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Adomian's decomposition method for eigenvalue problems

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We extend the *Adomian's decomposition method* to work for the general eigenvalue problems, in addition to the existing applications of the method to boundary and initial value problems with nonlinearity. We develop the *Hamiltonian inverse iteration* method which will provide the ground state eigenvalue and the explicit form eigenfunction within a few iterations. The method for finding the excited states is also proposed. We present a space partition method for the case that the usual way of series expansion failed to converge.

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I. INTRODUCTION

Adomian's method solves nonlinear differential equations with decompositions. Neither linearization nor perturbation is applied to the nonlinear part. The method has been widely applied to various domains in science and engineering, but is less popular in physics. Actually, in Chap. 14 of Adomian's comprehensive book [1], he treated many physical topics, namely, the Navier-Stokes equations, onset of turbulence, Burger's equation, nonlinear transport, advection-diffusion equation, Korteweg–de Vries equation, nonlinear Schrödinger equation (NLSE), and classical *N*-body dynamics, etc. It shows that *Adomian's decomposition method* (ADM) is extremely versatile in nonlinear physical problems. For some other examples, Adomian and co-workers also formulated the solutions for Thomas-Fermi equation [2] and the Ginzburg-Landau equation [3]. Wazwaz employed ADM to give the soliton and periodic solutions of the Boussinesq equation [4]. Abbaoui *et al.* discussed the convergence of the ADM [5]. Guellal *et al.* gave the ADM explicit solution of the Lorenz system [6].

The ADM is generally applicable to nonlinear differential equations for either initial value problems or boundary problems. The basic theory is clearly described in Adomian's book [1]. On the other hand, we are not able to find out systematic treatment for the eigenvalue problems by ADM. Since the eigenvalue problem is fundamentally important for the structure of a system, the pursuit of ADM for the eigenvalue problem is a worthwhile work. It is, nevertheless, not straightforward. Also, the ADM gives an explicit form of solutions that the numerical grids method cannot do. Thus the ADM treatment of the eigenvalue problem is valuable to computational physics. In this paper, we develop the method for solving the eigenvalue problems by ADM. We will briefly describe the ADM first, and then present our method for the eigenvalue problem. Some paradigmatic examples of both linear and nonlinear eigenvalue equations are given.

The paper is organized as follows. In Sec. II, we introduce the Hamiltomian inverse iteration scheme for the ADM of eigenvalue problems. In Sec. III, we apply the method to the problem of a particle in a box. In Sec. IV, the method is applied to the simple harmonic oscillator. Section V is a treatment of the anharmonic oscillator, and in Sec. VI, we try to solve the nonlinear Gross-Pitaevskii equation that describes the Bose-Einstein condensate by the new scheme. We find that the straightforward way of ADM failed to converge. We introduce in Sec. VII the space partition method to overcome the trouble of divergence encountered in the previous section. Section VII is devoted to concluding remarks.

II. HAMILTONIAN INVERSE ITERATION

Consider the general eigenvalue problem

$$Hu(x) = \lambda u(x), \tag{1}$$

where

$$H = L + V(u(x)). \tag{2}$$

L is usually a differential operator such as $-1/2(d^2/dx^2)$ and $V(u(x))$ is the potential function, either dependent or independent of $u(x)$. The former case is a linear problem while the latter case is nonlinear and is called a nonlinear Schrödinger equation. We describe the difficulty in the ADM for eigenvalue problems first. Adomian wrote the solution as the sum of decompositions

$$u(x;\epsilon) = \sum_{n=0}^{\infty} \epsilon^n u_n(x). \tag{3}$$

Expand the potential $V(u(x;\epsilon))$,

$$V(u(x;\epsilon)) = \sum_{n=0}^{\infty} \epsilon^n A_n; \tag{4}$$

here we introduce the parameter ϵ to collect the coefficients of same order in both sides to find out the decompositions A_n for a general V. The introduced ϵ is set equal to 1 at last. Some examples of A_n can be found in Ref. [1] and will not be repeated here. Let L^{-1} be the inverse operator of L. Operating the L^{-1} to Eq. (1), we have

$$L^{-1}Lu = \lambda L^{-1}u - L^{-1}V. \tag{5}$$

The solution of decomposition orders are obtained systematically [1]. That is,

$$u_0(x) = c_0 + c_1 x,$$

$$u_{k+1}(x) = \lambda L^{-1} u_k - L^{-1} A_k, \tag{6}$$

where c_0 and c_1 are the integration constants generated by the double integration of L^{-1}. They can be determined either by symmetric property or by the boundary condition. For the eigenvalue problems, the value of λ is unknown, the scheme does not work mainly due to the accumulation of unknown λ into higher orders during the iterations. Thus the ADM to the eigenvalue problems is not straightforward.

The inverse iteration is a powerful procedure to compute the eigenfunctions and eigenvalues of a linear system [7]. The basic idea of our Hamiltonian inverse iteration follows that for linear system. Consider an initial trial solution Ψ_0 of Eq. (1), Ψ_0 in principle will be a linear combination of the eigenfunctions $\{\phi_n\}$:

$$\Psi_0 = \sum_{n=0}^{\infty} c_n \phi_n. \tag{7}$$

Repeatedly applying the inverse Hamiltonian operator to Ψ_0 leads to

$$[H^{-1}]^k \Psi_0 = \sum_{n=0}^{\infty} c_n \frac{\phi_n}{(\lambda_n)^k}. \tag{8}$$

Without loss of generality, we assume the eigenspectrum is $\{0 < \lambda_0 < \lambda_1 < \lambda_2 < \cdots\}$. For sufficiently large value of k, the remaining of iterations will be dominated by the ground state eigenfunction only.

Symbolically, given an initial Ψ_0, and denoting the approximate eigenvalue of the *kth* iteration by λ_k, we have, for the next iteration Ψ_{k+1},

$$H^{-1}\Psi_k = \frac{1}{\lambda_k}\Psi_{k+1}, \tag{9}$$

where the $\Psi's$ are renormalized at the end of each iteration.

Convergence of the inverse iteration toward an eigenstate can be estimated by the *Rayleigh Quotients* (RQ), which are given by

$$\lambda_k = \int \Psi_k H \Psi_{k+1} dx. \tag{10}$$

The eigenvalue of the ground state is the stationary point of $\lambda_k's$,

$$\lim_{k\to\infty} \Psi_k = \phi_0, \quad \text{and} \quad H\phi_0 = \lambda_0 u_0. \tag{11}$$

We call this procedure the *Hamiltonian inverse iterations* (HII). If we project out the obtained ground state ϕ_0 from the initial trial function Ψ_0, the HII will lead to the next higher eigenstate. The excited states can thus be found by HII, too.

Next, we will estimate the number of iteration steps to reach a given accuracy in the eigenvalue. Consider we are solving the nth eigenstate for a system. After k times HII iteration procedures, we may denote the result as a factor of $(1-\epsilon)$ in the component of ϕ_n and a factor of ϵ in the next higher state ϕ_{n+1}. The components in other higher states are negligible through the HII scheme. So the expectation value of the eigenvalue is

$$\langle \lambda_n \rangle_k = (1-\epsilon)\lambda_n + \epsilon \lambda_{n+1}, \tag{12}$$

hence the error is

$$\delta_k = \langle \lambda_n \rangle_k - \lambda_n = \epsilon(\lambda_{n+1} - \lambda_n). \tag{13}$$

Iterate one more time, the component in ϕ_n becomes $C_{k+1}(1-\epsilon)/\lambda_n$, and the component of ϕ_{n+1} is $C_{k+1}\,\epsilon/\lambda_{n+1}$, where C_{k+1} is the normalization constant. Assume ϵ is very small, then

$$C_{k+1} = \frac{1}{(1-\epsilon)/\lambda_n + \epsilon/\lambda_{n+1}} \simeq \lambda_n \left[1 + \epsilon\left(1 - \frac{\lambda_n}{\lambda_{n+1}}\right)\right]. \tag{14}$$

Thus the expectation of the eigenvalue will be

$$\langle \lambda_n \rangle_{k+1} \simeq \left(1 - \epsilon\frac{\lambda_n}{\lambda_{n+1}}\right)\lambda_n + \frac{\lambda_n}{\lambda_{n+1}}\epsilon\lambda_{n+1}. \tag{15}$$

The error now will be

$$\delta_{k+1} = \langle \lambda_n \rangle_{k+1} - \lambda_n = \epsilon\frac{\lambda_n}{\lambda_{n+1}}(\lambda_{n+1} - \lambda_n) = \left(\frac{\lambda_n}{\lambda_{n+1}}\right)\delta_k. \tag{16}$$

It means that when we iterate one more time, the error will be smaller by a factor of λ_n/λ_{n+1}. To reach the Nth decimal place accuracy in eigenvalue, the number of iterations can be estimated to be $N/\log(\lambda_{n+1}/\lambda_n)$.

III. EXAMPLE OF THE PARTICLE IN A BOX

We start to explore the method with the simple problem of a particle in a box. Its eigenstates are analytically known. The potential $V(x)$ is

$$V(x) = \begin{cases} \infty, & x < -\dfrac{\pi}{2}, \\ 0, & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}, \\ \infty, & \dfrac{\pi}{2} < x. \end{cases} \tag{17}$$

The Hamiltonian operator for the particle inside the box is

$$H = -\frac{1}{2}\frac{d^2}{dx^2}, \tag{18}$$

and the boundary conditions at the box ends are

$$\psi\left(x = \pm\frac{\pi}{2}\right) = 0. \tag{19}$$

Using the notation $L_{xx} = d^2/dx^2$ and $H^{-1} = -2L_{xx}^{-1}$, by the HII method, the relationship between consecutive iterations is

$$\Psi_1 = H^{-1}\Psi_0 = -2L_{xx}^{-1}\Psi_0. \tag{20}$$

Assuming the decomposition forms are

$$\Psi_0(x) = \sum_{n=0} a_{0,n} x^n, \quad \text{and} \quad \Psi_1(x) = \sum_{n=0} a_{1,n} x^n, \tag{21}$$

then the HII leads to the following coefficient relations:

$$a_{1,0} = C_1 \quad \text{(an unknown integration constant)},$$

$$a_{1,1} = C_2 \quad \text{(an unknown integration constant)},$$

$$\cdots$$

$$a_{1,k} = -\frac{2}{k(k-1)} a_{0,k-2}. \tag{22}$$

We can assign the unknown constants by the parity of the wave function in this case. For the ground state, $C_2 = 0$. we arbitrarily use the starting trial function $\Psi_0(x) = 1 - (2x/\pi)^2$. The Ψ_1 derived from Eq. (15) is used as the new trial function. We iterate to $k = 7$, the eigenvalue obtained is accurate to 15 decimal places. And the obtained coefficients give us an explicit form of the ground state eigenfunction. For the first excited state, symmetry requires $C_1 = 0$. We use $\Psi_0(x) = x[1 - (2x/\pi)^2]$, the iterations to $k = 16$ provides the eigenvalue accurate to 10 decimal places, and iterations to $k = 24$ leads to 15 decimal places accuracy.

IV. EXAMPLE OF THE SIMPLE HARMONIC OSCILLATOR

The next example is the classic case of the simple harmonic oscillator. Again, the eigenstates are analytically known. The potential $V(x)$ now is

$$V(x) = \frac{1}{2} x^2, \tag{23}$$

with the boundary condition

$$\psi(x \to \pm\infty) \to 0. \tag{24}$$

By the HII method, the relation of iteration is

$$\Psi_0 = -\frac{1}{2} L_{xx} \Psi_1 + \frac{1}{2} x^2 \Psi_1. \tag{25}$$

Assume the decomposition forms

$$\Psi_i(x) = \sum_{n=0} a_{i,n} x^n, \quad \text{with} \quad i = 0, 1 \tag{26}$$

and the coefficient relations from HII are

$$a_{0,0} = -\frac{2 \times 1}{2} a_{1,2},$$

$$a_{0,1} = -\frac{3 \times 2}{2} a_{1,3},$$

$$a_{0,2} = -\frac{4 \times 3}{2} a_{1,4} + \frac{1}{2} a_{1,0},$$

$$\cdots$$

$$a_{0,k} = -\frac{(k+2)(k+1)}{2} a_{1,k+2} + \frac{1}{2} a_{1,k-2}, \quad k > 2. \tag{27}$$

In terms of the power of x,

$$x^0: \quad a_{1,0} = C_1 \quad \text{(an unknown integration constant)},$$

$$x^1: \quad a_{1,1} = C_2 \quad \text{(an unknown integration constant)},$$

$$x^2: \quad a_{1,2} = -a_{0,0},$$

$$\cdots$$

$$x^k: \; a_{1,k} = \frac{1}{k(k-1)} (a_{1,k-4} - 2a_{0,k-2}). \tag{28}$$

To find the integration constants C_1 and C_2, we regroup the wave function Ψ_1 into three parts

$$\Psi_1 = \Psi_\alpha + C_1 \Psi_\beta + C_2 \Psi_\gamma, \tag{29}$$

where

$$\Psi_\alpha = \sum_{n=0} a_{\alpha,n} x^n, \quad \Psi_\beta = \sum_{n=0} a_{\beta,n} x^n, \quad \Psi_\gamma = \sum_{n=0} a_{\gamma,n} x^n. \tag{30}$$

Then we have the following relationships for the three series:

$$x^0: \; a_{\alpha,0} = 0,$$

$$x^1: \; a_{\alpha,1} = 0,$$

$$x^2: \; a_{\alpha,2} = \frac{-2}{2 \times 1} a_{0,0},$$

$$x^3: \; a_{\alpha,3} = \frac{-2}{3 \times 2} a_{0,1},$$

$$\cdots$$

$$x^k: \; a_{\alpha,k} = \frac{1}{k(k-1)} (a_{\alpha,k-4} - 2a_{0,k-2}); \tag{31}$$

and

$$x^0: \; a_{\beta,0} = 1,$$

$$x^1: \; a_{\beta,1} = 0,$$

$$x^2: \; a_{\beta,2} = 0,$$

$$x^3: \; a_{\beta,3} = 0,$$

$$\cdots$$

$$x^k: a_{\beta,k} = \frac{1}{k(k-1)} a_{\beta,k-4}; \tag{32}$$

and

$$x^0: a_{\gamma,0} = 0,$$

$$x^1: a_{\gamma,1} = 1,$$

$$x^2: a_{\gamma,2} = 0,$$

$$x^3: a_{\gamma,3} = 0,$$

$$\cdots$$

$$x^k: a_{\gamma,k} = \frac{1}{k(k+1)} a_{\gamma,k-4}\,. \tag{33}$$

We choose the boundary $\Psi_1(L)=0$ at $L=6.0$. For the ground state, we set $\Psi_0=e^{-x^2}$ and expand it to the power of 200 in x for iteration, at $k=12$ the eigenvalue is accurate to 10 decimal places; and at $k=15$ the accuracy is up to 15 decimal places. For the first excited state, we choose $\Psi_0 = xe^{-x^2}$, at $k=15$ the eigenvalue is accurate to 10 decimal places, and at $k=33$ the accuracy is up to 13 decimal places.

V. EXAMPLE OF THE ANHARMONIC OSCILLATOR

We consider in the following the anharmonic oscillator that was recently treated by the imaginary time propagation method [8]. It provides the calibrations of our method. The potential form is

$$V(x) = \frac{1}{2}x^2 + v_4 x^4 + v_6 x^6, \tag{34}$$

and the boundary condition is the same as the simple harmonic oscillator. By the HII method, the relationship between iterations is

$$\Psi_0 = -\frac{1}{2}L_{xx}\Psi_1 + \left(\frac{1}{2}x^2 + v_4 x^4 + v_6 x^6\right)\Psi_1; \tag{35}$$

as in the previous power series expansion, the coefficient relations are

$$a_{0,0} = -a_{1,2},$$

$$a_{0,1} = -\frac{3\times 2}{2} a_{1,3},$$

$$a_{0,2} = -\frac{4\times 3}{2} a_{1,4} + \frac{1}{2} a_{1,0},$$

$$a_{0,3} = -\frac{5\times 4}{2} a_{1,5} + \frac{1}{2} a_{1,1},$$

$$a_{0,4} = -\frac{6\times 5}{2} a_{1,6} + \frac{1}{2} a_{1,2} + v_4 a_{1,0},$$

$$a_{0,5} = -\frac{7\times 6}{2} a_{1,7} + \frac{1}{2} a_{1,3} + v_4 a_{1,1},$$

$$\cdots$$

$$a_{0,k} = -\frac{(k+2)(k+1)}{2} a_{1,k+2} + \frac{1}{2} a_{1,k-2} + v_4 a_{1,k-4} + v_6 a_{1,k-6}, \quad \text{when} \quad k \geqslant 6.$$

In terms of the power of x,

$$x^0: \quad a_{1,0} = C_1 \quad \text{(an unknown integration constant)},$$

$$x^1: \quad a_{1,1} = C_2 \quad \text{(an unknown integration constant)},$$

$$x^2: \quad a_{1,2} = \frac{-2}{2\times 12} a_{0,0},$$

$$x^3: \quad a_{1,3} = \frac{-2}{3\times 2} a_{0,1},$$

$$x^4: \quad a_{1,4} = \frac{1}{4\times 3}(a_{1,0} - 2a_{0,2}),$$

$$x^5: \quad a_{1,5} = \frac{1}{5\times 4}(a_{1,1} - 2a_{0,3}),$$

$$\cdots$$

$$x^k: \quad a_{1,k} = \frac{1}{k(k-1)}(a_{1,k-4} + v_4 a_{1,k-6} + v_6 a_{1,k-8} - 2a_{0,k-2}). \tag{36}$$

We again regroup the power series into Ψ_α, Ψ_β, and Ψ_γ. The integration constants C_1 and C_2 can be found by Eq. (29). For ground state with $v_4=0.02$, and $v_6=0.01$ [8], we choose $\Psi_0=e^{-x^2/2}$ and expand to the power of x up to 200. Iterations to $k=12$ and set cutoff at $L=4.5$, the eigenvalue is accurate to nine decimal places. For the first excited state, take $\Psi_0 = xe^{-x^2/2}$ and expand to the power of x up to 200. Iterations to $k=20$ gives the eigenvalue accurate up to eight decimal places.

VI. EXAMPLE OF NLSE, THE GROSS-PITAEVSKII EQUATION

In the recent Bose-Einstein condensation experiments of dilute alkali atomic gases [9], the condensate is well described by the mean-field approximation, that is, the Gross-Pitaevskii equation (GPE). Furthermore, if the trap potential is cigar shaped, the GPE is effectively one dimensional [10]. Following the pseudospectral method [11], we have developed an efficient and accurate numerical scheme for solving the GPE. Our pseudospectral method has been calibrated with published calculations [12] and provides a way to justify our new ADM of eigenvalue problems.

Consider the eigenvalue problem of the GPE,

$$\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+g|\Psi|^2\right)\Psi=\mu\Psi, \tag{37}$$

with the order parameter normalized to the number of condensed atoms

$$\int_{-\infty}^{\infty}|\Psi|^2dx=N. \tag{38}$$

By the HII, the iteration relationships are

$$\Psi_0=\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+g|\Psi_0|^2\right)\Psi_1. \tag{39}$$

We expand $|\Psi_0|^2=\rho=\Sigma_i\rho_i x^i$, and the nonlinearity into decompositions

$$g\rho\Psi_1=\sum_i A_i x^i\,. \tag{40}$$

The coefficient relationships are

$$a_{0,0}=A_0-\frac{2\times 1}{2}a_{1,2},$$

$$a_{0,1}=A_1-\frac{3\times 2}{2}a_{1,3},$$

$$\cdots$$

$$a_{0,k}=A_k-\frac{(k+2)(k+1)}{2}a_{1,k+2}+\frac{1}{2}a_{1,k-2}. \tag{41}$$

In terms of the power of x,

$$x^0:\quad a_{1,0}=C_1\quad\text{(an unknown integration constant)},$$

$$x^1:\quad a_{1,1}=C_2\quad\text{(an unknown integration constant)},$$

$$x^2:\quad a_{1,2}=A_0-a_{0,0},$$

$$x^3:\quad a_{1,3}=\frac{2}{3\times 2}(A_1-a_{0,1}),$$

$$\cdots$$

$$x^k:\quad a_{1,k}=\frac{2}{k(k-1)}\left(A_{k-2}-a_{0,k-2}+\frac{1}{2}a_{1,k-4}\right). \tag{42}$$

We again regroup it into Ψ_α,Ψ_β, and Ψ_γ to find the integration constants C_1 and C_2, by Eq. (29). The three nonlinear terms are expanded as

$$A_\alpha=g\rho\Psi_\alpha=\sum_{i=0}A_{\alpha,i}x^i, \tag{43}$$

and

$$A_\beta=g\rho\Psi_\beta=\sum_{i=0}A_{\beta,i}x^i, \tag{44}$$

and

$$A_\gamma=g\rho\Psi_\gamma=\sum_{i=0}A_{\gamma,i}x^i. \tag{45}$$

Then the relationships become

$$x^0:\ a_{\alpha,0}=0,$$

$$x^1:\ a_{\alpha,1}=0,$$

$$x^2:\ a_{\alpha,2}=A_{\alpha,0}-a_{\alpha,0},$$

$$x^3:\ a_{\alpha,3}=\frac{2}{3\times 2}(A_{\alpha,1}-a_{0,1}),$$

$$\cdots$$

$$x^k:\ a_{\alpha,k}=\frac{2}{k(k-1)}\left(A_{\alpha,k-2}-a_{0,k-2}+\frac{1}{2}a_{\alpha,k-4}\right); \tag{46}$$

and

$$x^0:\ a_{\beta,0}=1,$$

$$x^1:\ a_{\beta,1}=0,$$

$$x^2:\ a_{\beta,2}=A_{\beta,0},$$

$$x^3:\ a_{\beta,3}=\frac{2}{3\times 2}(A_{\beta,1}),$$

$$\cdots$$

$$x^k:\ a_{\beta,k}=\frac{1}{k(k-1)}(2A_{\beta,k-2}+a_{\beta,k-4}); \tag{47}$$

and

$$x^0:\ a_{\gamma,0}=0,$$

$$x^1:\ a_{\gamma,1}=1,$$

$$x^2:\ a_{\gamma,2}=A_{\gamma,0},$$

$$x^3:\ a_{\gamma,3}=\frac{2}{3\times 2}(A_{\gamma,1}),$$

$$\cdots$$

$$x^k:\ a_{\gamma,k}=\frac{1}{k(k-1)}(2A_{\gamma,k-2}+a_{\gamma,k-4}). \tag{48}$$

For the trap with parameters $g=0.017\,84$ and $N=100$, by using the trial function $e^{-x^2/2}$, and cutoff at $L=2.38$. We obtain $\lambda=1.1684$ for iterations up to $k=16$. However, if we extend the cutoff L to larger value, the direct use of power series expansion in ADM does not converge, and no solution will be found. So the eigenvalue $\lambda=1.1684$ is not the correct ground state. This happens if the cutoff lies beyond the ra-

dius of convergence for the power series expansion. To overcome the trouble, we develop the space partition method in the next section.

VII. SPACE PARTITION METHOD FOR ADM EIGENVALUE PROBLEMS

To justify the convergence of series expansion, we divide the space $[0,L]$ into p partitions $[X_{m-1},X_m], m=1,2,3,\ldots,p$. And the iteration function Ψ_k in the mth partition is denoted as $\Psi_{k,m}(x)$ with $x\in[X_{m-1},X_m]$. The global solution is given by

$$\Psi_k(x)=\sum_{m=1}^{p}\Psi_{k,m}(x)\chi_m(x),\quad x\in[0,L], \tag{49}$$

where

$$\chi_m(x)=\begin{cases}1, & x\in[X_{m-1},X_m],\\ 0, & x\neq[X_{m-1},X_m],\end{cases} \tag{50}$$

and

$$[0,L]=\bigcup_{m=1}^{p}[X_{m-1},X_m]. \tag{51}$$

In mth partition, note that the potential $V(x)$ become $V(X_{m-1}+x)$.

We choose $L=6$, $p=6$, and $X_m=m$, the connection condition at $x=X_m$ is

$$M_m(1)C_m=M_{m+1}(0)C_{m+1},\quad 1\leqslant m\leqslant p-1, \tag{52}$$

$$M_p(1)C_p=B, \tag{53}$$

where

$$M_m(x)=\begin{bmatrix}1 & 0 & 0\\ \Psi_{\alpha,m}(x) & \Psi_{\beta,m}(x) & \Psi_{\gamma,m}(x)\\ \Psi'_{\alpha,m}(x) & \Psi'_{\beta,m}(x) & \Psi'_{\gamma,m}(x)\end{bmatrix}, \tag{54}$$

where $M_m(0)=I$ and

$$C_m=\begin{bmatrix}1\\ C_{m,1}\\ C_{m,2}\end{bmatrix},\quad B=\begin{bmatrix}1\\ 0\\ \Psi'_k(L)\end{bmatrix}. \tag{55}$$

with unknown $\Psi'_{k,m}(L)$.

For the even parity ground state, $C_{1,1}=0$. we have $2p-1$ unknown constants. The order parameter and its first derivative must be continuous at the junctions of partitions, and $\Psi_k(L)=0$, so the number of constraints is also equal to $2p-1$.

To find the $M_m(1)$,

TABLE I. The eigenvalues of the Gross-Pitaevskii equation calculated by the space partition method for N bosons.

N	Number of iterations	Chemical potential λ
100	10	1.127825
500	16	2.88870
1000	28	4.522398
5000	93	13.1060

$$\begin{aligned}B&=M_p(1)C_p=M_p(1)M_{p-1}(1)C_{p-1}\\ &=M_p(1)M_{p-1}(1)M_{p-2}(1)\cdots M_1(1)C_1\\ &=MC_1,\end{aligned} \tag{56}$$

where $M=M_p(1)M_{p-1}(1)M_{p-2}(1)\cdots M_1(1)$. We determine $C_{1,2}$ and $\Psi_{1,1}$ by the Eq. (56), and the next neighbor order parameter by the Eq. (52). The total Ψ_1 can then be obtained.

With the space partition method, we overcome the problem of convergence. The results of some calculations are shown in Table I.

VIII. CONCLUSION

Adomian's decomposition method together with the Hamiltonian inverse iteration technique provide a simple and powerful tool for the eigenvalue problems. Unlike other numerical grids methods, the ADM technique gives explicit form of solution. Moreover, although we showed only the decomposition solutions for differential equations, the method is applicable to algebraic equations, too.

The inverse iteration for finding the eigenfunctions and eigenvalues is well known in matrix theory, but applications to differential equations are not common. The difficulty lies particularly in writing down the inverse of the Hamiltonian. But with the ADM, we have found a way to do it. Combining the Hamiltonian inverse iteration and Adomian's decomposition method, we have developed an efficient algorithm to solve the general eigenvalue problem. We show that the new method works well for both the linear and nonlinear problems. Even for the nonlinear case, only a few iterations are required to achieve high accurate results, while using our pseudospectral method for Gross-Pitaevskii takes far more steps of iterations.

A drawback of the ADM is the constraint of the boundary condition, especially when the size of boundary is larger than the radius of convergence of the expansion series of the wave function. We develop the partition of the space method and overcome this trouble.

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