Two New Theorems of Motion Instability
with Applications to the Stability of
Free Gyroscopes and Others

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（　Chetaev　）
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Since 1934 Chetaev improved Lyapunov's motion instability theorem and obtained Chetaev motion instability theorem, it will be the first time that this report gives two new motion instability theorems applicable both to autonomous and nonautonomous systems, which are different with Chetaev theorem, and successfully remedy the defect of Chetaev theorem. With the aid of the derivatives of higher order of V function, the motion instability can be determined. By using the first new theorem, the well-known gimbal walk phenominon of free gyroscope can be proved rigorously by direct method. This problem has remained unsolved by direct method for more than sixty years. Also, twelve other applications are also given in this report.

Keywords: motion instability, free gyroscope, Lyapunov stability, direct method

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