行政院國家科學委員會專題研究計畫　期中進度報告

亞洲大學與商學院評比系統設計----達爾菲排序分組法

(1/3)

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亞洲大學與商學院評比系統設計--達爾菲排序分組法(1/3)

Ranking Asia's Universities and Business School by a Delphi Ranking & Grouping Method (1/3)

計畫編號：NSC 93-2213-E-009-071
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一、中英文摘要

政府為提昇大學之國際競爭力，正積極推展 "亞洲第一，世界一百" 計畫，希望在五年內有國內大學院系中心，成為亞洲第一，並有國內的大學名列世界前一百名大學。

國際大學之排名結果頗為各界所重視，惟目前之排名系統(如:US Business Week, Financial Times, Asia Inc.等) 均採簡單加權法，各準則之權重設定缺乏理論。本研究擬發一達爾菲排序分組法(Delphi Ranking & Grouping Method, 簡稱 DRG method)，以評定亞洲地區大學與商學院。DRG method 首先為整理亞洲前一百大大學之客觀資料，接著以通訊調查各大學校、院長對各大學之初步分組(分為 Top 10%, Top 20%, Average, Below 20%, Below 10%)，之後，依調查結果做出排序與分組後，再調查校、院長之意見。DGM 法的優點為依據受測者對各校之粗略分組即可排序，排序完之結果又可再讓受調者再做分組。最後可得到一致性之排序，並可計算出各準則之加權值。

本研究將與遠見雜誌合作試行調查部分。若試行成功，則往後交大將與遠見長期合作，年年發表亞洲地區大學及商學院之評比與排名，建立一有國際公信力之評比系統。

關鍵詞：競爭力, 評比, 達爾菲排序分組法

Abstract

In order to improve the international competitiveness of our universities, R.O.C. Government has initialized the vision of "Asia's best and World's Top 100". It is expected that some of Taiwan's universities may be ranked as top in Asia, and ranked as top 100 around the world. The rank of universities has attracted much attention; however, all current ranking systems (such as US Business Weeks, Asia.com, Asia Inc., Financial Time) use Simple Weighting Method to rank universities. It lacks solid theoretical support for the weights on decision criteria. This study develops a Delphi Ranking & Grouping (DRG) Method to rank the universities and Business Schools in Asia region.

DRG method first lists all related hard data for top 100 Asia universities. Then send questionnaires to the presidents and deans of these universities to ask them to group roughly 100 universities into Top 10%, Top 20%, Average, below 20% and below 10%. The initial ranking and grouping report on these universities is then computed based on the survey. The results will then be sent to the presidents and deans for further evaluation. Finally all universities are ranked and grouped and weights on criteria are computed.

This study will coorporate with Vision Magazine to perform the survey. If the outcome of this study is promising, we hope can build an internationally credited system for ranking Asia's universities and Business Schools.

Keywords: Competitiveness, Evaluation, Delphi Ranking & Grouping Method

二、前言

Every year, many high school graduates and university graduates purchase University Ranking Guides to help them select the right undergraduate program or graduate program that is best suited for them. Although among the quarter million freshmen who participated in the survey done by the Higher Education Research Institute, only 8.6% responded that the rankings were very important to them when selecting colleges or universities (Crissey, 1997). The reasons may lie on the question of ranking methodology. How do we know these rankings are right for the students and rank universities in the way the students needed? How do we know the criteria participated in the ranking system are what the ones students consider important? These are some of the key concerns which should be solved.

Currently, there are many publishers which release various kinds of ranking each year. US News and World Report, for example, started releasing university ranking in with the October issue in late 1980’s. They have realized that in the subsequent years, the October issue had sold many more copies than any other issues. Hence, they decided to start publishing an independent issue for university ranking. In the 1990’s, many other publishers like Time, Newsweek, Money Magazine, and many more have also realized that the market for university ranking is enormous and have started to create their own rankings and publish them. Similarly, Canada, Asia, and Europe all have magazines that do rankings for universities in different regions.

三、研究目的

The ranking guides currently in the market are heavily criticized by many people ranging from educational field to people in the publishing industry. Some of these criticisms are as follow:

(1) To increase the sales, publishers may introduce new measures or change the weightings of measures from year to the next (Gater, 2003).

(2) Some of the factors are highly manipulable, and, as a result, the ranking outcome is meaningless (Leiter, 2003).

(3) Ranking formula and factors participated in the ranking process are constantly changing, so the results are high in variation (Levin, 1997).

In this study, we propose a new ranking method that can help the Decision Makers (DM) rank Decision Making Units (DMUs). The characteristics are listed below:

(1) The model can automatically generate weightings with minimal human influence.

(2) Ranking can still be done with minimum information from Decision Makers, i.e. preferences.

(3) 3D ball representation gives clear view on the correlations.

(4) This model allows DM to add preferences through out the ranking process.

(5) DM can specify groupings for DMUs.

四、文獻探討

Ranking Methodology

There are several rankings published in the market. Each of them has different methodology to rank universities. They vary in criteria selection, assignment of weightings, and raw data, just to name a few. Let us look at few of the more popular ranking systems and their methodology.

- **U.S. News and World Report** (Source: www.usnews.com)

U.S. News ranks business colleges in United States in 2004 and listed 82 of them. They have used three major sections with total of eight criteria for the entire ranking process. These criteria are listed below with their weightings and descriptions.

(1) Quality Assessment (total 40%):

I. *Peer Assessment (25%) – Deans and directors from business schools of accredited programs were asked to rate programs from marginal (1) to outstanding (5). Notice that 56% of them have returned the survey.*

II. *Recruiter Assessment (15%) – Corporate recruiters were also asked to rank the programs which they have hired employee from in the previous year. However, only 32% of them replied the survey.*

(2) Placement Success (total 35%):

I. *Average Starting Salary and Bonus (14%) – This is the mean of starting salary and bonus.*

II. *Percentage of Graduates Employed at Graduation (7%) – The percentage of emplacement rate is measure before the students actually graduate from full-time MBA program.*

III. *Percentage of Graduates Employed 3 Months after Grad (14%) – The percentage of employed graduates three months after completing the full-time MBA program.*

(3) Student Selectivity (total 25%):

I. *Average Undergrad GPA (7.5%) – The average GPA of new students.*

II. *Average GMAT (16.25%) – Average GMAT score of new students who are accepted to the full-time MBA program.*

III. *Acceptance Rate (1.25%) – Percentage of accepted applications.*

From their hard data, we have tried to duplicate their ranking formula and have found a very similar ranking result with identical overall scores. The formula should be very close to

$$Score = \sum_{k=1}^{n} \left(w_k * \frac{C_k - \underline{C_k}}{\overline{C_k} - \underline{C_k}} \right)$$

where n is the total number of criteria and Ck is the value of kth criterion and $\overline{C_k}$ and $\underline{C_k}$ are the maximum and minimum values of kth criteria.

◆ **Financial Times (**Source: www.ft.com)

Unlike U.S. News & World Report, Financial Times (FT) has ranked business schools from all over the world and has listed 100 of them. FT has also selected twenty criteria for the ranking process. The following are those criteria and their weightings.

(1) *Weighted Salary (20%)* – This is the average salary today with adjustment for different industries. Also, this figure is the average salary three years after graduation. (in US dollars)

(2) *Salary Percentage Increase (20%)* – The percentage increase in salary from beginning of MBA program to three years after graduation.

(3) *Value for Money (3%)* – This is calculated by the salary earned by MBA graduates three years after graduation with the course costs and the opportunity cost, while still in school and not employed.

(4) *Career Progress (3%)* – The degree to which alumni have moved up the career ladder three years after graduating. Progression is measured through changes in level of seniority and the size of company in which they are employed.

(5) *Aims Achieved (3%)* – The extent ot which alumni fulfilled their goals or reasons for doing an MBA. This is measured as a percentage of total returns for a school and presented as a rank.

(6) *Placement Success (2%)* – The percentage of 2000 alumni that gained employment with the help of career advice. The data is presented as rank.

(7) *Alumni Recommendation (2%)* – Alumni of 2000 were asked to name three business schools from which they would recruit MBA graduates. The figure represents the number of votes received by each school. The data is presented as a rank.

(8) *International Mobility (6%)* – A rating system that measures the degree of international mobility based on the employment movements of alumni between graduation and today.

(9) *Employed at Three Months (2%)* – the percentage of the most recent graduating class that had gained employment within three months.

(10) *Women Faculty (2%)* – Percentage of female faculty.

(11) *Women Students (2%)* – Percentage of female students.

(12) *Women Board (1%)* – Percentage of female members in the advisory board.

(13) *International faculty (4%)* – The percentage of international students.

(14) *International Students (4%)* – Percentage of the board whose nationality differs from their country of employment.

(15) *International board (2%)* – Percentage of the board whose nationality differs from their country of employment.

(16) *International Experience (2%)* – Weighted average of three criteria that measure international exposure during the course.

(17) *Languages (2%)* – Number of additional languages required on completion of the MBA. Where a proportion of students required another language due to an additional diploma or degree chosen that figure is included in the calculations but not presented in the final table.

(18) *Faculty with Doctorates (5%)* – Percentage of faculty with a doctoral degree.

(19) *FT Doctoral Rating (5%)* – Number of doctoral graduates from the last three academic years with additional weighting for those graduates taking up a faculty position at one of the top 50 school in this year's ranking.

(20) *FT Research Rating (10%)* – a rating of faculty publications in 40 international academic and practitioner journals. Points are accrued by the business school at which the author is presently employed. Adjustment is made for faculty size.

The results and hard data of both U.S. News and World Report and Financial Times are attached in the Appendix section. Both publishers have worked with other companies for data collection. However, they did not explain how the weightings for the criteria were decided. Moreover, perhaps because U.S. News and World Report is the most recognized publisher in university ranking, it receives many criticisms on both the changes on weightings from year to year and the correctness of hard data. On the contrary, Financial Times has fixed their weightings. However the way hard data is presented has been modified from year to year. For example, the criterion "value for money" was a score ranging from 1 to 5 in year 2002 and 2003 ranking. In 2004, this criterion has been changed into "value for money rank". When it was a score from 1 to 5, there can be only 50 different scores and is unlikely that all the variation of the score will be assigned. Hence there are many schools with the same scores. When it changed to rank, only few schools are being ranked as the same, so the variation is larger. This problem arises on more than one criterion in Financial Times' ranking.

Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a method for evaluating the activity performance, especially for organizations such as business firms, government agencies, hospitals, educational institutions, and etc (Cooper etc. 1999). A commonly used measure for efficiency is the output-input ratio. Number of items sold in a store will be an example of the output; number of sales clerk in the store will be the input. Hence, the efficiency of this store, basing on only these two criteria, will simply be NumberOfGoodsSold / NumberOfClerk. These comparable entities are often called Decision Making Units (DMUs).

The purpose of DEA is to empirically estimate the efficient frontier based on the set of available DMUs and assumes that each performance measure can be categorized as either an input or an output (Schrage, 1997). It provides the user information about both efficient and inefficient units along with the efficiency scores and reference sets for inefficient units (Halme etc, 1999). An Efficient Frontier is a line that has at least one DMU point touching it. The DMUs, who touch the EF line, are the most efficient DMUs. The idea of Production Frontier is first discussed by Farrell in 1975 which has three assumptions. The attractive feature of DEA is that it produces efficiency score between 0 and 1.

In 1978, Charnes, Cooper, and Rhodes proposed a DEA model called the CCR model basing on Farrell's single input-output model in 1975. CCR model is designed to measure the cases of multi input and multi output. The following is the pseudo-code for the CCR model.

Ur represents the weighting for rth output criterion and Vi represents the weighting for ith input criterion. They are automatically generated when the score of kth DMU is maximized. Yr and Xi are the output and input criteria.

For each DMU *k*

$$MAX \quad Score_k = \frac{\sum_{r=1}^{s} U_r Y_r}{\sum_{i=1}^{m} V_i X_i}$$

such that

$$Score_k \leq 1$$

$$U_r > 0$$

$$V_i > 0$$

Where

Y_r is the r^{th} output of DMU

X_i is the i^{th} input of DMU

U_r is the weighting for r^{th} output

V_i is the weighting for i^{th} input

In this CCR model, it will calculate the score of each DMU based on the weightings that can maximize the score of current DMU, which means that the nth DMU can obtain the best score with nth set of weightings. Hence, if there are n numbers of DMUs, then there will have n set of weightings. kth set of weighting is determined under the condition that they can maximize the Scorek. All the scores have to be between 0 and 1. Once score of each DMU is determined, it then compares all of them again with their score. The DMU with highest score is the most efficient one.

Analytic Hierarchy Process

The Analytical Hierarchy Process (AHP) was proposed by Saaty in 1980 and his collaborators as a method for establishing priorities in multi-criteria decision making contexts based on variables that do not have exact numerical consequences (Genest, 1996). It also helps people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best.

AHP can be conducted in three steps:

Step1 Perform pairwise comparisons between each DMU on every criterion

In this step, the goal is to obtain the priorities between DMUs for each criterion. To do so, a pairwise comparison has to take place between each DMU with respect to each criterion.

For each criterion, a m by m matrix, where m is the number of DMUs, will be generated and the priority vector will be calculated from this matrix. Priority vector displays the preference orders for each DMU with respect to criteria. Since there are n numbers of criteria, n number of priority vector will be generated at the end.

Step 2 Perform pairwise comparison between each criterion

In the decision making process, not every criterion is quantitatively measurable, so a pairwise comparison between each criterion has to take place in order to specify the importance between each criterion. From the comparison, a set of weightings can be found for score calculation at the last step.

Step 3 Compute final scores for DMUs

With the priority vectors and the weightings for criteria, DM can now calculate the score for each DMU. DMU with the higher score should be the better alternative for the Decision Maker.

Intransitivity

When Decision Makers are making decisions, some do a pairwise comparison with AHP before they make the actual decision. However, AHP does not have a means for detecting an intransitivity situation. An intransitivity is when A > B, B > C, but C > A. This situation is also called logically inconsistent. When there is a cycle exists in the decision process and is not very logical. Hence, the intransitivity detection is a very important process before the any decision is made.

In Gass' study (1998), he presented a way to detect the intransitivity with simple matrix operation.

Theorem

Let **P** be the preference matrix of a preference diagram **D**. Then in $\mathbf{P}^k$**, the (i,j) entry, denoted by** $\mathbf{P}_{i,j}^{(k)}$, is the number of sequences in **D** of length k from node v_i to node v_j. ($\mathbf{P}^k$ is the k^{th} power of **P**)

The theorem states that $P_{i,j}^k$ denotes the number of cycles, with different sequence. Take a preference graph shown in Figure 4.1 as an example. We can generate a tournament matrix from this preference graph. The preference matrix P, Table 4.1, has values of 0 or 1. $P_{i,j}$ is set to 1 if i is smaller than j.

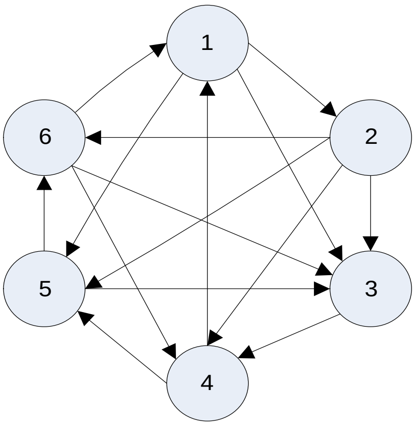

Figure 4.1 Preference Graph of six nodes

Table 0.1 Preference matrix on six nodes

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	0	0	0	1	0	1
P_2	1	0	0	0	0	0
P_3	1	1	0	0	1	1
P_4	0	1	1	0	0	1
P_5	1	1	0	1	0	0
P_6	0	1	0	0	1	0

From this preference matrix, we can apply the theorem to this matrix and look for the cycles. Since the theorem said that the value of P_{ij}^k means there are the same numbers of combinations of sequences in the preference graph of length k from node i to node j. Similarly, if we look at P_{ii}^k, then this will mean the sequence start at node i and come back to node i with the length of k. Hence, we can simply check the diagonal of each P^k for k = 3 up to k = n, where n is the number of nodes.

Table 4.1a to Table 4.1d are the power of preference matrix from P^3 to P^6. In Table 4.1a, we can see that the diagonal has nonzero values. P_{11}^3 is 4, so there are four cycles with the length of 3 and the starting and ending node is P_1. The cycles are (P_1, P_2, P_4, P_1), (P_1, P_3, P_4, P_1), (P_1, P_2, P_6, P_1), and (P_1, P_5, P_6, P_1). With the same technique, it is very easy to find the existence of cycles for any given preference graph. From Table 4.1b to Table 4.1d, it is clear that there are cycles with the length of 4, 5, and 6.

Table 4.1 Preference Matrixes

(a) P^3 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	4	3	0	1	2	1
P_2	0	2	1	0	1	1
P_3	3	4	2	3	1	4
P_4	4	3	0	4	1	2
P_5	2	4	1	1	3	3
P_6	1	1	1	2	0	3

(b) P^4 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	5	4	1	6	1	5
P_2	4	3	0	1	2	1
P_3	7	10	3	4	6	8
P_4	4	7	4	5	2	8
P_5	8	8	1	5	4	4
P_6	2	6	2	1	4	4

(c) P^5 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	6	13	6	6	6	12
P_2	5	4	1	6	1	5
P_3	19	21	4	13	11	14
P_4	13	19	5	6	12	13
P_5	13	14	5	12	5	14
P_6	12	11	1	6	6	5

(d) P^6 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	25	30	6	12	18	18
P_2	6	13	6	6	6	12
P_3	36	42	13	30	18	36
P_4	36	36	6	25	18	24
P_5	24	36	12	18	19	30
P_6	18	18	6	18	6	19

Clustering

Clustering involves dividing a set of data points into non-overlapping into groups, where points in each group are more similar to each other than to points in other groups (Faber, 1994). When a set of data is clustered, every point is assigned to a group and every group can be characterized by a single reference point, normally the average of points in the same group.

There are several techniques in the field of clustering. General clustering techniques are Hierarchical clustering, K-Mean clustering, Incremental clustering, and Probability-based clustering. K-mean clustering is also called Iterative Distance-based clustering. The character "k" in the name of K-mean is the number of groups, or clusters, DM wants to make. The basic idea for K-mean is randomly start with k number of points and assign each data point to one of the reference point in k by calculating the minimal total distance. Once the groups are determined, it then tries to adjust the position of the reference points so that it will locate in the center of corresponding group. The algorithm for the k-mean clustering is shown below.

> **Algorithm for K-mean Clustering:**
>
> (1) Choose k centroid points.
> (2) Calculate the distance of each point to all centroids.
> (3) Get the minimum distance. This data is said belong to the cluster that has minimum distance from this data
> (4) Adjust the centroid location based on the current data updated data.
> (5) Assign all the data to this new centroid.
> (6) Repeat until no data is moving to another cluster anymore.

五、研究方法

The proposed model will be able to generate a set of weightings for criteria based on the preferences given by the decision makers. The model has applied similar idea from Data Envelopment Analysis. In DEA, it is trying to measure the efficiency based on maximizing the score of DMU. However, in the proposed model, it will try to maximize the rank for each DMU instead of score. The concept from Analytic Hierarchy Process is also used to create tournament matrix for ranking by doing pairwise comparison. Gass' technique is also used to ensure the non-existence of intransitivity. Last but not least, the concept from K-mean clustering will be modified to help this ranking method to present the data points on a 3D ball to help DM make decisions.

Ranking and Grouping Models

In this chapter, the ranking and grouping process can be break down into two major parts. First part will deal with the actual ranking and score calculation. The second part is mapping each school onto a 3D ball and clustering these data points. Figure 5.1 shows the entire process of proposed ranking and grouping model.

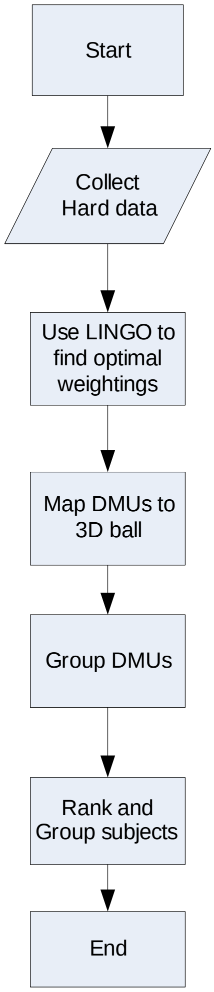

Figure 5.1 Flowchart

◆ **Common Weight Model**

DEA is mainly used for efficiency measurement. The concept of DEA is to calculate the ratio between inputs and outputs, and rank each DMU (Data Making Unit) by their maximized scores. In this ranking objective, however, DEA is not the perfect tool for the ranking process because the most efficient DMU might not be the best choice for DM (Decision Maker). Moreover, , sometimes criteria are hard to distinguish from input or output, the proposed method has modified the traditional DEA method to meet the DMs' requirement without the need to identify inputs and outputs for criteria. This model will automatically ranks and groups the DMUs based on the absolute dominance relationships found in the hard data, so the DMs do not need to worry about assigning weightings for each criterion. This is a big improvement from the traditional ranking systems, which often have controversy on weighting settings.

In the experiments, Lingo8.0 is used as the optimization tool. Given the correct model and inputs, the system will calculate the ideal weights for each criterion, which will allow us to rank the DMUs and map each DMU to a coordinate on 3D ball to help DM visualize the relationships between DMUs, as well as the correlation between DMUs. In this section, the mathematical model and the concept behind it will be discussed in detail and the model will be applied on an example of 20 universities. Before the mathematical model is being discussed, Table 5.1 lists and describes the variables, following is the model.

Table 5.2 Variables for Common Weight Model

Variables	**Descriptions**
m	Total number of DMUs
n	Total number of criteria
$t_{i,j}$	$t_{i,j}$ = 1 if DMU *j* is better than DMU *i*, *else* $t_{i,j}$ = 0
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
M	A large constant number

Common Weight Model (Model 1)

$$Min \quad \sum_{i=1}^{m}\sum_{j\neq i}^{m} t_{i,j} \qquad \textbf{(5.1)}$$

Subject to

$$\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k}-\underline{C_k}}{\overline{C_k}-\underline{C_k}}\right)\right)+\left(M * t_{i,j}\right) \geq \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k}-\underline{C_k}}{\overline{C_k}-\underline{C_k}}\right)\right). \quad \forall\ i, j\ and\ j \neq i \qquad \textbf{(5.2)}$$

$$\sum_{k=1}^{n} w_k = 1 \qquad \textbf{(5.3)}$$

$$w_k \geq \varepsilon, \quad \forall\ k \qquad \textbf{(5.4)}$$

$$t_{i,j} \in \{0,1\} \qquad \textbf{(5.5)}$$

$$t_{i,j} + t_{j,i} \leq 1, \quad \forall\ i, j < i \qquad \textbf{(5.6)}$$

In this model, Lingo will generate a set of weightings for the ranking process. This model ranks the DMUs without DMs worrying about the numbers (weightings). Moreover, these weightings could be more convincing for some DM because these numbers are generated by the system automatically based only on the absolute dominance relationships.

After this model is run by Lingo, Lingo will return a matrix with the size of m by m. This matrix will consist values of only 0 and 1. For t_{ij}, if $t_j > t_i$, then t_{ij} will be set to 1. The sum of each row will represent their rank correspondingly. The objective function (5.1) is trying to maximize the rank of each DMU by minimizing the sum of *t* for each row. Note that the DMU with lower the sum of *t*, the higher rank it will get. Constraint 5.2 is for determining the values of $t_{i,j}$. If $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$ is greater than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$, then $t_{i,j}$ will be 0, since we are minimizing the sum of $t_{i,j}$. On the other hand, if $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$ is smaller than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$, in order to satisfy constraint 5.2, the value of $M * t_{i,j}$ must not be 0, so $t_{i,j}$ will be set to 1.

Constraint 5.3 is to make sure that the sum of weights of all the criteria will be equal to 1. Also, constraint 5.4 ensures that the weights are all non-zero, so every criterion will be taken into account in this ranking process. Constraint 5.5 specifies that $t_{i,j}$ is a binary variable, which can only be 0 or 1. The last constraint is to insure that if *i* is better than *j*, then j can not be better than *i* at the same time.

Once the weights for each criterion are automatically generated by the model, score of each DMU will be calculated by equation 5.7 for future ranking purposes. This score function ensures that the scores are all between 0 and 1 by normalizing the hard data. This will help DM to see the differences in the scores.

$$SCORE_i = \sum_{k=1}^{n}\left(w_k * \frac{(C_{i,k} - \underline{C_k})}{(\overline{C_k} - C_k)}\right) \qquad \textbf{(5.7)}$$

Table 5.2 shows the original hard data of the first twenty universities listed on the Financial Times' 2004 Global MBA Ranking. The data has been normalized so that 1 is the maximum score and 0 is the minimum score. Notice that we have only chosen six criteria that have the heaviest weightings.

Table 5.3 Normalized hard data from Financial Times' 2004 Ranking

Rank in 2004	School name	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT doctoral rank	FT research rank
1	University of Pennsylvania: Wharton	0.836865335	0.855670103	0.74157303	1	1	0.987805
2	Harvard Business School	1	0.525773196	0	0.888889	0.92	1
3	Columbia Business School	0.696863457	1	0.39325843	0.888889	0.88	0.939024
4	Insead	0.553465223	0.257731959	1	0.888889	0.373333	0.890244
4	London Business School	0.42117949	0.680412371	0.87640449	0.888889	0.56	0.780488
4	University of Chicago GSB	0.658188819	0.855670103	0.6741573	0.888889	0.773333	0.963415
7	Stanford University GSB	0.814405559	0.402061856	0.35955056	0.944444	0.866667	0.97561
8	New York University: Stern	0.408235773	0.886597938	0.4494382	0.944444	1	0.865854
9	MIT: Sloan	0.645918112	0.463917526	0.17977528	0.777778	0.973333	0.902439
10	Dartmouth College: Tuck	0.725693358	0.773195876	0.30337079	0.777778	0	0.829268
11	Northwestern University: Kellogg	0.640330558	0.494845361	0.78651685	0.833333	0.746667	0.95122
12	IMD	0.694437488	0	0.47191011	0.722222	0	0.097561
13	Iese Business School	0.018985162	0.907216495	0.97752809	0.944444	0.346667	0.146341
13	Yale School of Management	0.485553747	0.979381443	0.04494382	0.888889	0.12	0.560976
15	Instituto de Empresa	0	0.515463918	0.95505618	0	0	0.04878
16	Cornell University: Johnson	0.490624804	0.618556701	0.28089888	0.666667	0.16	0.743902
17	Georgetown Uni: McDonough	0.359716396	0.824742268	0.53932584	0.5	0	0.402439
17	Uni of N Carolina: Kenan-Flagler	0.303355663	0.659793814	0.69662921	0.555556	0.64	0.853659
19	University of Virginia: Darden	0.606570463	0.742268041	0.23595506	0.888889	0.12	0
20	Duke University: Fuqua	0.375430414	0.505154639	0.68539326	0.555556	0.453333	0.878049

After applying the hard data to the Common-Weight Model, Tables 5.3a and 5.3b displays the results. Table 5.3a shows the new score and the new rankings for these twenty universities along with the original rankings and Table 5.3b shows the new weightings. Please note that due the number of the original criteria, only five were selected from the original twenty criteria. Hence the result varied greatly.

Table 5.4 Results from Common-Weight Model

(a) New scores and rankings

Schools	score	Original Ranking	New Ranking	Change in Rankings
University of Pennsylvania: Wharton	0.845614	1	1	0
Harvard Business School	0.594397	2	11	-9
Columbia Business School	0.726394	3	3	0
Insead	0.668107	4	6	-2
London Business School	0.692608	5	4	0
University of Chicago GSB	0.758528	6	2	2
Stanford University GSB	0.615344	7	10	-3
New York University: Stern	0.6338	8	7	1
MIT: Sloan	0.50519	9	17	-8
Dartmouth College: Tuck	0.6338	10	8	2
Northwestern University: Kellogg	0.688126	11	5	6
IMD	0.435994	12	19	-7
Iese Business School	0.632058	13	9	4
Yale School of Management	0.543776	14	16	-3
Instituto de Empresa	0.390719	15	20	-5
Cornell University: Johnson	0.501724	16	18	-2
Georgetown Uni: McDonough	0.543776	17	13	4
Uni of N Carolina: Kenan-Flagler	0.562208	18	12	5
University of Virginia: Darden	0.543776	19	13	6
Duke University: Fuqua	0.543776	20	13	7

(b) New weightings obtained from Common-Weight Model

	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT research rank
Original Weightings	0.2	0.2	0.06	0.05	0.1
Normalized original weightings	0.303030303	0.303030303	0.09090909	0.07575758	0.151515
New weightings	0.291382783	0.243472234	0.27496259	0.13661036	0.053572
Change (%)	-1.16%	-5.96%	18.41%	6.09%	-9.80%

By studying both tables, it is clear that the criterion “International Mobility Rank” has increased its weighting by more than double of its original weightings and criteria other than “Weighted Salary” has changed about 6% to 10% each. These changes have effected the new extremely. In the new ranking, half of the universities have shifted their rankings for more than 4 spots. Harvard and MIT have shifted 9 spots and 8 spots accordingly. Harvard has dropped 9 spots in ranking due to the fact that it has the lowest value in “International Mobility Rank”, which is accounted for 27.50% of the total score. MIT has dropped 8 spots because it has the second lowest score on “International Mobility Rank” and fourth lowest score on “Salary Increase %”, which accounted for 24.35%.

After applying the statistical t-test, the P value was found to be 0.8919, which means the differences between the original rankings and the new rankings are considered to be not statistically significant. Hence the result from the Common-Weight Model is acceptable statistically.

- **3D Spherical Model**

In last section, the weights for each criterion were generated by the model, as well as the rankings. The model will calculate the coordinates of each DMU based on the weightings and project them onto a 3D ball. To insure the correctness of the mapping and the correlations between each DMU, the concept of dissimilarity is used in the calculation of the coordinates. Dissimilarity is the degree of difference between subjects. The general calculation method for dissimilarity will be discussed later in this section.

Table 5.4 lists the variables used in 3D Spherical Model and their meanings. Note that all the radius of the 3D balls is set to 1, and an ideal solution will be projected onto the North Pole. Ideal solution is an imaginary DMU that has the maximum value for each of its criterion. The purpose of this ideal DMU, as the standard, is to help the comparison process.

Table 5.5 Variables and descriptions

Variables	**Descriptions**
m	Total number of DMUs
n	Total number of criteria
S_i	Score of i^{th} DMU
$D_{i,j}$	The dissimilarity between DMU *i* and DMU *j*
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
X_i, Y_i, Z_i	The X,Y, and Z coordinates of DMU *i*

The X_i, Y_i, and Z_i are the actual coordinates of the DMUs on the 3D ball. Also, because the distances between DMUs on the 3D ball are not exactly the same as the values of dissimilarities, we minimize the error between these two values to obtain the closest solution (Equation 5.8). With this solution, the projection of the points on the ball will be able to represent the relationships of the DMUs.

3D Spherical Model (Model 2)

$$MIN \qquad \sum_{i=1}^{m}\sum_{j>i}^{m}\left|(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2 - D_{i,j}^2\right| \qquad \textbf{(5.8)}$$

Subject to**:**

$$S_i = \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right) \qquad \textbf{(5.9)}$$

$$D_{i,j} = \sqrt{2} * \sum_{k=1}^{n}\left(w_k * \left(\frac{\left|C_{i,k} - C_{j,k}\right|}{\overline{C_k} - \underline{C_k}}\right)\right) \qquad \textbf{(5.10)}$$

$$X_i^2 + Y_i^2 + Z_i^2 = 1 \ , \ \forall \ i \qquad \textbf{(5.11)}$$

$$Y_i = 2S_i - S_i^2 \ , \ \forall \ i \qquad \textbf{(5.12)}$$

The objective of this model is to let the dissimilarity between two DMUs represents the distance between two DMUs. This is accomplished by minimizing the difference between the straight line distance of two DMUs and their dissimilarity value.

Equation 5.9 is the function to calculate score, which is the same as equation 5.7. Equation 5.10 calculates the dissimilarity between DMU *i* and DMU *j*. The largest possible value for $D_{i,j}$ is $\sqrt{2}$, because when one DMU is the ideal solution, which have all the maximum value for each criterion, and the other DMU is the worst possible DMU, which must have minimum value for each criterion. Since the ideal solution will be at the North Pole and the worst possible solution will be on the equator. The straight line distance from the North Pole to the Equator on a ball with radius of 1 will be $\sqrt{2}$. Similarly, if two DMUs are exactly the same, thought it is not likely to happen, the numerator will become 0, and so the $D_{i,j}$ will be 0.

Equation 5.11 is to ensure that every point is on the surface of the ball. And equation 5.12 defines the relationship between the Y coordinates and the score. To explain this equation, there is a proposition to discuss, as stated below.

Proposition 1:

$$Y_i = 2 * S_i - S_i^2 \ , \ \forall \ i \qquad \textbf{(5.13)}$$

Proof:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = (\sqrt{2} * D_{i,*})^2 = 2(1 - S_i)^2 \qquad \textbf{(5.14)}$$

$$2 - 2Y_i = 2(1 - 2S_i + S_i^2) \qquad \textbf{(5.15)}$$

$$Y_i = 2S_i - S_i^2 \qquad \textbf{(5.16)}$$

In this proposition, $D_{i,*}$ in equation 5.14 represent the dissimilarity between DMU i and the ideal solution. The original equation that calculates the distance between two points was changed to the current form, $(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$, since the ideal solution has the coordinate of (0, 1, 0). Equation 5.14 can be verified with (ideal solution, worst possible solution) pair and (ideal solution, best possible solution) pair. When these two pairs of DMUs are plugged in 5.15, they both hold. Hence, equation 5.14 is further simplified to 5.15 and finally 5.16. The simplification processes are shown as below.

LHS:	**RHS:**
$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$	$2(1 - S_i)^2$
$\Rightarrow X_i^2 + Y_i^2 - 2Y_i + 1 + Z_i^2$	$\Rightarrow 2(1 - 2S_i + S_i^2)$
$\Rightarrow (X_i^2 + Y_i^2 + Z_i^2) - 2Y_i + 1$	$\Rightarrow 2 - 4S_i + 2S_i^2$
$\Rightarrow 1 - 2Y_i + 1$	
$\Rightarrow 2 - 2Y_i$	

LHS = RHS:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = 2(1 - S_i)^2$$

$$\Rightarrow \quad 2 - 2Y_i = 2 - 4S_i + 2S_i^2$$

$$\Rightarrow \quad Y_i = 2S_i - S_i^2$$

By applying the model to the example from section 5.1, we obtain the result shown in Table 5.5.

Table 5.6 Coordinates for each universities

Schools	score	New Ranking	x	y	z
Ideal Solution	1		0	1	0
University of Pennsylvania: Wharton	0.845613979	1	-0.21658096	0.97616496	0.013952
Harvard Business School	0.594397421	11	-0.41597168	0.83548655	0.359068
Columbia Business School	0.726394479	3	-0.36376656	0.92514002	0.108581
Insead	0.66810701	6	-0.32914496	0.88984704	-0.31597
London Business School	0.692608172	4	-0.32199465	0.90551026	-0.27635
University of Chicago GSB	0.758528351	2	-0.33056332	0.94169144	-0.06281
Stanford University GSB	0.615343904	10	-0.49832929	0.85203969	0.160301
New York University: Stern	0.633799985	7	-0.45945342	0.86589755	-0.1978
MIT: Sloan	0.505189925	17	-0.65293543	0.75516299	0.058345
Dartmouth College: Tuck	0.633799985	8	-0.49305753	0.86589755	0.084355
Northwestern University: Kellogg	0.688125846	5	-0.41447314	0.90273451	-0.11525
IMD	0.435994334	19	-0.73131613	0.68189761	0.0139
Iese Business School	0.63205834	9	-0.12768982	0.86461893	-0.48593
Yale School of Management	0.543776095	16	-0.58187571	0.79185975	0.185415
Instituto de Empresa	0.390719143	20	-0.36325638	0.62877684	-0.68752
Cornell University: Johnson	0.501723624	18	-0.65438051	0.75172065	-0.08187
Georgetown Uni: McDonough	0.543776095	13	-0.53405605	0.79185975	-0.29621
Uni of N Carolina: Kenan-Flagler	0.562207931	12	-0.49102552	0.8083381	-0.32478
University of Virginia: Darden	0.543776095	13	-0.60957381	0.79185975	0.037127
Duke University: Fuqua	0.543776095	13	-0.52498427	0.79185975	-0.31201

As previously mentioned, the ideal point is a point formed by setting the value of each of its criterion to the maximum value found from hard data. This point will lie on the North Pole with coordinates of (0, 1, 0) and score of 1. The worst point will be A4, with coordinates of (0.99127, 0, 0) and score of 0. With this example, it is coincident that the ideal solution is same as A1 and the worst point A4 is lying on the equator. Despite these facts, the distances between each point are shown in Table 5.6. These numbers also represent the dissimilarity between each DMU.

Table 5.7 Dissimilarity matrix

	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20
A0	0	0.22	0.57	0.39	0.47	0.43	0.34	0.54	0.52	0.7	0.52	0.44	0.8	0.52	0.65	0.86	0.7	0.65	0.62	0.65	0.65
A1	0.22	0	0.49	0.27	0.45	0.32	0.12	0.33	0.32	0.48	0.3	0.26	0.58	0.52	0.51	0.81	0.49	0.43	0.4	0.43	0.43
A2	0.57	0.49	0	0.45	0.67	0.65	0.52	0.27	0.56	0.27	0.35	0.48	0.59	0.99	0.42	1.03	0.41	0.7	0.68	0.4	0.6
A3	0.39	0.27	0.45	0	0.55	0.42	0.18	0.28	0.2	0.31	0.15	0.36	0.47	0.61	0.26	0.91	0.32	0.37	0.47	0.26	0.49
A4	0.47	0.45	0.67	0.55	0	0.26	0.38	0.42	0.5	0.45	0.55	0.22	0.44	0.52	0.67	0.57	0.48	0.57	0.43	0.55	0.35
A5	0.43	0.32	0.65	0.42	0.26	0	0.25	0.48	0.26	0.47	0.41	0.21	0.59	0.34	0.47	0.49	0.33	0.31	0.2	0.41	0.23
A6	0.34	0.12	0.52	0.18	0.38	0.25	0	0.35	0.22	0.36	0.23	0.19	0.49	0.47	0.39	0.74	0.36	0.3	0.3	0.3	0.31
A7	0.54	0.33	0.27	0.28	0.42	0.48	0.35	0	0.38	0.2	0.23	0.29	0.34	0.8	0.5	0.86	0.31	0.53	0.51	0.34	0.43
A8	0.52	0.32	0.56	0.2	0.5	0.26	0.22	0.38	0	0.38	0.26	0.39	0.53	0.43	0.25	0.74	0.25	0.2	0.29	0.29	0.31
A9	0.7	0.48	0.27	0.31	0.45	0.47	0.36	0.2	0.38	0	0.19	0.26	0.37	0.81	0.34	0.8	0.19	0.47	0.46	0.22	0.37
A10	0.52	0.3	0.35	0.15	0.55	0.41	0.23	0.23	0.26	0.19	0	0.34	0.41	0.68	0.31	0.85	0.19	0.35	0.41	0.17	0.43
A11	0.44	0.26	0.48	0.36	0.22	0.21	0.19	0.29	0.39	0.26	0.34	0	0.4	0.55	0.56	0.57	0.35	0.43	0.29	0.4	0.21
A12	0.8	0.58	0.59	0.47	0.44	0.59	0.49	0.34	0.53	0.37	0.41	0.4	0	0.83	0.66	0.79	0.43	0.51	0.57	0.42	0.48
A13	0.52	0.52	0.99	0.61	0.52	0.34	0.47	0.8	0.43	0.81	0.68	0.55	0.83	0	0.62	0.34	0.66	0.44	0.44	0.61	0.53
A14	0.65	0.51	0.42	0.26	0.67	0.47	0.39	0.5	0.25	0.34	0.31	0.56	0.66	0.62	0	0.92	0.27	0.38	0.53	0.25	0.55
A15	0.86	0.81	1.03	0.91	0.57	0.49	0.74	0.86	0.74	0.8	0.85	0.57	0.79	0.34	0.92	0	0.68	0.54	0.44	0.78	0.43
A16	0.7	0.49	0.41	0.32	0.48	0.33	0.36	0.31	0.25	0.19	0.19	0.35	0.43	0.66	0.27	0.68	0	0.28	0.28	0.21	0.28
A17	0.65	0.43	0.7	0.37	0.57	0.31	0.3	0.53	0.2	0.47	0.35	0.43	0.51	0.44	0.38	0.54	0.28	0	0.19	0.35	0.22
A18	0.62	0.4	0.68	0.47	0.43	0.2	0.3	0.51	0.29	0.46	0.41	0.29	0.57	0.44	0.53	0.44	0.28	0.19	0	0.46	0.09
A19	0.65	0.43	0.4	0.26	0.55	0.41	0.3	0.34	0.29	0.22	0.17	0.4	0.42	0.61	0.25	0.78	0.21	0.35	0.46	0	0.48
A20	0.65	0.43	0.6	0.49	0.35	0.23	0.31	0.43	0.31	0.37	0.43	0.21	0.48	0.53	0.55	0.43	0.28	0.22	0.09	0.48	0

The dissimilarity values represent the degree dissimilarity between any two DMUs. If the value is 1, then the DMUS are totally different. If the value is 0, then the two DMUs are exactly the same, so the coordinates of these two DMUs will be the same as well. The school name has been replaced by variables due to the size of the dissimilarity matrix. A0 represents the Ideal Solution, A1 represents UPenn, A2 represents Harvard, and so on. Figure 5.2 is the projection of these points on a 3D ball by using the coordinates in Table 5.5.

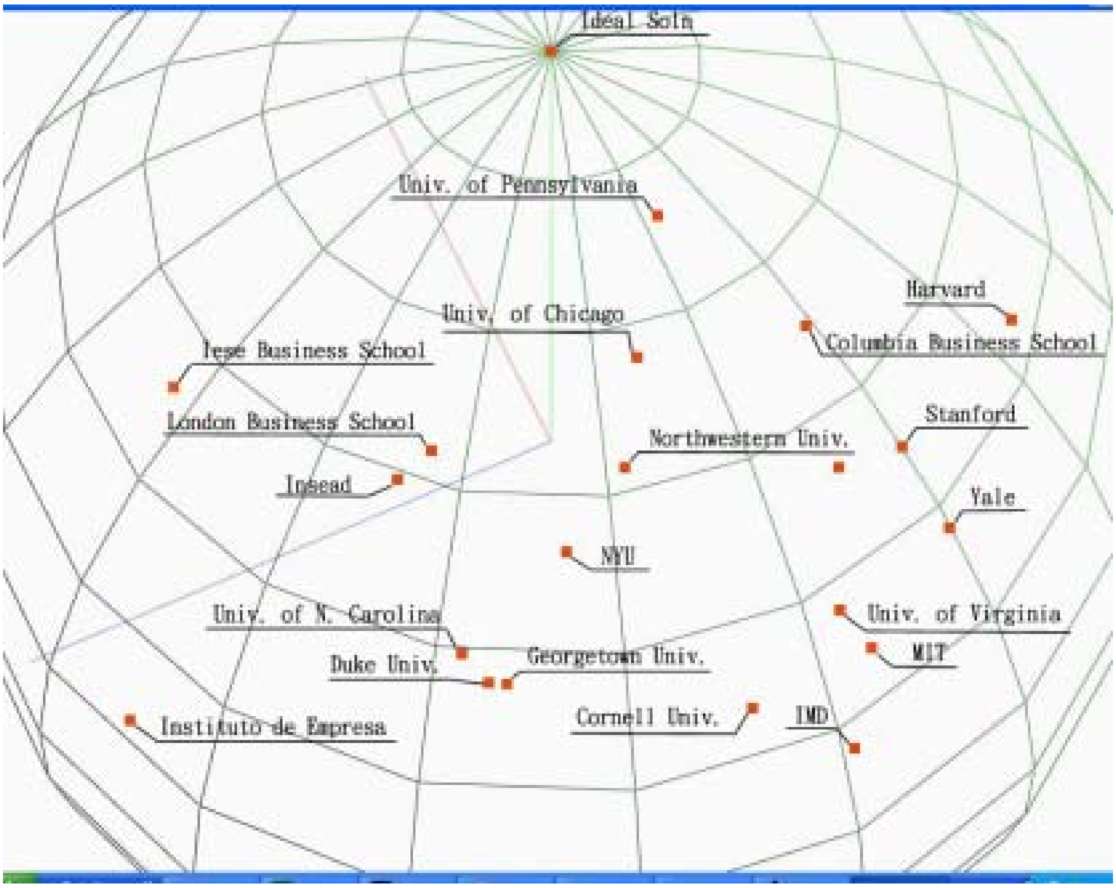

Figure 5.2 3D ball with DMUs projected on the surface

Notice that the North Pole is the ideal point. The points with higher altitudes are points with higher rankings. Universities that are closer to the equator are the ones with lower ranking and scores. Figure 5.2 clearly shows that Instituto de Empresa has the lowest ranking and IMD has the second lowest ranking, where University of Pennsylvania still has the best score.

◆ **Clustering**

In this step, the Clustering Model will assign each data point to a best fitting group. The DM can specify the number of groups he/she wants. The model will make sure that every group will have at least one data points.

Table 5.8 Variables and descriptions for Clustering Model

Variables	**Descriptions**
m	Total number of DMUs
g	Total number of groups DM wants.
$Tdist_i$	Total distance between data points to their center point in a group
grp_{ij}	Binary variable. $grp_{ij} = 1$ if DMU i belongs to group j.
pt_{ij}	Coordinate of DMU i. j = x, y, or z.
$ctpt_{ij}$	Coordinate of Center Point i. j = x, y, z.

Clustering Model (Model 3)

$$Min\left(\sum_{i=1}^{g} tdist_i - \sum_{j=1}^{g}\sum_{k=1}^{g}\left((x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2\right)\right) \quad \textbf{(5.17)}$$

Subject to:

$$tdist_i = \sum_{j=i}^{m}\left(grp_{ji} * \left((x_{pt_j} - x_{ctpt_i})^2 + (y_{pt_j} - y_{ctpt_i})^2 + (z_{pt_j} - z_{ctpt_i})^2\right)\right) \quad \textbf{(5.18)}$$

$$grp_{ij} \in \{0,1\} \quad \textbf{(5.19)}$$

$$\sum_{j=1}^{g} grp_{ij} = 1 \quad , \quad \forall\ i \quad \textbf{(5.20)}$$

$$\sum_{j=1}^{m} grp_{ij} \geq 1 \quad , \quad \forall\ i \quad \textbf{(5.21)}$$

$$(x_{ctpt_i})^2 + (y_{ctpt_i})^2 + (z_{ctpt_i})^2 = 1 \quad , \quad \forall\ i \quad \textbf{(5.22)}$$

$$(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \leq \sqrt{2} \quad , \quad \forall\ i,\ j \quad \textbf{(5.23)}$$

Equation 5.17 is the objective function, which tries to minimize the sum of distance between center points and data points in their group. Also, the distance between each center point has to be maximized to ensure that the clusters will be as far from each other as possible. Equation 5.18 calculates the distance between data points and center points in each cluster for every group. Equation 5.20 limits each DMU to belong to only one cluster. Equation 5.21 is to ensure every group has at least one DMU. Equation 5.22 is to force the center point to fall on the surface of the 3D ball. Finally, Equation 5.23 is to ensure that the longest distance between any two center points will be $\sqrt{2}$.

From the 3D ball, we can group the DMUs by using the Clustering Model. The By running the Clustering Model on this example, the grouping result is shown in Table 5.8. These twenty universities were grouped into three groups, where Harvard was grouped as the only member for group 1. Group 2 has 12 members and group 3 has 7. The number of members in a group was determined by the model automatically, but the user can specify the number of clustering groups.

Table 5.9 Groupings for universities

	Group 1	Group 2	Group 3
University of Pennsylvania: Wharton	0	0	1
Harvard Business School	1	0	0
Columbia Business School	0	0	1
Insead	0	1	0
London Business School	0	1	0
University of Chicago GSB	0	1	0
Stanford University GSB	0	0	1
New York University: Stern	0	1	0
MIT: Sloan	0	0	1
Dartmouth College: Tuck	0	0	1
Northwestern University: Kellogg	0	1	0
IMD	0	1	0
Iese Business School	0	1	0
Yale School of Management	0	0	1
Instituto de Empresa	0	1	0
Cornell University: Johnson	0	1	0
Georgetown Uni: McDonough	0	1	0
Univ. of N Carolina: Kenan-Flagler	0	1	0
University of Virginia: Darden	0	0	1
Duke University: Fuqua	0	1	0

The grouping situation is shown as Figure 5.3.

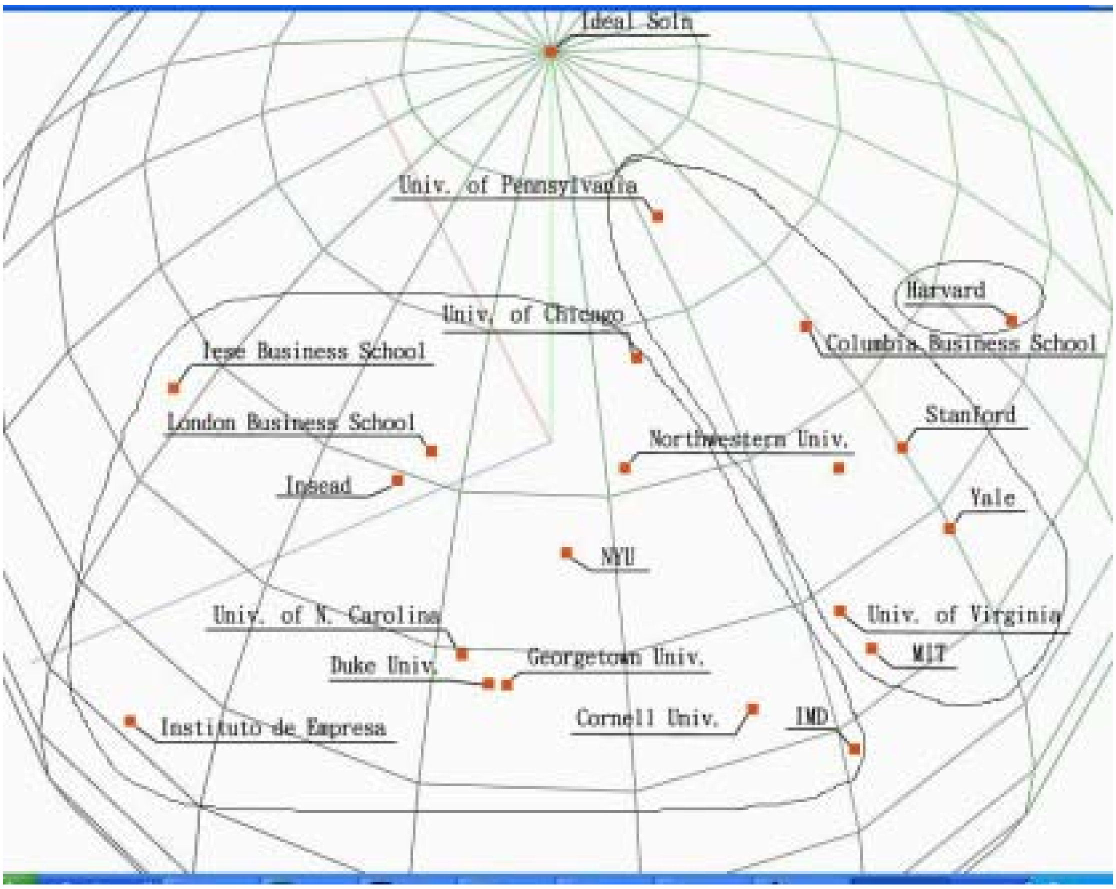

Figure 5.3 Groupings for twenty universities

六、結果與討論

People have being ranking DMU to show their importance and priorities since long ago. There are many ways to rank and each method has their strengths and weaknesses. From this study, we have proposed a method to help Decision Makers rank DMUs with out the needs to specify weightings for each criteria, which often is the most controversy and difficult in the whole ranking process. Using the techniques from Linear Programming, this model can produce a set of weightings for DMUs based on the absolute dominances relationships and preferences relationships, given by the Decision Makers. The 3D Ball representation not only has given Decision Makers the views they can not have by only looking at the table, but also allows them to categorize the DMUs and change the groupings for DMUs.

This model has focused on the mathematical models. There are still many issues can be studied in this area. Following are some suggestions for future works:

- Efficiency and validity in data collection and criteria selection.
- Although this model provides the function of changing groupings for DMUs, the clustering function can be improved. Certain clustering technique could be applied and help the groupings to be more accurate.
- The mathematical model can be modified to produce a more profound model, which can reduce the computation time and returns globally optimized solution.