行政院國家科學委員會專題研究計畫　成果報告

綜觀系統中量子波函數之研究：利用微型雷射之橫模特性

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Study of quantum wave functions in mesoscopic systems using transverse patterns in microcavity lasers

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一、中文摘要

本計畫設計各式雷射系統，研究其同調波(coherent waves)的特性。其研究主題包含(1)同調波與古典周期軌道間的關聯性，(2)同調波與腔體邊界的相關性，(3)介觀系統系統中同調波的選擇法則(selection rules)。另外，我們也將進行理論分析與實驗做一比較並找出其基本機制。

關鍵詞：雷射、橫模形態、量子系統

Abstract

We construct an exact connection between the quantum wave functions and the classical periodic orbits to analyze the relation between classical nonlinear dynamics and quantum theory for the mesoscopic systems with quantum Fermi resonance. In particular, the high efficiency of wave extension within the classical caustics is found to be an intriguing coherent phenomenon in mesoscopic systems with nonlinear coupling resonances. Furthermore, we give a first verification that the laser cavity is a promising analogous experiment for visualizing the quantum wave functions associated with Fermi resonance. This verification also indicates that the laser resonator can be designed to simulate a wide range of physical phenomena.

Keywords: laser, transverse pattern, pattern formation, classical-quantum correspondence

二、緣由與目的

The exploration of the relation between classical nonlinear dynamics and quantum theory is a central problem in modern physics. The phenomenon of the Fermi resonance [1] plays a significant role for understanding this quantum-to-classical transition because this type of resonance has been observed to be important in experimental studies of molecular excitations, tunneling effect, stellar trajectories, as well as other experimental and theoretical works [2-6]. Deeper understandings of the Fermi resonance are, therefore, strongly desired. Recently, Cushman et al. [5,6] show that the singularities introduce defects into the ensemble of quantum eigenstates, but they also organize the structure of those defects. As suggested by the authors [5], it is now tempting to find feasible experiments for visualizing the quantum manifestations of Fermi resonances.

On the other hand, the theoretical studies [2] reveal that a single trajectory in the coupled Fermi resonance system often sweeps out a region similar to that described by an ensemble of periodic orbits in the uncoupled system. This finding signifies that the quantum effect of classical nonlinear resonance can be manifested with the quantum wave functions related to classical periodic orbits in the zero-order systems. Accordingly, the connection between the quantum wave functions and the simple classical periodic orbits in the zero-order systems is not only important for understanding the quantum-to-classical transition but also informative for revealing the quantum features of nonlinear classical dynamics [7,8].

In this work, we provide analytic insights into the quantum Fermi resonance based on the construction of the correspondence between the quantum wave functions and the classical periodic orbits for two dimensional harmonic oscillators with commensurate frequencies. We employ this analytical connection to make an important verification that a degenerate laser resonator with an intracavity saturable absorber can be utilized as an analogous system for simulating the physical phenomena of the Fermi resonance in quantum mesoscopic systems.

The conventional eigenstates of a 2D harmonic oscillator with commensurate frequencies do not reveal the characteristics of classical periodic orbits even in the correspondence limit of large quantum number. Even though the numerical analysis has been performed to investigate the wave functions localized on the periodic orbits [9], a clear quantum-classical connection has not been constructed as yet. Here we analytically derive the quantum wave functions related to the classical periodic orbits for 2D harmonic oscillators with commensurate frequencies. The exact wave functions can be straightforwardly applicable to the analysis of the quantum systems with Fermi resonance.

Since the Hamiltonian is separable, the Schrödinger coherent state [10] for a 2D harmonic oscillator can be expressed as:

$$|\alpha,\beta\rangle = \left(\sum_{n_x=0}^{\infty} \frac{|\alpha|^{n_x} e^{i n_x \phi_x}}{\sqrt{n_x!}} e^{-\frac{|\alpha|^2}{2}} |n_x\rangle e^{-i\left(n_x+\frac{1}{2}\right)\omega_x t} \right) \times \left(\sum_{n_y=0}^{\infty} \frac{|\beta|^{n_y} e^{i n_y \phi_y}}{\sqrt{n_y!}} e^{-\frac{|\beta|^2}{2}} |n_y\rangle e^{-i\left(n_y+\frac{1}{2}\right)\omega_y t} \right) \quad (1)$$

It is clear that the center of the wave packet follows the motion of a classical 2D isotropic harmonic oscillator, i.e.,

$$x(t) = \sqrt{\frac{2\eta}{m\omega_x}} |\alpha| \cos(\omega_x t - \phi_x) \quad (2)$$
$$y(t) = \sqrt{\frac{2\eta}{m\omega_y}} |\beta| \cos(\omega_y t - \phi_y)$$

It is clear that the Schrödinger coherent state for a 2D harmonic oscillator with commensurate frequencies is a wave packet with its center generally moving along a Lissajous trajectory. Consider the general case of the ratio $\omega_x : \omega_y = q : p$, where p and q are coprime integers, the set of states with indices (n_x, n_y) can be divided into subsets characterized by a pair of indices (u_x, u_y) given by $n_x \equiv u_x \pmod{p}$ and $n_y \equiv u_y \pmod{q}$. In terms of these subsets, the Schrödinger coherent state in Eq. (1) can be rewritten as

$$|\alpha,\beta\rangle = \left(\sum_{u_x=0}^{p-1} \sum_{N_x=0}^{\infty} \frac{|\alpha|^{pN_x+u_x} e^{i(pN_x+u_x)\phi_x}}{\sqrt{(pN_x+u_x)!}} e^{-\frac{|\alpha|^2}{2}} |pN_x+u_x\rangle e^{-i\left(pN_x+u_x+\frac{1}{2}\right)q\omega t} \right) \times \left(\sum_{u_y=0}^{q-1} \sum_{N_y=0}^{\infty} \frac{|\beta|^{qN_y+u_y} e^{i(qN_y+u_y)\phi_y}}{\sqrt{(qN_y+u_y)!}} e^{-\frac{|\beta|^2}{2}} |qN_y+u_y\rangle e^{-i\left(qN_y+u_y+\frac{1}{2}\right)p\omega t} \right) \quad (3)$$

where ω is the common factor of the frequencies by ω_x and ω_y. With the representation of the Cauchy product [11], the terms $|pN_x+u_x\rangle |pN_y+u_y\rangle$ in Eq. (3) can be arranged diagonally by grouping together those terms for which $N_x + N_y = N$:

$$|\alpha,\beta\rangle = \sum_{N=o}^{\infty} e^{-\frac{|\alpha|^2+|\beta|^2}{2}} \sum_{u_y=0}^{q-1} \sum_{u_x=0}^{p-1} \left\{ \sum_{K=0}^{N} \frac{\left(|\alpha| e^{i\phi_x}\right)^{pK+u_x} \left(|\beta| e^{i\phi_y}\right)^{q(N-K)+u_y}}{\sqrt{(pK+u_x)!}\sqrt{[q(N-K)+u_x]!}} |pK+u_x\rangle |q(N-K)+u_y\rangle \right\} \times e^{-i\left[Npq+\left(u_x+\frac{1}{2}\right)q+\left(u_y+\frac{1}{2}\right)p\right]\omega t} \quad .(4)$$

After some algebra, Eq. (4) can be written as

$$|\alpha,\beta\rangle = \sum_{N=o}^{\infty} e^{-\frac{|\alpha|^2+|\beta|^2}{2}} \sum_{u_y=0}^{q-1} \sum_{u_x=0}^{p-1} C_{N,u_x,u_y} |A,\phi\rangle_{N,u_x,u_y} \times \left(|\alpha| e^{i\phi_x}\right)^{u_x} \left(|\beta| e^{i\phi_y}\right)^{qN+u_y} e^{-i\left[Npq+\left(u_x+\frac{1}{2}\right)q+\left(u_y+\frac{1}{2}\right)p\right]\omega t} \quad , (5)$$

where

$$|A,\phi\rangle_{N,u_x,u_y} = \frac{1}{C_{N,u_x,u_y}} \sum_{K=0}^{N} \frac{\left(A e^{i\phi}\right)^K}{\sqrt{(pK+u_x)!}\sqrt{[q(N-K)+u_y]!}} |pK+u_x\rangle |q(N-K)+u_y\rangle \quad (6)$$

$$C_{N,u_x,u_y} = \left[\sum_{K=0}^{N} \frac{A^{2K}}{(pK+u_x)!\,[q(N-K)+u_y]!} \right]^{1/2} , \quad (7)$$

$$A = |\alpha|^p / |\beta|^q , \quad \phi = p\phi_x - q\phi_y . \quad (8)$$

Eq. (5) indicates that the Schrödinger coherent state for a 2D harmonic oscillator with commensurate frequencies can be expressed as a superposition of the quantum stationary states $|A,\phi\rangle_{N,u_x,u_y}$. Since the Schrödinger coherent state is a wave packet with its center moving along a classical trajectory, the quantum stationary states $|A,\phi\rangle_{N,u_x,u_y}$ are expected to be associated with the classical periodic orbit. The amplitude factor A and the phase factor ϕ in the quantum stationary state are explicitly related to the amplitude ratio and the phase difference between two independent oscillators, respectively. Since u_x and u_y essentially do not affect the intensity distribution of $|A,\phi\rangle_{N,u_x,u_y}$, the condition of $u_x = u_y = 0$ is used for the following analysis unless otherwise specified. Figure 1 depicts the dependence of the wave pattern $\left|\langle x,y|A,\phi\rangle_{N,u_x,u_y}\right|^2$ on the parameter A for the frequency of 1:2, $N = 20$, and $\phi = \pi/2$. For comparison, the corresponding classical periodic orbits are also shown in Fig. 1. The characteristic patterns can be found to reveal a clear quantum-classical correspondence. Since the stationary coherent states $|A,\phi\rangle_{N,u_x,u_y}$ are well localized on the periodic orbits, we call them "localized states".

As mentioned earlier, a classical trajectory in the weakly perturbed systems can be characterized by an ensemble of the unperturbed periodic orbits. The quantum features of classical nonlinear resonance can be manifested with a coherent superposition of an ensemble of the localized states. Figure 2 displays the variation of the coherent wave patterns with the number of participant localized states. It is found that only a few localized states are already sufficient to form a well extended pattern within the classical caustics, similar to a kaleidoscopic pattern. The high efficiency of the wave extension comes from the fact that the encompassing region of each localized state covers a width of several de Broglie wavelengths. Since the de Broglie wavelength is inversely proportional to $\sqrt{N}$, the critical number of localized states for a well extended pattern is of order $\sqrt{N}$. In other words, the highly efficient extension of the wave pattern is a salient quantum phenomenon in mesoscopic systems with nonlinear resonances. Accordingly, it is tempting to find feasible experiments for visualizing the wave patterns in mesoscopic systems with Fermi resonances.

三、結果與討論

In recent years, microwave cavities have been used to perform analog studies of transport in open quantum dots [12-14]. On the other hand, we demonstrated that it is promising to explore the high-order quantum wave function from the pattern formation of the laser resonators [15,16]. This demonstration is based on the fact that the wave equation for the transverse mode of the laser resonators in the paraxial approximation is in analogy with the Schrödinger equation for the 2D quantum confined systems [17,18]. More recently, we have observed the kaleidoscope of laser patterns in a near-hemispheric microchip laser with an intracavity saturable absorber [19]. However, the origin of the salient pattern formation was not clearly understood at that moment. Here we confirm that the quantum coherent states in a 1:2 bend-stretch Fermi resonance can be analogously observed from laser pattern formation.

The experimental configuration in Ref. [19] is a near-hemispheric cavity in which the transverse mode spacing and the longitudinal mode spacing are very close to be commensurable, i.e. : ≈1:2. The inherent commensurability between and have a dramatic effect on the formation of laser patterns, as shown in the internal nonlinear resonances. In other words, the coupling of a 1:2 transverse-longitudinal resonance is identical in form to the well-known phenomenon of Fermi resonance in molecular systems. As shown in Fig. 3, the formation of the kaleidoscopic laser patterns can be well explained with the quantum coherent states of a 1:2 intrinsic Fermi resonance. Note that the bright spot near the center of the kaleidoscopic patterns arises from the quantum-classical correspondence that all figure-eight classical orbits pass through the focal point near origin. The excellent agreement between the experimental and theoretical patterns confirms that the coupling of a 1:2 transverse-longitudinal resonance in a near-hemispheric laser resonator is analogous to the well-known phenomenon of Fermi resonance in molecular systems. The present analysis also provides a further indication that laser resonators can be designed to demonstrate the quantum phenomenon in mesoscopic physics.

四、 結論

In conclusion, the quantum manifestations of classical nonlinear resonance have been clearly demonstrated by making the connection between the quantum wave functions and the classical periodic orbits for the unperturbed systems. Intriguingly, it is found that the high efficiency of wave extension within the caustics is a significant quantum phenomenon in mesoscopic systems with nonlinear coupling resonances. Furthermore, we have theoretically and experimentally verified, for the first time to our knowledge, that a degenerate laser resonator with an intracavity saturable absorber forms a useful analogous system for visualizing the quantum wave functions associated with Fermi resonance. This verification indicates that the modern laser resonator now provides a concrete optical system which simulates a wide range of physical phenomena.

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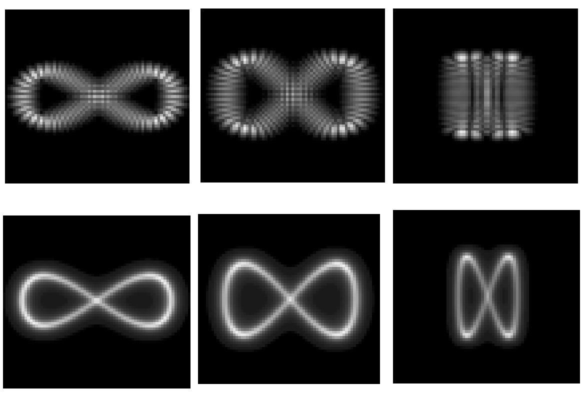

Fig. 1

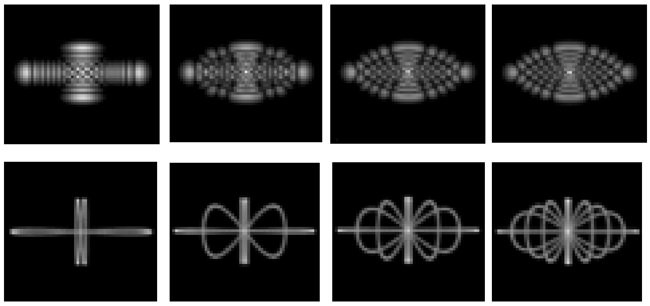

Fig. 2

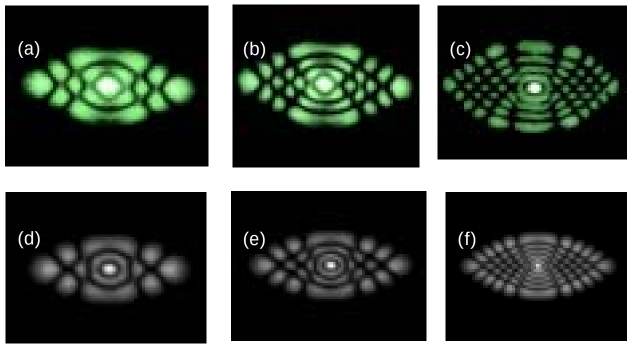

Fig. 3