Expert Systems with Applications 38 (2011) 3155–3160

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Exploration of efficiency underestimation of CCR model: Based on medical sectors with DEA-R model

Ching-Kuo Wei [a], Liang-Chih Chen [b], Rong-Kwei Li [b], Chih-Hung Tsai [c,*]

[a] Department of Health Care Administration, Oriental Institute of Technology, 58, Section 2, Sihchuan Road, Pan-Chiao City, Taipei County 22061, Taiwan
[b] Department of Industrial Engineering and Management, National Chiao-Tung University, Hsinchu, Taiwan
[c] Department of Information Management, Yuanpei University, No. 306, Yuanpei Street, Hsin-Chu, Taiwan

ARTICLE INFO

Keywords:
Data Envelopment Analysis
Weight restriction
Decision Making Unit

ABSTRACT

Data Envelopment Analysis (DEA) is one of the best-known efficiency evaluation methods due to its advantages in selection of weights. Many research papers have extensively discussed the issue of weight restrictions, rather than those implied in the model itself. However, this often leads to a failure to represent the relations of certain weights, as well as underestimation of the efficiency of Decision Making Units (DMUs). When analyzing the medical sectors of Taiwan with the developed models and CCR, it is found that efficiency underestimation by efficient DMUs is more serious than that of inefficient DMUs. In addition, underestimation occurs when weights are concentrated in the same output, however, every output of referenced DMU is the same times of corresponding output of targeted DMU.

1. Introduction

Efficiency is an important topic, and Data Envelopment Analysis (DEA) is one of the most famous efficiency evaluation methods. A mathematical model is established in DEA to judge efficient frontiers, and evaluate if the Decision Making Unit (DMU) is efficient. In addition, DEA permits to propose an improved package for inefficient DMU. The concept of a non-dominated solution proposed by Pareto and an index-based efficiency representation concept proposed by Farrell provide the basis for DEA (Cooper, Seiford, & Tone, 2002). With the introduction of the concept of non-dominated solutions and indices, Charnes, Cooper, and Rhodes (1978) developed a group of optimal mathematical equations for judging the efficient frontier, and calculating efficiency, which was called DEA, and the first group of mathematical expressions was named CCR, the abbreviated name of the authors.

Many studies have focused on the analysis of weight restrictions, since selection of weight represents one of DEA's advantages (e.g. Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997; Liu & Chuang, 2009; Pedraja-Chaparro, Salinas-Jimenez, & Smith, 1997; Podinovski, 2007). Tracy and Chen (2005) first proposed that weight hypothesis may lead to underestimation from additional weight restrictions. However, CCR, based on $(\sum vx)/(\sum uy)$ or $(\sum uy)/(\sum vx)$, implies inherent weight restrictions, and has never been extensively discussed. Such restrictions may lead to a failure to represent the relations of certain weights, thus, output-oriented DEA-R was developed to address such problems (Despic, Despic, & Paradi, 2007). Since unnecessary and unreasonable weight hypothesis would cause CCR to underestimate the efficiency of a DMU, an input-oriented DEA-R model was developed. Another research pointed out that, this hypothesis not only underestimated efficiency, but also resulted in false low efficiency solutions (an efficient DMU was judged as an inefficient DMU). Therefore, this paper aims to further discuss the underestimation issues of an efficient DMU, and provide a deeper understanding of the instance when underestimation occurs. Andersen and Petersen (1993) and Seiford and Zhu (2003) developed a super-efficient model and a dependent model, respectively, to discuss efficient and inefficient DMUs. As both the super-efficient and dependent models were developed based on CCR, the problem of efficiency underestimations occur. Thus, this research intends to develop a pro-rated super-efficient evaluation model in an attempt to study how the efficient DMU was underestimated, and the instance when underestimation occurred.

The remainder of this paper is organized as follows: Section 1 describes the issues of efficiency underestimation, as well as two subjects that have not been discussed, which are underestimation of an efficient DMU, and when exactly does the instance of underestimation occur. Regarding the underestimation of an efficient DMU, this section discusses two high-efficiency models, with/without weight restrictions. Section 2 reviews the super-efficient model based on CCR, and proposes a super-efficient model based on DEA-R (excluding weight restrictions). Taking medical centers in Taiwan as an example, Section 3 compares the efficiency and optimal weights of CCR and DEA-R-based super-efficient models,

* Corresponding author. Tel.: +886 3 6102338; fax: +886 3 6102343.
E-mail addresses: ietch@thit.edu.tw, imtch@mail.ypu.edu.tw (C.-H. Tsai).

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doi:10.1016/j.eswa.2010.08.108

and gains insight into the underestimation issues of an efficient DMU, as well as possible underestimation instances. The time point of underestimation is further discussed in Section 4. Finally, results and discussions are presented in Section 5.

2. Mathematical model

This research attempts to develop a new evaluation model according to a super-efficient concept proposed by Andersen and Petersen (1993), which could distinguish the advantages/disadvantages of both efficient and inefficient DMUs. In evaluating low efficiency, this development model differs little from previous models. In evaluating high efficiency, this development model evaluates the targeted DMUs evolution from high efficiency to low efficiency. First, the high-efficiency evaluation model based on CCR, an input-oriented high-efficiency model (Super-CCR-I), developed by Andersen and Petersen (1993), is described as follows:

$$\max \quad \bar{\theta}_o = \sum_{r=1}^{s} u_r \times y_{ro} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i \times x_{ij} \geqslant \sum_{r=1}^{s} u_r \times y_{rj}, \quad j = 1, \ldots, n, \ j \neq o \tag{2}$$

$$\sum_{i=1}^{m} v_i \times x_{io} = 1 \tag{3}$$

$$v_i, u_r \geqslant \varepsilon > 0 \tag{4}$$

According to previous research, CCR may lead to underestimations due to excessive weight restrictions, which is inherit in the high-efficiency model based on CCR. Hence, a DEA-R-based high-efficiency model is proposed in this paper. In the next chapter, two high-efficiency models are compared to discuss the underestimation of an efficient DMU. The following is a DEA-R-based input-oriented high-efficiency model, i.e. Super-DEA-R-I:

$$\max \quad \theta_o \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} \frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \theta_o, \quad j = 1, \ldots, n \ j \neq o \tag{6}$$

$$\sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} = 1 \tag{7}$$

$$W_{ir} \geqslant 0, \ \ \theta_o \geqslant 0 \tag{8}$$

3. Case study and comparison of efficiency

3.1. Case study

To evaluate the possible underestimation of an efficient DMU with CCR, this research evaluates one case, and compares the results using both Super-CCR-I and super-DEA-R-I. Medical centers in Taiwan (the highest level of medical institutions in Taiwan) 2005, were selected for case study (Table 1). Many hospitals have been upgraded to medical centers through accreditation in order to receive increased budgets and payment for medical research. However, this surge of medical centers cannot concentrate their resources to support key research, thus, the advantages/disadvantages of an efficient DMU should be evaluated as an accurate evaluation by an efficient DMU could assist the Bureau of National Health Insurance in controlling the outlay of various medical services, and prevent such excessive outlay from crippling the entire health insurance system. Moreover, upon evaluation of the medical system by DEA, the outputs, such as efficiency, weights, and improvement packages, can provide reasonable explanations of practical applications. For instance, Chen, Hwang, and Shao (2005) and Katharaki (2008) evaluated the medical system by DEA.

This research selected all medical centers (21) as evaluation subjects, including seven public hospitals (33%) and private hospitals (67%). Two inputs and three outputs were selected, of which the total inputs and outputs were less than half of all DMUs in conformity with empirical rules. The inputs include: sickbeds and physicians, outputs include: out-patients, in-patients, and surgeries. Take DMU 4 for example, it serviced 2,596,143 out-patients, and 855,467 in-patients, and conducted 75,348 surgeries in 2005, with 2902 sickbeds and 973 physicians. The relevant coefficients of inputs and outputs are listed in Table 2, wherein the coefficient is no less than 0.7. There are no problems in selection of variables according to empirical rule.

3.2. Comparison of efficiency between Super-CCR and Super-DEA-R

First, efficiency between Super-CCR and Super-DEA-R models is compared. An input-oriented model was used in this research since a global budget payment system was adopted in Taiwan. The software for efficiency calculation was Excel. The efficiencies of CCR, DEA-R, Super-CCR, and super-DEA-R are listed in Table 3. If the efficiency of DMU is larger than 1, it represents that this DMU is efficient, and thus, inputs can be increased and the DMU maintains efficiency. The available input increment is equal to previous inputs multiplied by efficiency, namely, a higher efficiency means inputs can be increased, while maintaining an efficient state. When evaluating DMU 8 with Super-CCR, the evaluated efficiency is

Table 2
Correlation of input and output variables.

	I-1	I-2	O-1	O-2	O-3
I-1	1.000	0.956	0.774	0.990	0.828
I-2	0.956	1.000	0.775	0.945	0.781
O-1	0.774	0.775	1.000	0.769	0.719
O-2	0.990	0.945	0.769	1.000	0.863
O-3	0.828	0.781	0.719	0.863	1.000

Table 1
The input and output variables of Taiwan medical centers in 2005.

DMU	Sickbed	Physician	Out-patient	In-patient	Surgeries	DMU	Sickbed	Physician	Out-patient	In-patient	Surgeries
01	2618	1106	2,029,864	680,136	38,714	11	920	316	334,090	268,723	15,130
02	1212	473	1,003,707	297,719	18,575	12	3236	1023	1,954,775	920,215	56,167
03	1721	531	1,592,960	408,556	36,658	13	495	130	332,741	136,351	23,423
04	2902	973	2,596,143	855,467	75,348	14	1759	491	1,465,374	430,407	35,599
05	1389	447	1,116,161	337,523	23,803	15	1357	390	1,277,752	368,174	36,006
06	1500	547	1,476,282	378,658	22,503	16	2468	675	1,825,332	668,467	32,275
07	340	145	1,300,016	55,003	5,614	17	962	316	550,700	247,961	15,618
08	571	305	1,052,992	199,780	26,026	18	745	272	1,277,899	217,371	11,671
09	1168	369	1,849,711	326,109	30,967	19	1662	590	1,916,888	418,205	21,551
10	921	372	1,089,975	209,323	23,847	20	898	275	698,945	209,134	11,748
						21	1708	537	1,702,676	470,437	32,218

Table 3
Efficiency of medical centers.

	CCR	DEA-R	Difference		Super-CCR	Super-DEA-R	Difference
1	0.8137	0.8137	0.0000	1	0.8137	0.8137	0.0000
2	0.7913	0.7920	0.0007	2	0.7913	0.7920	0.0007
3	0.8352	0.8432	0.0080	3	0.8352	0.8432	0.0080
4	0.9980	1.0000	0.0020	4	0.9980	1.0022	**0.0042**
5	0.8347	0.8417	0.0070	5	0.8347	0.8417	0.0070
6	0.8349	0.8423	0.0074	6	0.8349	0.8423	0.0074
7	1.0000	1.0000	**0.0000**	7	2.1699	2.1699	**0.0000**
8	1.0000	1.0000	**0.0000**	8	1.3149	1.5396	**0.2247**
9	1.0000	1.0000	**0.0000**	9	1.0753	1.2771	**0.2018**
10	0.7356	0.7465	0.0109	10	0.7356	0.7465	0.0109
11	0.9814	0.9814	0.0000	11	0.9814	0.9814	0.0000
12	0.9802	0.9802	0.0000	12	0.9802	0.9802	0.0000
13	1.0000	1.0000	**0.0000**	13	1.9516	1.9516	**0.0000**
14	0.8840	0.9082	0.0242	14	0.8840	0.9082	0.0242
15	0.9717	0.9865	0.0148	15	0.9717	0.9865	0.0148
16	0.9750	0.9797	0.0047	16	0.9750	0.9797	0.0047
17	0.8782	0.8782	0.0000	17	0.8782	0.8782	0.0000
18	1.0000	1.0000	**0.0000**	18	1.0047	1.0235	**0.0188**
19	0.8495	0.8551	0.0056	19	0.8495	0.8551	0.0056
20	0.8146	0.8220	0.0074	20	0.8146	0.8220	0.0074
21	0.9585	0.9675	0.0090	21	0.9585	0.9675	0.0090

1.3149, which indicates DMU 8 could increase its input and maintain high efficiency; sickbeds can be increased from 571 to 751, and physicians from 305 to 401. If evaluating DMU 8 with Super-DEA-R, the evaluated efficiency is 1.5396, which indicates DMU 8 can increase its input and maintain high efficiency under the same output. This result suggests that Super-CCR may have underestimated the efficiency of the DMU.

It is found that the mean difference of CCR and DEA-R is 0.0048; that of 15 inefficient DMUs is 0.0066; and that of six efficient DMUs is 0.0003. If DMU 4 is assumed to have judged a false inefficiency by CCR, the mean difference of 5 efficient DMUs is 0. This indicates that, for efficient DMUs, DEA-R and CCR have little difference unless false inefficiency exists. The mean difference of Super-CCR and Super-DEA-R is 0.0262; that of 15 inefficient DMUs is 0.0066; and that of 6 efficient DMUs is 0.0749. First, the efficiency of Super-DEA-R is larger than or equal to that of Super-CCR, indicating that the evaluation model based on CCR underestimates efficiency. Moreover, the difference in efficiency, among the efficiency models, is larger than that among the common models, indicating that comparisons of super-efficiency models could create higher visibility of the efficiency underestimation problems. More importantly, it is necessary to develop a DEA-R-based super-efficient model due to more serious underestimations of efficient DMUs.

4. Comparison of optimal weights

After confirmation of underestimation, this research re-confirms the causes of underestimation by comparing the optimal weights of the two models. The efficiency differences and optimal weights of efficient DMUs (DMU 4, 7, 8, 9, 13, and 18) are listed in Table 4. Next, DMUs instances of underestimation and correct estimations are discussed.

First, underestimated efficient DMUs (04, 08, 09, and 18) are discussed, starting from DMU 08. As for CCR, the significance ratio of input 1 to input 2 is, $v_1x_1 : v_2x_2 = 1:0$, the significance ratio of output 1, output 2, and output 3 is $u_1y_1 : u_2y_2 : u_3y_3 = 0.262 : 0.751 : 0.301 = 0.199 : 0.571 : 0.229$; as for DEA-R, the significance ratio of input 1 to input 2 is $(w_{11} + w_{12} + w_{13}) : (w_{21} + w_{22} + w_{23}) = 1:0$, the significance ratio of output 1, output 2, and output 3 is $(w_{11} + w_{21}) : (w_{12} + w_{22}) : (w_{13} + w_{23}) = 0.324 : 0.000 : 0.676$. As seen, for DMU 8, CCR and DEA-R have inconsistent viewpoints on output weights, despite a consistent viewpoint of input weights. Secondly, the efficiency difference of DMU 04 is only up to 0.004. However, as shown in Table 3, efficiency judged by super-CCR and super-DEA-R is 0.998 and 1.002, respectively, indicating that CCR considers DMU 4 inefficient, but DEA-R considers it efficient. In the case study, downgrading may occur if inefficiency is judged. The underestimation of efficiency cannot be neglected since minor differences may lead to obviously different judgments. With respect to optimal weights for CCR, the significance ratio of input 1 to input 2 is $v_1x_1 : v_2x_2 = 0.689 : 0.311$, the significance ratio of output 1, output 2, and output 3 is $u_1y_1 : u_2y_2 : u_3y_3 = 0 : 0.998 : 0 = 0 : 1 : 0$; as for DEA-R, the significance ratio of input 1 to input 2 is $(w_{11} + w_{12} + w_{13}) : (w_{21} + w_{22} + w_{23}) = 0.635 : 0.365$, the significance ratio of output 1, output 2, and output 3 is $(w_{11} + w_{21}) : (w_{12} + w_{22}) : (w_{13} + w_{23}) = 0.064 : 0.936 : 0$. It can thus be seen that, as for DMU 4, the optimal weights selected by CCR and DEA-R are fully inconsistent.

According to past studies, three factors led to underestimations by CCR, which are the amount of selectable weights, hypothesis of restricted weights, and sum of ratios. First, the case study discussed the influence of weight amounts on the efficiency of both DEA-R and CCR. In the case study, only 5 weights could be changed by CCR, and 6 by DEA-R. In other words, DEA-R has a broader space for searching optimal solutions than CCR, thus, the efficiency of DEA-R is greater than CCR. Some scholars may cast doubt on the validity of DEA-R, which was validated in another research paper. In addition, some scholars suggested that a broader search ability requires more time to search the optimal solution. However, since

Table 4
The optimal weight of efficient DMU.

	$\Delta\theta_o$	v_1x_1	v_2x_2	u_1y_1	u_2y_2	u_3y_3	w_{11}	w_{12}	w_{13}	w_{21}	w_{22}	w_{23}
04	0.004	0.689	0.311	0.000	0.998	0.000	0.064	0.571	0.000	0.000	0.365	0.000
07	0.000	0.816	0.184	2.170	0.000	0.000	0.816	0.000	0.000	0.184	0.000	0.000
08	0.225	1.000	0.000	0.262	0.751	0.301	0.324	0.000	0.676	0.000	0.000	0.000
09	0.202	0.113	0.887	0.375	0.701	0.000	0.072	0.000	0.000	0.495	0.194	0.240
13	0.000	0.000	1.000	0.000	0.000	1.952	0.000	0.000	0.000	0.000	0.000	1.000
18	0.019	0.638	0.362	0.301	0.704	0.000	0.649	0.022	0.000	0.000	0.329	0.000

there are few DMUs or variables for most cases, and both CCR and DEA-R belong to linear LP, the influences caused by the variables or the DMU were insignificant. Therefore, it is believed that underestimation exists primarily due to the narrow field of selectable weight amounts.

Next, the second reason for underestimation is discussed as weight restrictions. Take the simplified DMU 4, for example, as for CCR, the significance ratio of output 1, output 2, and output 3 is u_1y_1:u_2y_2:u_3y_3 = 0:1:0; as for DEA-R, the significance ratio of output 1, output 2, and output 3 is $(w_{11}+w_{21})$:$(w_{12}+w_{22})$:$(w_{13}+w_{23})$ = 0.064:0.936:0. In other words, both CCR and DEA-R judge the DMU has no advantage in terms of output 3. Thus, output 3 is eliminated to recalculate efficiency, and it is found that the efficiency remains unchanged. As seen, even if the amount of changeable weights is the same, the efficiency of a DEA-R-based model is greater than that of a CCR-based model. The weights selected by DEA-R were analyzed in detail, of which w_{11} = 0.064, w_{21} = 0, indicating input 1 has a small advantage over output 1, but input 2 has no advantage over output 1. Among the weights selected by CCR, u_1y_1 = 0, indicates output 1 has no advantage. As for a single output, CCR cannot treat input 1 and input 2 separately, thus, the advantage disappears when a favorable input 1 vs. output 1 is combined with unfavorable input 2 vs. output 1. Therein lies the restriction of CCR, inputs cannot be individually analyzed; whereas, DEA-R can analyze the advantages of every input. Take DMU 18 for example, the hypothesis of weight restriction influences efficiency from another perspective. After output 3 is removed, the recalculated efficiency does not change. As for CCR, the ratio of input 1 to input 2 is v_1x_1:v_2x_2 = 0.638:0.362; as for DEA-R, the ratio of input 1 to input 2 is $(w_{11}+w_{12})$:$(w_{21}+w_{22})$ = 0.671:0.329. These two models have similar viewpoints on input: input 1 has a comparative advantage, followed by input 2. However, the viewpoint of the output is very inconsistent. CCR judges output 2 as having greater advantages (u_1y_1:u_2y_2 = 0.301:0.704), whereas, DEA-R judges output 1 as having greater advantages ($(w_{11}+w_{21})$:$(w_{12}+w_{22})$ = 0.649:0.351). Concerning the details of the weights of DEA-R, for input 1, DEA-R judges production output 1 has the greater advantage (w_{11} = 0.649), and production output 2 has little advantage (w_{12} = 0.022); for input 2, production output 1 has no advantage (w_{21} = 0.0), production output 2 has certain advantages (w_{22} = 0.329). DEA-R separately evaluates the influence of input 1/input 2 on output, the advantage of input 1 lies in production output 1, and that of input 2 lies in production output 2. However, CCR places consistent restrictions on the significance of output 1 and output 2, and the advantage of individual input/output cannot be presented. In the DEA-R model, w_{11}:w_{12} = w_{21}:w_{22} can be used to express this weight restriction. The efficiency rating of CCR is smaller than that of DEA-R due to this redundant weight restriction. The above-specified is the second reason for the greater efficiency of DEA-R. Such a weight restriction is both redundant and unreasonable in terms of medical practice. Despite the cooperation and mutual influence of sickbed management and physician management systems (different management systems), a model without such a weight restriction (e.g. Super-DEA-R) could evaluate efficient solutions in a more effective and reasonable manner.

Finally, this research finds that the influence of the sum of ratios accounts for the greater efficiency of DEA-R-I. However, proper DMUs cannot be found to address this issue. In sum, lesser weight amounts and redundant weight restrictions are major influential factors in the underestimating efficiency of CCR.

Next, optimal weights, without instances of underestimating DMUs, are discussed to increase understanding of the estimating procedures of CCR in order to judge inefficiency, according to the weights of DEA-R. Efficient DMUs without efficiency differences include: DMU 7 and DMU 13. As for these two groups of DMUs, both CCR and DEA-R judged the same most favorable weights. The optimal weight of DMU 7 was analyzed first. As for CCR, the significance ratio of input 1 to input 2 is v_1x_1:v_2x_2 = 0.816:0.184, the significance ratio of output 1, output 2, and output 3 is u_1y_1:u_2y_2:u_3y_3 = 2.170:0:0 = 1:0:0. As for DEA-R, the significance ratio of input 1 to input 2 is $(w_{11}+w_{12}+w_{13})$:$(w_{21}+w_{22}+w_{23})$ = 0.816:0.184, the significance ratio of output 1, output 2, and output 3 is $(w_{11}+w_{21})$:$(w_{12}+w_{22})$:$(w_{13}+w_{23})$ = 1:0:0. It can thus be seen that, for DMU 7, the optimal weights selected by CCR and DEA-R are consistent. Next, DMU 13 is analyzed. As for CCR, the significance ratio of input 1 to input 2 is v_1x_1:v_2x_2 = 0:1, the significance ratio of output 1, output 2, and output 3 is u_1y_1:u_2y_2:u_3y_3 = 0:0:1.952 = 0:0:1. As for DEA-R, the significance ratio of input 1 to input 2 is $(w_{11}+w_{12}+w_{13})$:$(w_{21}+w_{22}+w_{23})$ = 0:1, the significance ratio of output 1, output 2, and output 3 is $(w_{11}+w_{21})$:$(w_{12}+w_{22})$:$(w_{13}+w_{23})$ = 0:0:1. It is estimated that, "consistent weight selection of CCR and DEA-R" is possibly associated with "non-estimated CCR efficiency".

As for DMU with the same efficiency of CCR-I and DEA-R-I, it is found that all optimal weights have a characteristic, namely, the weights greater than zero are related to the same output, and the weights not related to this output are zero. Take DMU 13 for example, when the optimal weights of different outputs are compared, $(w_{11}+w_{21})$:$(w_{12}+w_{22})$:$(w_{13}+w_{23})$ = 0:0:1, the weights greater than zero are related to output 3, and the weights related to output 1 and output 2 are smaller than zero. This circumstance is similar to DMU 7, $(w_{11}+w_{21})$:$(w_{12}+w_{22})$:$(w_{13}+w_{23})$ = 1:0:0, the weights greater than zero are related to output 1, and the weights related to output 2 and output 3 are smaller than zero. Unlike DMU 7 and 13, the weights of DMU 4, 8, 9, and 18 are correlated with over one output, and the efficiency of CCR-I also differs from DEA-R-I. Two hypotheses were made according to this phenomenon. The first is: when DEA-R-I exists and the weights are concentrated on an output, the efficiency of DEA-R is the same as that of CCR-I, namely, no underestimation occurs. The second hypothesis is: when DEA-R-I exists but the weights are not concentrated on an output, the efficiency of DEA-R is greater than that of CCR-I, namely, underestimation occurs.

5. Comparison of mathematical models

Based on the observations of optimal weights, this paper proposes two hypotheses. In the following, mathematical models are applied to compare and validate the accuracy of the two hypotheses.

H_1. When DEA-R-I weights are concentrated on one output, the CCR-I efficiency, $\bar{\theta}_o$, and DEA-R-I efficiency, θ_o, are the same.

Proof 1. This paper first states that when there is only one output, CCR-I model can be converted into DEA-R-I model, which indicates that when there is one output, CCR-I efficiency, $\bar{\theta}'_o$, and DEA-R-I efficiency, θ'_o, are the same. In addition, when weights are concentrated on one output, as other outputs are ignored, the efficiency of the multiple output model is the same as that of a single output model. Based on the above concepts, the first hypothesis can be proved. When there is one output, the CCR-I model can be converted into the DEA-R-I model; Eq. (1) of CCR model is $\bar{\theta}'_o = u'_1 \times y_{1o}$, and this relationship can be rewritten as $u'_1 = \frac{\theta'_o}{y_{1o}}$. When substituted into Eq. (2) with only one output, thus, $\sum_{i=1}^{m} v_i \times x_{ij} \geqslant \frac{\theta'_o}{y_{1o}} \times y_{1j}$. Let $v'_i = \frac{W'_{i1}}{x_{io}}$, and substitute the above equation to have Eq. (10). Substitute $v'_i = \frac{W'_{i1}}{x_{io}}$ into Eq. (3) with only one output, we can have Eq. (11). Thus, the CCR model with only one output can be converted to Eqs. (9)–(12)

$$\max \quad \theta'_o \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} W'_{i1} \frac{x_{ij}}{x_{io}} \geqslant \theta'_o \frac{y_{1j}}{y_{1o}}, \quad j = 1, \ldots, n, \ j \neq o \tag{10}$$

$$\sum_{i=1}^{m} W'_{i1} = 1 \tag{11}$$

$$W'_{i1} \geqslant \varepsilon > 0, \quad \theta_o \geqslant \varepsilon > 0 \tag{12}$$

Comparing Eqs. (9)–(12) with DEA-R-I equations (Eqs. (5)–(8)), it is found that when there is only one output, such as in DEA-R-I, CCR-I is equal to DEA-R-I. In other words, CCR-I efficiency, $\bar{\theta}'_o$, equals to DEA-R-I efficiency, θ'_o. Moreover, it is known that the efficiency, $\bar{\theta}'_o$, of CCR-I with only one output is the same as the CCR-I efficiency, $\bar{\theta}_o$, in which the weights are concentrated on one output, while DEA-R-I efficiency, θ'_o, with one output, is the same as the DEA-R-I efficiency, θ_o, in which the weights are concentrated on one output. Based on the above, it is known that weight-concentrated CCR-I efficiency, $\bar{\theta}_o =$ CCR efficiency with one output, $\bar{\theta}'_o$; DEA-R-I efficiency with one output, θ'_o = weight-concentrated DEA-R-I efficiency, θ_o. Therefore, it is proven that when the weights of DEA-R-I are concentrated on one output, CCR-I efficiency, $\bar{\theta}_o$, and DEA-R-I efficiency, θ_o, are the same. □

H$_2$. When DEA-R-I weights are not concentrated on multiple outputs, DEA-R-I efficiency, θ_o, and CCR-I efficiency, $\bar{\theta}_o$, are not the same, with the exception of, every output of the referenced DMU is the same times of corresponding output of targeted DMU.

Proof 2. First, since CCR efficiency is not subjected to the condition of output multiplying or dividing one number, the original x_{ij} is divided by the input of target DMU, x_{io}, and is equal to x'_{ij}, the original y_{rj} is divided by the input of target DMU, y_{ro}, and is equal to y'_{rj}. If the optimal weight of CCR is expressed in the concept of ratio, it can be written as $v_i = p_i t''$, $u_r = a_r t'$. Since constraint (3) $\sum_{i=1}^{m} v_i \times x_{io} = 1$ is divided by x_{io}, and converted to obtain $\sum_{i=1}^{m} v_i = 1$. The above relation is substituted into $v_i = p_i t''$ to have $t'' \sum_{i=1}^{m} p_i = \sum_{i=1}^{m} v_i = 1$, and converted to $v_i = \frac{p_i}{\sum_{i=1}^{m} p_i}$.

Moreover, since constraint (2) $\sum_{i=1}^{m} v_i \times x'_{ij} \geqslant \sum_{r=1}^{s} u_r \times y'_{rj}$, $j = 1, \ldots, n, \ j \neq o$ is true to the referring DMU, thus, $1 = \frac{\sum_{r=1}^{s} u_r \times y'_{rj}}{\sum_{i=1}^{m} v_i \times x'_{ij}} j = \textit{reference DMU}$, and is substituted into $v_i = \frac{p_i}{\sum_{i=1}^{m} p_i}$ and $u_r = a_r t'$ to have $t' = \frac{\left(\sum_{i=1}^{m} p_i x'_i / \sum_{i=1}^{m} p_i\right)}{\sum_{r=1}^{s} a_r y'_{rj}}$. Substitute $t' = \frac{\left(\sum_{i=1}^{m} p_i x'_i / \sum_{i=1}^{m} p_i\right)}{\sum_{r=1}^{s} a_r y'_{rj}}$ into CCR efficiency, $\bar{\theta}_o$, we have: $\sum_{r=1}^{s} u_r \times y'_{ro} = \sum_{r=1}^{s} u_r = t' \sum_{r=1}^{s} a_r = \frac{\sum_{r=1}^{s} a_r \times \left(\sum_{i=1}^{m} p_i x'_i / \sum_{i=1}^{m} p_i\right)}{\sum_{r=1}^{s} a_r y'_{rj}} = \frac{\sum_{r=1}^{s} a_r}{\sum_{i=1}^{m} p_i} \times \frac{\sum_{i=1}^{m} p_i x'_i}{\sum_{r=1}^{s} a_r y'_{rj}}$.

Next, in the discussion of DEA-R-I efficiency, corresponding to the optimal of CCR, $v_i = p_i t''$, $u_r = a_r t'$, in the previous section, it is known that the corresponding weight of DEA-R is $w_{ir} = p_i a_r t$, thus, when substituted into constraint (7) $\sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} = 1$, we have $t = \frac{1}{\sum_{i=1}^{m} \sum_{r=1}^{s} p_i a_r} = \frac{1}{\sum_{i=1}^{m} p_i \times \sum_{r=1}^{s} a_r}$. Substitute t into constraint (7), then $\sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} \frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \theta_o$, $j = 1, \ldots, n, \ j \neq o$. Similar to CCR, the equation for DMU is valid, thus, θ_o can be expressed as $\frac{\sum_{i=1}^{m} \sum_{r=1}^{s} p_i x'_{ij} \times a_r \frac{1}{y'_{rj}}}{\sum_{i=1}^{m} p_i \times \sum_{r=1}^{s} a_r} = \frac{\sum_{i=1}^{m} p_i x'_{ij} \times \sum_{r=1}^{s} a_r \frac{1}{y'_{rj}}}{\sum_{i=1}^{m} p_i \times \sum_{r=1}^{s} a_r} j = \textit{reference DMU}$.

Lastly, CCR efficiency, $\bar{\theta}_o$, and corresponding DEA-R-I efficiency, θ_o, are compared. $\frac{\theta_o}{\bar{\theta}_o} = \left[\frac{\sum_{i=1}^{m} p_i x'_{ij} \times \sum_{r=1}^{s} a_r \frac{1}{y'_{rj}}}{\sum_{i=1}^{m} p_i \times \sum_{r=1}^{s} a_r}\right] \Big/ \left[\frac{\sum_{r=1}^{s} a_r}{\sum_{i=1}^{m} p_i} \times \frac{\sum_{i=1}^{m} p_i x'_i}{\sum_{r=1}^{s} a_r y'_{rj}}\right]$

$$= \frac{\sum_{r=1}^{s} a_r \frac{1}{y'_{rj} \times \sum_{r=1}^{s} a_r y'_{rj}}}{\sum_{r=1}^{s} a_r \times \sum_{r=1}^{s} a_r} = \frac{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} \frac{y'_{\hat{r}j}}{y'_{rj}}}{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}}}$$

$$= \frac{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} - \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} \frac{y'_{\hat{r}j}}{y'_{rj}}}{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}}}$$

$$= \frac{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} \frac{-2y'_{rj} y'_{\hat{r}j} + y'^2_{rj} + y'^2_{\hat{r}j}}{y'_{rj} y'_{\hat{r}j}}}{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}}}$$

$= \frac{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} + \sum_{r=1}^{s} \sum_{\hat{r}=r+1}^{s} a_r a_{\hat{r}} \frac{(y'_{rj} - y'^2_{\hat{r}j})}{y'_{rj} y'_{\hat{r}j}}}{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}}}$. When the optimal weights are concentrated on one output, $\frac{\theta_o}{\bar{\theta}_o} = \frac{a_{\tilde{r}}^2}{a_{\tilde{r}}^2} = 1$, or when $y'_{rj} = y'_{\hat{r}j}$, $r = 1, \ldots, s$, $\hat{r} = r + 1, \ldots, s$ $\frac{\theta_o}{\bar{\theta}_o} = \frac{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}} + 0}{\sum_{r=1}^{s} a_r^2 + \sum_{r=1}^{s} \sum_{\hat{r} \neq r, \hat{r}=1}^{s} a_r a_{\hat{r}}} = 1$. Otherwise, $\frac{\theta_o}{\bar{\theta}_o} > 1$. Thus, it is proven that, when DEA-R-I weights are not concentrated on multiple outputs, DEA-R-I efficiency, θ_o, and CCR-I efficiency, $\bar{\theta}_o$, are not the same, with the exception of, every output of the referenced DMU is the same times of corresponding output of the targeted DMU. □

6. Conclusions

Based on past studies on underestimation, this paper suggests that the CCR-based model has occasional underestimations when evaluating high efficiency DMUs, and thus, proposes to use Super-DEA to discuss the underestimation problems of DMUs. There are four main findings: (1) in this case, since the underestimation problem of a high efficiency DMU is more serious than that in the low efficiency DMU, this model is necessary; (2) based on the comparison and analysis of optimal weights, it is known that the number of weights is less, and the unreasonable weight limitation assumption leads to underestimations in efficiency when using $(\sum vx)/(\sum uy)$ or $(\sum uy)/(\sum vx)$ as the base of the model; (3) to determine exactly when underestimation will occur, this study analyzes the DMU that was not underestimated from another perspective, and finds that when DEA-R-I weights are concentrated on one output, and DEA-R-I efficiency, θ_o, and CCR-I efficiency, $\bar{\theta}_o$, are the same, there is no underestimation; (4) it is proven that when DEA-R-I weights are not concentrated on one output, DEA-R-I efficiency, θ_o, and CCR-I efficiency, $\bar{\theta}_o$, are not the same; with the exception of every output of referenced DMU is the same times of corresponding output of the targeted DMU. Therefore, it is demonstrated that after executing DEA-R, whether CCR is underestimated can be determined. In other words, DEA-R not only can be used to replace CCR to evaluate efficiency, and thus, avoid underestimations and weight limits, but also to predict whether CCR underestimation would occur.

The examples of special subsidies for further education in Taiwan, merger of DRAM factories, and evaluation of hospitals have demonstrated the practical value of this study. In the first example, the Taiwanese government, with the goal of assisting the colleges of universities to become top colleges recognized worldwide, proposed the Five-year Fifth Billion plan to fund the development or research facilities of colleges. However, since different colleges can excel in their own domains, the scope of subsidy may be too broad. Therefore, it is no longer necessary to discuss whether schools should possess a leading position in their respective domains, instead, the focus should be the magnitude of the achievements. Only by funding the colleges with the greatest magnitude could the colleges be further advanced. As a result, a more accurate model is needed to calculate the efficiency of DMUs. In the second example, due to the financial crisis and long-term fluctuation, the DRAM industry is facing serious obstacles. Thus, the government aims to integrate DRAM factories in order to reach an appropriate scale. However, almost all DRAM companies are experiencing losses. Thus, it is necessary to identify the strengths of each company and the leading magnitude in order to determine the main body of mergers.

From another perspective, the government would assist inefficient companies to merge with efficient companies to improve efficiency. Similarly, in DMU 4, if the efficient DMU is underestimated as an inefficient DMU, it would cause the company to be merged. In such case, a more accurate model is needed to calculate the efficiency of DMU. Lastly, to effectively control the national health insurance budget, in addition to limiting the global budget, the government also plans to downgrade inefficient hospitals to reduce expenditures. If a hospital is indeed efficient, but determined as inefficient, it would be downgraded, causing serious problems for the hospital. After downgraded, the payment for each outpatient visit and hospitalization would be reduced, thus, the total revenue of the hospital would also be reduced. The hospital may need to downsize the staffs or increase the physicians' working hours to maintain the total revenue. As a result, doctors would not have sufficient time for research, and the management strategies of the hospital may need to be drastically changed (transforming from a research and medical care institution to a sole medical care institution). As seen, accurate evaluations of DMU efficiency are very important in practice. This study developed a new DEA model based on Super-high and ratio concepts. This model not only can avoid underestimating the efficiency of efficient DMU, but can also be used to determine whether underestimation would occur. In the future, this model can be employed to evaluate efficiency and prevent problems caused by underestimation.

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