_{t_i}^{t_B} - M_{t_i}^{t_B}) = 0 \\
&\vdots \\
&F_{t_m}(R,D,t_B) = A_{t_m} - (D_{t_m}^{t_B} - M_{t_m}^{t_B}) = 0 \\
&\sum_{i=1}^{m} D_{t_i}^{t_B} = I^{t_B}
\end{aligned}
\tag{21}
$$

Here, the value of A_{ti} at t_B can be found by the solution methods from Kutner (1988) and Hwang and Jou (1994):

$$A_{t_i}(R,D,t_B) = R_{t_i}^{t_B} N(k_{1_{t_i}}) - M_{t_i}^{t_B} N(k_{2_{t_i}}) \tag{22}$$

where

$$k_{1_{t_i}} = \frac{\ln\left(R_{t_i}^{t_B} \big/ M_{t_i}^{t_B} \right) + \left(\sigma^2 \big/ 2 \right)\ (t_i - t_B)}{\sigma\sqrt{(t_i - t_B)}}$$

$$k_{2_{t_i}} = \frac{\ln\left(R_{t_i}^{t_B} \big/ R_{t_i}^{t_B} \right) - \left(\sigma^2 \big/ 2 \right)\ (t_i - t_B)}{\sigma\sqrt{(t_i - t_B)}} = k_{1_{t_i}} - \sigma\sqrt{(t_i - t_B)}$$

Substituting A_{t_i} into (21) gives the following (m + 1) equations:

$$
\begin{aligned}
&R_{t_1}^{*t_B} N(k_{1_{t_1}}) - M_{t_1}^{t_B} N(k_{2_{t_1}}) - (D_{t_1}^{t_B} - M_{t_1}^{t_B}) = 0 \\
&R_{t_1}^{*t_B} e^{(-\mu+\alpha)(t_2 - t_1)} N(k_{1_{t_2}}) - M_{t_2}^{t_B} N(k_{2_{t_2}}) - (D_{t_2}^{t_B} - M_{t_2}^{t_B}) = 0 \\
&\vdots \\
&R_{t_i}^{*t_B} e^{(-\mu+\alpha)(t_i - t_1)} N(k_{1_{t_i}}) - M_{t_i}^{t_B} N(k_{2_{t_i}}) - (D_{t_i}^{t_B} - M_{t_i}^{t_B}) = 0 \\
&\vdots \\
&R_{t_m}^{*t_B} e^{(-\mu+\alpha)(t_m - t_1)} N(k_{1_{t_m}}) - M_{t_m}^{t_B} N(k_{2_{t_m}}) - (D_{t_m}^{t_B} - M_{t_m}^{t_B}) = 0 \\
&\sum_{i=1}^{m} D_{t_i}^{t_B} = I^{t_B}
\end{aligned}
\tag{23}
$$

These equations have (m + 1) unknowns, namely $R^{*t_B}_{t_1}$ and $D^{t_B}_{t_i}$ $i = 1 \sim m$. The first m equations can be summed up as:

$$R^{*t_B}_{t_1} \sum_{i=1}^{m} e^{(-\mu+\alpha)(t_i - t_1)} N(k_{1_{t_i}}) + \sum_{i=1}^{m} M^{*t_B}_{t_1} (1 - N(k_{2_{t_i}})) - I^{t_B} = 0 \qquad (24)$$

$N(\cdot)$ is the one-dimensional cumulative normal distribution function. Since the values of k_1 and k_2 are dependent on $R^{*t_B}_{t_1}$, equation (24) is nonlinear, and thus $R^{*t_B}_{t_1}$ can only be found numerically. Once $R^{*t_B}_{t_1}$ is found, the amortized costs $D^{t_B}_{t_i}$ can be calculated by (23), the critical asset prices $R^{*t_B}_{t_i}$ can be calculated by $e^{(-\mu+\alpha)(t_i - t_1)} R^{*t_B}_{t_1}$, and F_{t_i} can be obtained by (18).

Now, let's examine the special case when the MRG is absent from the foregoing formulation. First, if $M = 0$, then $k_1 \rightarrow \infty$, $k_2 \rightarrow \infty$, $N(\cdot) = 1$, and (24) can be simplified as:

$$R^{*t_B}_{t_1} \sum_{i=1}^{m} e^{(-\mu+\alpha)(t_i - t_1)} - I^{t_B} = P^{t_B} - I^{t_B} = 0 \qquad (25)$$

Here, the calculation of $R^{*t_B}_{t_1}$ is straightforward, and the amortized costs $D^{t_B}_{t_i}$ are equal to their corresponding critical asset values $R^{*t_B}_{t_1}$, which can further be found by:

$$\begin{bmatrix} R^{*0}_{t_1} \\ R^{*0}_{t_2} \\ \vdots \\ R^{*0}_{t_i} \\ \vdots \\ R^{*0}_{t_m} \end{bmatrix} = \begin{bmatrix} D^{0}_{t_1} \\ D^{0}_{t_2} \\ \vdots \\ D^{0}_{t_i} \\ \vdots \\ D^{0}_{t_m} \end{bmatrix} = \frac{I^0}{\sum_{i=1}^{m} e^{(-\mu+\alpha)(t_i - t_1)}} \begin{bmatrix} 1 \\ e^{(-\mu+\alpha)(t_2 - t_1)} \\ \vdots \\ e^{(-\mu+\alpha)(t_i - t_1)} \\ \vdots \\ e^{(-\mu+\alpha)(t_m - t_1)} \end{bmatrix} \qquad (26)$$

In equation (17), if $M = 0$, then $b_{1_{t_i}} \to \infty$, $b_{2_{t_i}} \to \infty$, and the two-dimensional cumulative bivariate normal distribution function can be written as:

$$B(a_{1_{t_i}}, \infty; \sqrt{t_B / t_i}) = N(a_{1_{t_i}}) \qquad B(a_{2_{t_i}}, \infty; \sqrt{t_B / t_i}) = N(a_{2_{t_i}})$$

Equation (17) can thus be simplified as equation (19). Since $R^{*t_B}_{t_i} = R^{*t_B}_{t_1} e^{(-\mu+\alpha)(t_i - t_1)}$, $a_{1_{t_i}}$ and $a_{2_{t_i}}$ can be calculated by:

$$a_{1_{t_1}} = a_{1_{t_2}} = \ldots = a_{1_{t_i}} = \ldots = a_{1_{tm}},$$

$$a_{1_{t_i}} = \frac{ln\left(R^0_{t_i} \big/ D^0_{t_i}\right) + \left(\sigma^2 \big/ 2\right) t_B}{\sigma\sqrt{t_B}} = \frac{ln\left(P^0 \big/ I^0\right) + \left(\sigma^2 \big/ 2\right) t_B}{\sigma\sqrt{t_B}} = k_1$$

and

$$a_{2_{t_1}} = a_{2_{t_2}} = \ldots = a_{2_{t_i}} = \ldots = a_{2_{tm}},$$

$$a_{2_{t_i}} = \frac{ln\left(R^0_{t_i} \big/ D^0_{t_i}\right) - \left(\sigma^2 \big/ 2\right) t_B}{\sigma\sqrt{t_B}} = \frac{ln\left(P^0 \big/ I^0\right) - \left(\sigma^2 \big/ 2\right) t_B}{\sigma\sqrt{t_B}} = k_2$$

As a result, the price formula is given by:

$$\begin{aligned}\sum_{i=1}^{m} F_{t_i}(R, D, 0)\,|_{M=0} &= \sum_{i=1}^{m} R^0_{t_i} N(a_{1_{t_i}}) - \sum_{i=1}^{m} D^0_{t_i} N(a_{2_{t_i}}) \\ &= N(k_1) \sum_{i=1}^{m} R^0_{t_i} - N(k_2) \sum_{i=1}^{m} D^0_{t_i} \\ &= P^0 N(k_1) - I^0 N(k_2)\end{aligned} \qquad (27)$$

This result is the same as equation (5). In fact, when the MRG is not provided, the compound and the single option pricing formulas will always lead to the same valuation so long as the amortization schedule satisfies the following conditions:

$$\frac{R^0_{t_1}}{D^0_{t_1}} = \frac{R^0_{t_2}}{D^0_{t_2}} = \ldots = \frac{R^0_{t_i}}{D^0_{t_i}} = \ldots = \frac{R^0_{t_{m-1}}}{D^0_{t_{m-1}}} = \frac{R^0_{t_m}}{D^0_{t_m}} = \frac{\sum_{i=1}^{m} R^0_{t_i}}{\sum_{i=1}^{m} D^0_{t_i}} = \frac{P^0}{I^0}$$

(28)

A Numerical Case

The Data

Taiwan High Speed Rail BOT Project is now used as a numerical case to apply the foregoing pricing formulas. The Project's detailed financial data and projections can be obtained from the authors. As summarized in Table 1, the Project's total concession period is 36 years, the present value of the total capital investment cost is NT$ 242,422 M, and the present value of the operating costs is NT$ 101,429 M. The variation of the Project's revenues is small, so the Generalized Wiener process can be applied. The average growth rate is thus estimated at 6.02%, and we use 6% as an approximation. The volatility of Taiwan's stock market price varies from 0.2 to 0.4, and we use 0.3 as the volatility of the growth rate. Finally, the Project's discount rate is estimated at 12% by the financial consultant.

For the purpose of this application, we suppose the Project has an MRG, and the annual MRG levels are equal to the corresponding annual operating costs. In addition, although the construction commencement date is not specified in the concession contract, suppose the pre-construction period of the project is one year after the contract signing date. Table 1 includes a pre-construction cost and its present value is denoted as J^0. The cost is required to keep the Project alive until t

$= t_B$. It will reduce the Project's NPV, or $NPV = P^0 - I^0 - J^0$, and affect the value of the investment option. Therefore, the price formula for F should be adjusted. When the MRG is not available, the price formula is as the following:

$$F = P_0 N(k_1) - I_0 N(k_2) - J_0 \tag{29}$$

When the MRG is available, the price formula for F_M is adjusted as:

$$\begin{aligned} F_M &= \sum_{i=1}^{m} \left[R_{t_n}^0 B(a_{1_{t_n}}, b_{1_{t_n}}; \sqrt{t_B/t_n}) - M_{t_n}^0 B(a_{2_{t_n}}, b_{2_{t_n}}; \sqrt{t_B/t_n}) - (D_{t_n}^0 - M_{t_n}^0) N(a_{2_{t_n}}) - J^0/m \right] \\ &= \sum_{i=1}^{m} F_{t_i} - J^0 \end{aligned} \tag{30}$$

Results and Interpretations

The results of this numerical case are shown in Table 2, from which we have the following observations:

(a) The Project's NPV is NT$139,508 M, so the Project is financially feasible.

(b) The option to abandon creates values. The value of the option, or f, is NT$7,436 M, calculated by either equation (5) or (28). By equation (5), for example, the shortfall rate is calculated at 6% from the growth rate and the discount rate. $N(k_1) = 0.9047$, $N(k_2) = 0.8434$, and $F =$ NT$ 146,944 M. F is greater than the Project's NPV, and the amount of f is obtained by subtracting the NPV from it.

(c) The MRG also creates values. The option value of the MRG, or Q, is

$7,716 M, calculated by (9).

(d) When the options are combined, the foregoing valuations change. The value of the option to invest with the MRG, or F_M, is NT$152,897 M, calculated by the adjusted compound option pricing formula in (29). The MRG value or Q_f is NT$5,953 M, calculated by subtracting F from F_M by (19). And the value of the option to abandon is $5,673 M, calculated by subtracting the NPV and the value of the MRG from F_M by (21).

(e) As a result, when the MRG and the option to abandon are combined, their values are reduced. The values disappeared are called joint value, which is equal to NT$1,763 M in this numerical case. This result is consistent with Trigeorgis' observation. When options intended to control down-side risks are exercised at the same time, they will counteract each other, and thus reduce their own values (Trigeorgis, 1993). There are two counteracting forces in our compound option. On one hand, the value of the MRG cannot be realized if the option to abandon is exercised at t_B. So long as there is a chance that the project will be abandoned, the value of the MRG will not be fully realized. On the other, the presence of the MRG will increase the value of the underlying assets of the option to abandon, and thus reduce the value of the option itself.

(f) Although the option values are reduced, the total value created by the compound option is still substantial. The value is NT$13,389 M, calculated by subtracting the joint value from the original values of the MRG and the option to abandon in the single option models.

Policy Implications

Both the MRG and the option to abandon are valuable policy tools as we have shown, but the government should check if they will produce intended policy effects when combined in the same BOT package. For example, Table 3 shows if the pre-specified level of the MRG is increased by 250%, then the value of the option to abandon will decrease to NT$10M, which is no longer substantial compared to the Project's NPV.

In general, the investment value of BOT projects increases with the MRG level, but the increase of the MRG will decrease the value of the option to abandon. In terms of risk allocation, the higher the MRG level is, the more the down-side risk is transferred to the government, and the less likely the concessionaire will exercise the option to abandon. When both the MRG and the option to abandon are proposed at the same time, the government should check if the proposed MRG level will render the option to abandon worthless.

In practice, the MRG may be valuable from an investor's perspective, but it requires substantial budgetary commitments. The benefits and costs of the commitments should be justified, and the pricing formulas can be used as valuation tools. When the project is not bankable due to high revenue risks, the lender may ask the government to provide MRG for a minimum level of debt coverage. In this case, MRG can also be justified by credit enhancement benefits to the concessionaire, such as reduced financing costs. However, to know how much the financing costs can be saved, MRG should be valued from lender's perspective. This appears to be another open issue.

Whenever there are budgetary constraints, the option to abandon is a preferable, more easily justified policy choice; especially it can only be used in the pre-construction phase. If the option can be used during construction and operation, abandonment will cause greater disruptions and disturbances, and this will make the justification more difficult.

Conclusions

Two single option pricing models were first developed for the valuation of the MRG and the option to abandon in the pre-construction phase. The MRG was further combined with the option to abandon to develop a compound option model.

Taiwan High-Speed Rail Project was used as a numerical case to apply the derived option pricing formulas. The results indicated that both MRG and the option to abandon could create substantial values. When the MRG and the option to abandon were combined, they counteracted each other and their values were reduced. If the level of the MRG were high enough, the option to abandon would be rendered valueless.

MRG involves substantial budgetary commitments, and its benefits and costs should be carefully justified. The option to abandon is a preferable policy choice under budgetary constraints, and its justification is more straightforward.

Overall, the application of the real option approach in BOT project evaluation looks promising. The option pricing formulas developed in this paper can be used as valuation tools for the foregoing justifications as well as extensions of traditional project evaluation approaches, such as the discounted cash flow model. But the presented formulas are limited in scope. They do not consider the options to abandon during construction and operation. In addition, if the MRG is to be used as a credit enhancement tool, it should also be valued from the lender's perspective. Future researches are required to tackle these problems.

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Table 1 Data and assumptions.

<table>
<tr><td rowspan="3">The concession period (yrs)</td><td>Pre-construction</td><td>1</td><td rowspan="3">Estimated parameters</td><td>Expected growth rate α</td><td>0.06</td></tr>
<tr><td>Construction</td><td>5</td><td>Growth rate volatility σ</td><td>0.3</td></tr>
<tr><td>Operation</td><td>30</td><td>Discount rate μ</td><td>0.12</td></tr>
<tr><td rowspan="3">Investment costs (NT$ million)</td><td>Pre-construction cost J^0 :</td><td>3,400</td><td rowspan="3">Annual level of MRG, M_{t_i} , i=1~30</td><td colspan="2" rowspan="3">The corresponding annual operating cost, C_{t_i}</td></tr>
<tr><td>Construction cost B^0 :</td><td>242,422</td></tr>
<tr><td>Operating cost C^0 :</td><td>101,429</td></tr>
</table>

Table 2 Results.

Unit: million of NT$

Year (i)	R_{t_i}	C_{t_i}	M_{t_i}	$N(-d_2)$	$N(-d_1)$	Q_{t_i}
1	75,879	12,740	12,740	0.0321	0.0041	43
2	81,125	13,586	13,586	0.0463	0.0057	64
3	86,744	14,630	14,630	0.0633	0.0076	91
4	92,685	15,568	15,568	0.0798	0.0093	116
5	99,155	16,537	16,537	0.0964	0.0108	140
6	105,999	17,353	17,353	0.1109	0.0119	158
7	113,303	18,582	18,582	0.1291	0.0135	183
8	120,457	19,942	19,942	0.1489	0.0153	211
9	128,184	21,144	21,144	0.166	0.0165	230
10	136,347	22,229	22,229	0.181	0.0174	243
11	145,027	23,599	23,599	0.1978	0.0185	259
12	154,256	24,969	24,969	0.2135	0.0194	270
13	164,069	26,540	26,540	0.2299	0.0203	283
14	174,501	28,763	28,763	0.2505	0.0220	306
15	185,594	30,434	30,434	0.2651	0.0226	311
16	197,386	32,090	32,090	0.2785	0.0230	313
17	209,924	34,503	34,503	0.2961	0.0242	326
18	222,578	36,379	36,379	0.3094	0.0246	325
19	235,987	38,600	38,600	0.3238	0.0252	327
20	250,196	41,005	41,005	0.3382	0.0258	328
21	265,253	44,198	44,198	0.3556	0.0269	337
22	281,206	46,728	46,728	0.3681	0.0272	332
23	298,110	49,493	49,493	0.3807	0.0275	328
24	316,018	52,620	52,620	0.3938	0.0279	325
25	334,993	56,671	56,671	0.4096	0.0288	329
26	355,096	59,865	59,865	0.4205	0.0289	321
27	376,393	63,756	63,756	0.4331	0.0292	316
28	398,341	67,404	67,404	0.4439	0.0294	308
29	416,926	71,081	71,081	0.4565	0.0298	300
30	434,757	75,295	75,295	0.4705	0.0305	295

Table 2 Continued.

Year (i)	$R^{*0}_{t_i}$	$D^{0}_{t_i}$	$B(a_2,b_2)$	$B(a_1,b_1)$	$N(a_2)$	F_{t_i}
1	22,433	22,516	0.8473	0.9185	0.8669	10,675
2	21,272	21,392	0.8367	0.9172	0.8669	10,103
3	20,174	20,337	0.8236	0.9157	0.8669	9,560
4	19,118	19,319	0.8104	0.9143	0.8669	9,042
5	18,140	18,375	0.7970	0.9130	0.8669	8,564
6	17,199	17,457	0.7849	0.9121	0.8669	8,109
7	16,305	16,597	0.7698	0.9107	0.8669	7,674
8	15,374	15,702	0.7532	0.9092	0.8669	7,221
9	14,511	14,860	0.7388	0.9080	0.8669	6,805
10	13,689	14,050	0.7260	0.9073	0.8669	6,414
11	12,914	13,292	0.7116	0.9063	0.8669	6,043
12	12,183	12,571	0.6981	0.9054	0.8669	5,695
13	11,493	11,892	0.6840	0.9045	0.8669	5,367
14	10,841	11,266	0.6663	0.9031	0.8669	5,053
15	10,226	10,653	0.6537	0.9025	0.8669	4,764
16	9,646	10,070	0.6420	0.9021	0.8669	4,491
17	9,099	9,534	0.6267	0.9010	0.8669	4,231
18	8,557	8,986	0.6150	0.9006	0.8669	3,977
19	8,046	8,473	0.6024	0.9001	0.8669	3,738
20	7,566	7,991	0.5899	0.8995	0.8669	3,512
21	7,114	7,546	0.5746	0.8985	0.8669	3,299
22	6,689	7,112	0.5636	0.8982	0.8669	3,101
23	6,289	6,703	0.5526	0.8979	0.8669	2,914
24	5,913	6,320	0.5410	0.8975	0.8669	2,739
25	5,560	5,968	0.5272	0.8967	0.8669	2,573
26	5,227	5,622	0.5175	0.8966	0.8669	2,418
27	4,914	5,301	0.5064	0.8962	0.8669	2,273
28	4,612	4,987	0.4968	0.8961	0.8669	2,133
29	4,282	4,645	0.4858	0.8957	0.8669	1,979
30	3,960	4,314	0.4734	0.8951	0.8669	1,829

Table 3 Comparative static.

Change of the MRG Level	NPV	F_M	Q_f	f_M	Joint Value
0% (no MRG)	139,508	146,944 (= F)	0	7,436 (= f)	0
50%	139,508	148,267	1,323	6,992	444
100%	139,508	152,897	5,953	5,673	1,763
150%	139,508	161,233	14,290	4,132	3,304
250%	139,508	187,708	40,715	883	6,602
350%	139,508	227,037	80,093	10	7,426

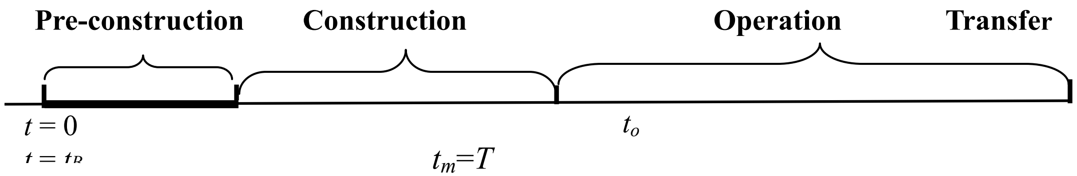

Figure 1 Typical BOT project life-cycle after contract signing.

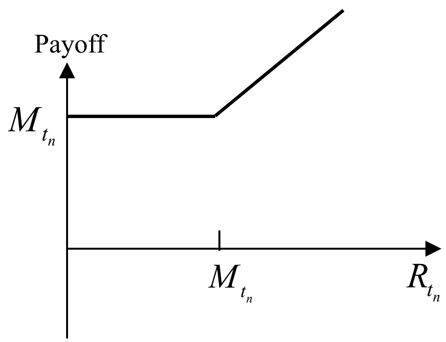

Figure 2 The n[th] period revenue payoff function.

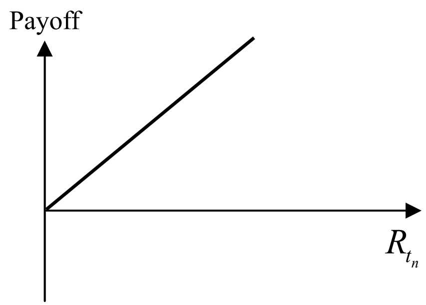

Figure 3 The n[th] period revenue payoff function from *R*.

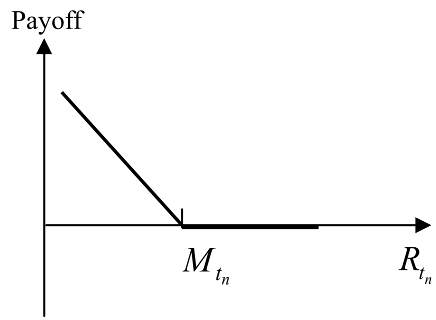

Figure 4 The n[th] period revenue payoff function from MRG.

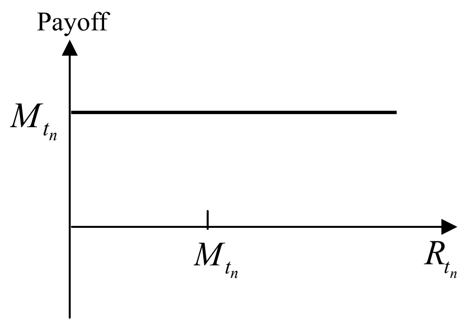

Figure 5 A constant cash flow

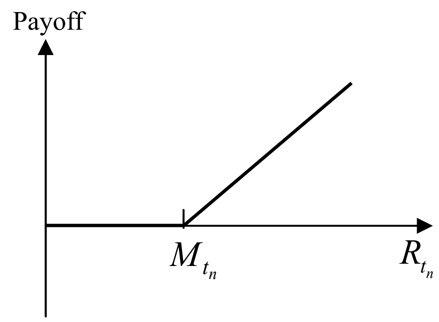

Figure 6 A European style call option.