低功率系統之設計及自動化

子計畫九：適用於晶片上電力傳輸分析之階層模組簡化技術(2/3)

Power Delivery Network Analysis with

Hierarchical Model Order Reduction Techniques

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一、中文摘要

隨著超大型積體電路(VLSI)複雜度的增加，以及晶片元件特徵尺寸(feature size)的持續縮減，導線的數量將超過上億個區段，因此電力傳輸設計與分析成為極具挑戰性的工作，不良的電力傳輸設計將會降低電路的效能、雜訊邊際，及可靠度。

為確認電力傳輸的設計品質, 必須使用暫態模擬去分析電力傳輸的波動。同時，由於電力傳輸網路的分析矩陣很大，傳統的電路模擬工具如 SPICE/HSPICE 無法有效的分析這類系統。因此，發展一個有效的電力傳輸分析工具是非常重要的。前期的研究中，我們考慮電感（自感及互感）的效應並結合了階層式（hierarchical）分析與模組簡化（model order reduction）技術，發展出一個計算負載與電流源數目無關的快速電力傳輸分析工具。然而由於我們忽略不同分割電路區塊間的互感效應，如此將引進人為的誤差。在本子計畫的第一年中，我們整合了模組簡化技術與階層式分析，在數個不同的頻率下做係數對應比較（moment matching）以發展有效率的快速電力傳輸分析工具。然而，如何選擇適當的展開頻率是很困難的。

因為電力傳輸網路的分析目的是擷取傳輸網路中每個端點壓降的波形，所以選擇在時域空間求解是較為直觀的。因此，在本子計畫的第二年中，我們專注於研究與發展數種時域下模擬電力傳輸網路的方法。

關鍵詞：供電/接地 網路，電力傳輸網路，階層式分析，模組簡化

二、英文摘要

The increase in the complexity of VLSI (Very Large Scale Integrated) chips, and the decrease in the feature size of chips demand large metal resources for the power delivery network. The number of wire segments will be over one billion in nanometer designs. This causes the designing and verifying of the power delivery network to become a challenging task. The inferior design of the power distribution network can degrade the circuit performance, noise margin, and reliability.

To ensure the design quality of power delivery, extensive transient power grid simulations

need to be performed to analyze the power delivery fluctuation. However, due to the large size of power delivery network, the traditional circuit simulators, such as SPICE/HSPICE, do not perform well and often take days to complete the full simulation and need many gigabytes of memory space. Hence, in order to facilitate the design of large scale power grids, it is crucial to develop an efficient transient simulation engine for the power delivery network analysis. In the first year of this sub-project, we have integrated a model order reduction method based on matching the solution moments at several desired frequencies, with the hierarchical analysis to develop an efficient power delivery network simulation engine. However, it is hard to choose the suitable expanded frequencies. Since the goal of analyzing power delivery network is to get the voltage drop waveform at each node, it is more straightforwardly to solve its system equation in the time domain. Therefore, we focus on studying and developing several time domain methods to simulate the power distribution network in the second year of this sub-project.

Keywords：Power/Ground Network, Power Delivery Network, Hierarchical Analysis, Model Order Reduction

三、研究計畫之背景及目的

With the UDSM (Ultra Deep Sub-Micron) technology, several features of today's chips (higher operating frequencies, larger number of transistors, smaller feature size and lower power supply voltage) have pushed the power delivery noise analysis onto the designers' list of high priority concerns [1~4]. Basically, the power delivery noise consists of IR drop, Ldi/dt drop and resonance fluctuations. The IR drop has been widely discussed and extensively studied in the literatures [5~8]. Due to the roaring clock frequency, increasing current consumption, and even the clock gating feature, Ldi/dt noise is quickly emerging as another power fluctuation concern [6]. Power delivery noise causing the power voltage to deviate from the ideal value can severely degrade the performance and even make the gate function erroneously. Therefore, the extensive analysis of RLC/RLKC (resistance, inductance, mutual inductance, and capacitance) power delivery system is required to ensure them to meet the target performance and reliability goals.

Generally speaking, one of the major difficulties for the power delivery analysis is size explosion. Tens of millions of devices and parasitics are required to be modeled and simulated over a long time period. However, it is computationally expensive to simultaneously simulate all transistors with the power delivery structure. To enhance the simulation speed, it has been proposed to decouple the power delivery structure simulation and transistors' simulation [6]. First, the current profiles of transistors can be estimated by the current extraction methods. After that, the power delivery network can be modeled as a suitable RLC/RLKC circuit attached by current

sources. In this way, the simulation can be effectively done since there are fewer elements in the circuit, and a RLC/RLKC circuit can be simulated with one LU-decomposition. However, due to the large size of linear circuit, traditional circuit simulators, such as SPICE/HSPICE, do not perform well and often take days to complete the full simulation and need many gigabytes of memory space. For this reason, the hierarchical simulation technique has been applied by [6] to speed up the power delivery network simulation.

The MOR (Model Order Reduction) technique is another efficient way which can be utilized to speed up the circuit analysis [8], and has been widely studied and improved over the last decade [5, 9~11]. Starting from AWE (Asymptotic Waveform Evaluation) [9] to PRIMA (Passive Reduced-order Interconnect Macromodeling Algorithm) [11], MOR techniques have been successfully extended to consider the inductance effects with reasonable accuracy. Later, an extended Krylov subspace method, EKS (Extended Krylov Subspace) [5], has been developed to simulate large scale power delivery circuits with many PWL (Piece Wise Linear) current sources. To resolve the source waveform modeling issues, EKS need to perform the moment shifting procedure to recover the proper moments. In [12], we proposed an improved EKS (IEKS) method such that it did not need to perform moment shifting for the source waveform modeling, and established a novel hierarchical power delivery macro-modeling methodology which integrated the multiple-port Norton equivalent theorem with the MOR algorithm to generate compact and accurate models and achieved significant runtime improvement. We not only considered the self inductance, but also included the effect of mutual inductance in each sub-circuit. The computation load of this method is independent of the number of sources.

In the first year of this sub-project, we have proposed a procedure of hierarchical analysis of power delivery system with including the coupling at the boundaries of different sub-circuits to approximate the coupling effects. We integrated a model order reduction method based on matching the solution moments at several desired frequencies, with the hierarchical analysis to develop an efficient power delivery network simulation engine. However, it is hard to choose the suitable expanded frequencies. Since the goal of analyzing power delivery network is to get the voltage drop waveform at each node, it is more straightforwardly to solve its system equation in the time domain.

As we mentioned above, one bottleneck of power network analysis is the tremendous amount of elements in the system equation. Therefore, the directly solving methods are prohibitive, and the iteratively analyzing methods are better for this large system. Although the iterative methods always converge very quickly for the errors of high-frequency components, they converge extremely slowly for the errors of low-frequency components. Hence, one issue for applying iterative methods to analyze the power delivery network is to accelerate convergent rate due to the slow convergence of low-frequency errors.

Recently, several stochastic methods [14, 15] were proposed to solve the power delivery network. A system-like method by using the stochastic approach was proposed by [14]. It first solved the impulse response of the power delivery network, and then used the random process method to calculate the voltage at each node. However, the procedure of finding auto-correlation

matrix and impulse response are very expensive. A random walk method was developed to analyze the power distribution network in [15]. This method viewed the procedure of solving the power network as a “travel game”. A traveler walks from a starting node to another with a probability which is proportional to the conductivity between them, and collects a voltage value contributed from the node which the traveler arrives. The traveler keeps walking until he/she reaches a node with constant voltage, and adds all the collected voltage values together as his/her reward of this travel. This walking procedure repeats again and again until the average reward/voltage converges. Thus, the flat method based random walk procedure may be too expensive. Although the authors in [15] proposed a hierarchical based method, the interface nodes between subsets must be vias. This hierarchical procedure might not be general if the number of nodes in each sub-network is too huge.

In the following section, two different time-domain methods are developed to analyze the power delivery network. First, a multilevel method based on an aggregation based algebraic multigrid (AAMG) method [16] is proposed to avoid the slow convergence of the basic iterative methods. Then, a semi-stochastic like method based on the Markov chain is developed to simulate the power delivery network. It works very similar with random walk process but we can find the limit behavior by using the techniques of linear algebra. Moreover, a Markov chain based hierarchical method can be generalized to speed up the analysis of power distribution network.

四、研究方法

The power delivery network can be modeled as a RLC/RLKC circuit attached by many current sources as illustrated in Figure 1. The top chip view of power delivery network is shown in Figure 1.(a). Figure 1.(b) is the layout view of the interconnection of power delivery network which the devices are modeled as many independent time-varying current sources. The main goal of power delivery network analysis engine is to calculate the voltage disturbance at each node of Figure 2.(b). By applying the extraction tools to Figure 1.(b), the power delivery network can be modeled as a RLKC mesh as illustrated in Figure 1.(c). For simplicity, we do not show the mutual inductances in Figure 1.(c). This RLC/RLKC circuit model can be represented as a set of MNA (Modified Nodal Analysis) equations,

$$Gx(t)+C\dot{x}(t) = Bu(t), \tag{1}$$

where $x(t)$ represents the vector of MNA variables consisting of nodal voltages, inductor source currents, and voltage source currents, $u(t)$ denotes the vector of port voltage sources and internal current sources. G is the conductance matrix, C is the susceptance matrix, and B is the input selector matrix mapping the sources to the internal states.

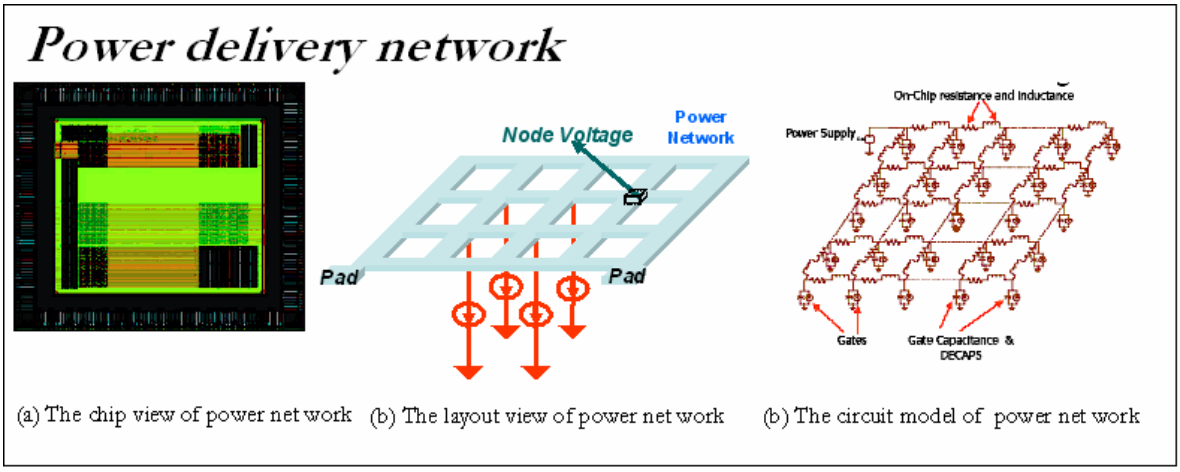


Figure 1. Modeling of power delivery networks

In the following two subsections, first, a non-symmetric aggregation-based algebraic multigrid method for power grid network analysis is developed to improve the convergence speed of the basic iterative methods. After that, a different approach based on the Markov chain is developed to analyze the power delivery network.

A. A Non-Symmetric Aggregation-Based Algebraic Multigrid Method for Power Delivery Network Analysis

Problem Transformation and Introduction of Multigrid Methods

The multigrid method (MG) is an efficient method widely used for solving partial differential equations (PDEs). Firstly, the trapezoidal numerical approximation is applied to Equation (1) resulting in a set of linear equations,

$$\left(G+\frac{2C}{h}\right)x(t+h)=-\left(G-\frac{2C}{h}\right)x(t)+Bu(t+h)+Bu(t) \tag{2}$$

where h is the time step.

Equation (2) can be represented as

$$Ax(t+h)=b(t+h), \tag{3}$$

where $A=G+2C/h$ and $b=-(G-2C/h)x(t)+Bu(t+h)+Bu(t)$.

The solution space of Equation (3) can be decomposed into two subspaces, the smooth (low frequency) subspace $\mathcal{A}$ and the oscillatory (high frequency) subspace $\mathcal{B}$, according to the eigenvectors of the associated system. The eigenvectors with small eigenvalues are according the smooth subspace $\mathcal{A}$, and the eigenvectors with large eigenvalues are according to the oscillatory subspace $\mathcal{B}$.

The concept of multigrid method consists of two complementary components: one is *relaxation*, and the other is *error correction* [17]. Relaxation acts as a good smoother which is shown in Figure 2. The relaxation step can dramatically reduce the high frequency error components by using iterative solvers such as Gauss-Seidel and Weighted Jacobi methods.

On the other hand, the low frequency error components can be reduced by the error correction which involves mapping the problem to coarser grids via the restriction operator, I_h^{2h},

solving the mapped problem which has fewer variables, and mapping the solution back to the original fine grids via the interpolation (prolongation) operator, I_{2h}^{h}. The low-frequency error components at the fine grids Ω^h become more oscillatory at the coarse grids Ω^{2h} as shown in Figure 3 [17], and the relaxation procedure at coarser grids can reduce those components more efficiently.

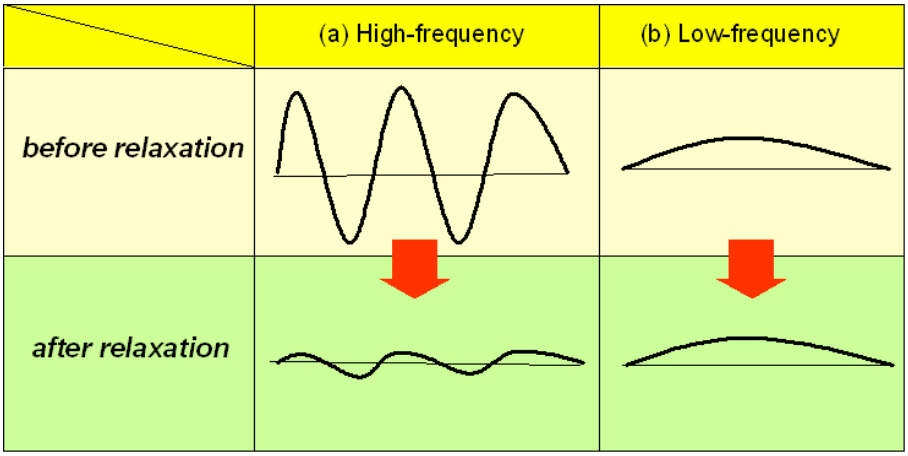


Figure 2. Relaxation involves several iterations. (a) High-frequency error components reduced effectively. (b) Slight reduction of error for low-frequency components.

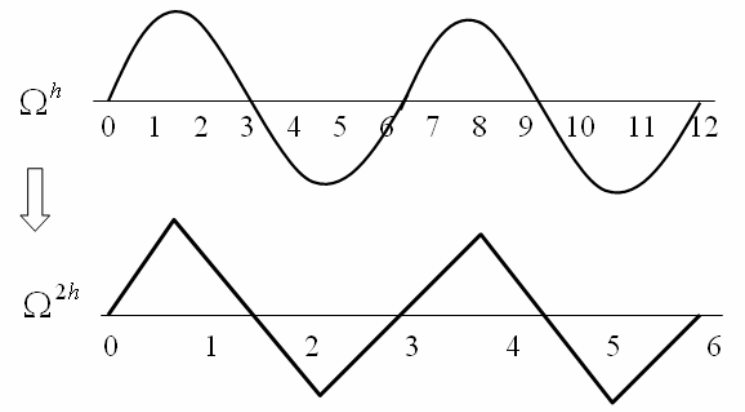


Figure 3. The low-frequency error components at the fine grids become more oscillatory at the coarse grids.

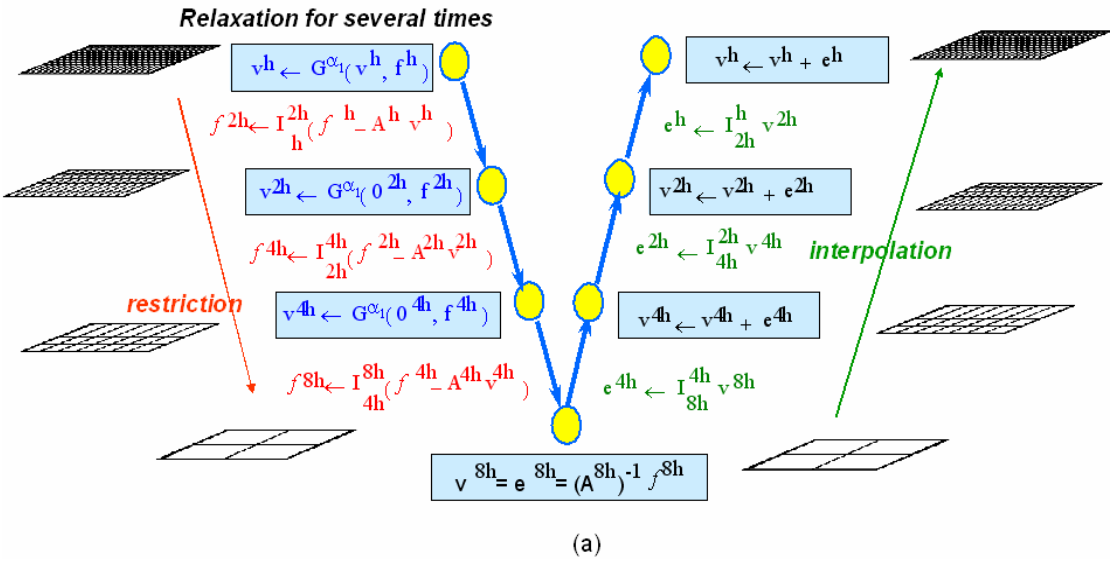
While solving the original problem, $A^h x^h(t+h)=b^h(t+h)$, the residual of fine grids is equal to $r^h= b^h(t+h)-A^h v^h$ after several times of relaxing Equation (3) with initial guess v_{ini}^h. The original problem is equivalent to solve the following problem.

$$A^h e^h=r^h \tag{4}$$

If we can solve Equation (4), the exact solution of Equation (3) is equal to v^h+e^h. In order to efficiently solve e^h, the restriction operator, I_h^{2h}, maps the residual of fine grids to the residual of coarse grids by $r^{2h} = I_h^{2h} r^h$, and Equation (4) is reduced to the following smaller system,

$$A^{2h} e^{2h}=r^{2h} \tag{5}$$

where A^{2h} is the reduced system matrix, $A^{2h} = \left(I_{2h}^{h}\right)^{T} A^{h} I_{2h}^{h}$, e^{2h} is the coarse-grid error, $\left(I_{h}^{2h}\right)^{T}$ is the interpolation operator, and $I_{2h}^{h} = \left(I_{h}^{2h}\right)^{T}$. If the size of reduced system matrix is small enough, the coarse-grid residual equation can be solved by direct methods. Otherwise, the similar steps, relaxation and restriction, are used again to further simplify the system matrix until it can be solved directly. Assuming the solution of the residual equation on the coarsest grids, Ω^{nh}, have been obtained, then the interpolation operator, $I_{nh}^{nh/2}$, is used to map the error term e^{nh} back to the finer level, $\Omega^{nh/2}$, by $e^{nh/2} \leftarrow e^{nh/2} + I_{nh}^{nh/2} e^{nh}$. This interpolation procedure is recursively performed. Finally the approximate solution of equation (3) is equal to $v^{h} \leftarrow v^{h} + I_{2h}^{h} e^{2h}$. Various recursive schemes can be utilized to obtain the approximated solution. The recursive V-cycle [17] of the multigrid method with several nested iterations is illustrated in Figure 4(a), where direct solvers are usually applied at the coarsest level to find the exact solution of the reduced system, and the interpolation operator is applied from the coarsest level back to the finest level. Also, the W-cycle or full multigrid V-cycle scheme [17] shown in Figure 4(b) and (c) can be used to obtain better results in some cases.



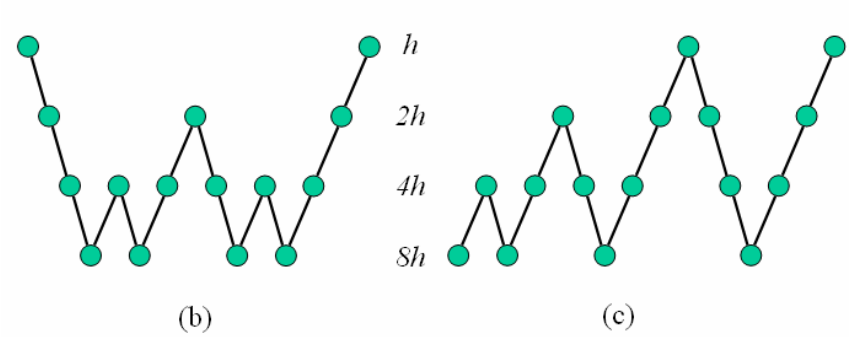


Figure 4. (a) The recursive V-cycle of the multigrid method with four nested iterations; (b) W-cycle scheme; (c) Full multigrid (FMG) scheme on four levels.

Given the system equation of the power delivery network as shown in Equation (3), $Ax(t+h)=b(t+h)$, our goal is to solve $x(t+h)$ at any specific time, $t+h$. For simplicity, we take '$(t+h)$' out at the rest of this subsection. Therefore, the equation can be rewritten as

$$Ax=b, \tag{6}$$

where the solution x is in R^N and the system matrix A is in $R^N \times R^N$, and N is the number of unknowns. As we mentioned before, the solution space $R^N = \mathcal{A} \oplus \mathcal{B}$, where $\mathcal{A} \in R^N$ is of dimension M<N, and $\mathcal{B} \in R^N$ is of dimension N − M. We can construct a basis $\{q_1, q_2, \ldots, q_M\}$ for the smooth subspace $\mathcal{A}$ with each $q_i \in R^N$, such that

$$\mathcal{A} = \text{span}\{q_1, q_2, \ldots, q_M\}$$

If the matrix Q is defined to be a N ×M matrix which is constructed by the column vectors q_i's, an arbitrary vector $v \in$ A can be uniquely specified as

$$v = \sum_{i=1}^{M} a_i q_i = Qa \tag{7}$$

where $Q=[q_1\ q_2\ \ldots\ q_M]$, $a=(a_1, a_2, \ldots, a_M)^t$. The matrix Q: $R^M \rightarrow R^N$ is the prolongation operator and its transpose, Q^t: $R^N \rightarrow R^M$, is the restriction operator.

Using the prolongation operator Q, we can construct a multilevel scheme and obtain a corrected solution approximation,

$$x^c = x^{v1} + Qa, \tag{8}$$

where x^{v1} is an approximated solution after $v1$ smoothing iterations.

Let the corrected residual $R^c = Ax^c - b$ be orthogonal to the smooth subspace $\mathcal{A}$: $\langle R^c, Q \rangle = 0$, where $\langle \cdot, \cdot \rangle$ is the standard inner product. We can derive the coarse-level equation,

$$A_0 a = -Q^t R^{v1}, \tag{9}$$

where $R^{v1} = Ax^{v1} - f$ is the residual after $v1$ smoothing iterations, and $A_0 = Q^t A Q$ is the coarse–level matrix.

After the above derivations, a multilevel scheme can be constructed with the prolongation operator Q which is composed of the eigenvectors with small eigenvalues of the system normal matrix. Because of the orthogonal property between the corrected residual R^c and smooth subspace $\mathcal{A}$, the high-frequency errors can be eliminated efficiently.

With applying a mesh-based aggregation technique, the system normal matrix can be divided into many aggregated sub-matrices by the following five principles:

- ***P1*** Select the node with maximum degree of strong connections in the graph as the first node in an aggregation.
- ***P2*** Every point must be included in the aggregation.
- ***P3*** Each point cannot belong to different aggregations.
- ***P4*** Let the points which have strong connection between them in the same aggregation.
- ***P5*** Let the points which have weak connection between them in the different aggregations

The eigenvectors and eigenvalues of the system normal matrix can be solved by using QR decomposition [18] to calculate the eigenvectors and eigenvalues of each aggregated sub-matrix. For each aggregation, those eigenvectors with smaller eigenvalues are assembled into the columns of prolongation matrix.

The aggregation-based algebraic multigrid method uses a multilevel scheme to eliminate the high-frequency errors, and the low-frequency errors are eliminated by the smoother at each level. The complementarities between the smoother and the coarse-level correction can make the convergence faster, and the convergence rate is nearly independent of the problem size.

B. Power Delivery Network Analysis Using Markov Chain

In this subsection, we first develop a Markov-chain like method to solve the power delivery network which only includes the RC components. Then, we extend this method to deal with the RLKC components.

Markov-Chain Like Solver for Flat RC Power Delivery Network

Without considering the inductance effect, the RLKC power delivery system in Figure 1 can be reduced to RC model as shown in Figure 2.

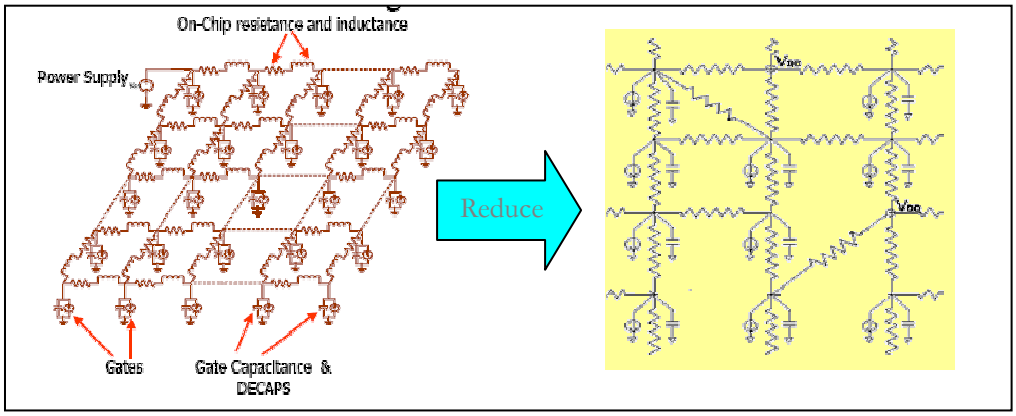


Figure 5. RC power delivery network

By performing KCL at each node x, its nodal equation can be rewritten as

$$v_x(t) = \sum_{i=1}^{\deg ree(x)} g_i v_i(t) - I_x(t) - c_x \frac{dv_x(t)}{dt} \tag{10}$$

where $v_i(t)$ is the nodal voltage of each adjacent node x, g_i is the corresponding conductance between node i and x, c_x is the capacitance at node x, and $I_x(t)$ is the independent current source at node x.

After applying backward Euler approximation to Equation (10), it can be simplified to

$$v_x(t) = \sum_{i \in adj(x)} \frac{g_i}{\left(\sum_{i \in adj(x)} g_i\right) + \frac{C_x}{h}} v_i(t) + \frac{\frac{C_x}{h}}{\left(\sum_{i \in adj(x)} g_i\right) + \frac{C_x}{h}} v_x(t-h) - \frac{I_x(t)}{\left(\sum_{i \in adj(x)} g_i\right) + \frac{C_x}{h}} \tag{11}$$

As you can see that the first two items' coefficients in the right hand side of Equation (10) are all positive, and their sum is equal to 1. Therefore, a probability model can be constructed for our Markov Chain model.

- ***Markov Chain Model for Flat Power Delivery Network Analysis***

According to Equation (11), each nodal voltage at time *t* is view as a transient state [19] (This kind of node is named as "*internal node*".), and the probability of transition, p_{xi}, from state *x* to state *i* is the coefficient of $v_i(t)$. The last two terms in Equation (11), and all power supplies are absorbing states [19] (This kind of node is named as "*boundary node*".) and their coefficients are the probabilities of transition form state *x* to tem. With the above configuration, The Markov Chain recursive equation can set up as

$$v_x(t) = \sum_{i=1}^{\deg ree(x)} p_{xi} v_i(t) + p_{xx\_last} v_x(t-h) - kI_x(t) \tag{12}$$

$$\begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix} = \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{R} & \mathbf{Q} \end{pmatrix} \begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P_x} \end{pmatrix} \begin{pmatrix} 0 \\ V_I(t-h) \end{pmatrix} - \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{I}_{indep} \end{pmatrix} \tag{13}$$

where

R: transition matrix form internal nodes to pure V_{DD} states

Q: transition matrix from internal nodes to internal nodes.

$\mathbf{P}_x$: transition matrix from internal nodes to their last sampling time voltage states.

K: contribution matrix of independent sources.

$\boldsymbol{V}_b$: the vector of pure V_{DD} states.

$\boldsymbol{V}_I(t)$: the vector of internal nodal voltages.

$\boldsymbol{V}_I(t\text{-}h)$: the vector of last sampling nodal voltage states.

From above, the related absorbing Markov chain [19] recursive equation (13) of the power delivery network bas been constructed. The limit-behavior of Equation (13) needs to be derived in order to calculate all nodal voltages.

- ***Limit Behavior of Flat RC Power Delivery Network Markov Chain System***

To solve $V_I(t)$ in Equation (13), we can perform an iterative procedure and find the solution of this recursive equation as following lemma,

Lemma:

Given a RC power delivery model system, the solution of its absorbing Markov chain recursive equation is

$$V_I(t) = \mathbf{B}_{I\to b} V_b(t) + \mathbf{B}_{I\to I\_last} V_I(t-h) - \mathbf{NKI}_{indep} \tag{14}$$

where

$\mathbf{B}_{I->b}$: Static-state probability matrix form $\boldsymbol{V}_I(t\text{-}h)$ to $\boldsymbol{V}_b$.

$\mathbf{B}_{I->I\_last}$: Static-state probability matrix from $\boldsymbol{V}_I(t)$ to $\boldsymbol{V}_I(t\text{-}h)$.

N: Static-state probability matrix from $\boldsymbol{V}_I(t)$ to $\boldsymbol{V}_I(t)$.

Proof:

The iterative method is used to find the limit behavior of Equation (13).

Let V^k be the k^{th} iterative nodal voltage vector.

Up to the k^{th} iteration, the absorbing Markov chain recursive equation is

$$\begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix}^{k+1} = \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{R} & \mathbf{Q} \end{pmatrix}^{k} \begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix}^{1} + \left(\sum_{i=0}^{k-1} \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{R} & \mathbf{Q} \end{pmatrix}^{i} \right) \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P_x} \end{pmatrix} \begin{pmatrix} 0 \\ V_I(t-h) \end{pmatrix} - \left(\sum_{i=0}^{k-1} \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{R} & \mathbf{Q} \end{pmatrix}^{i} \right) \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{I}_{indep} \end{pmatrix}$$

As the iteration number goes infinity, it becomes

$$\begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix}^{\infty} = \begin{pmatrix} \mathbf{I} & 0 \\ \mathbf{B} & \mathbf{Q}^{\infty} \end{pmatrix} \begin{pmatrix} V_b(t) \\ V_I(t) \end{pmatrix}^{1} + \begin{pmatrix} \mathbf{0} & 0 \\ \mathbf{0} & \left(\sum_{i=0}^{\infty} \mathbf{Q}^i \right) \mathbf{P_x} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ V_I(t-h) \end{pmatrix} - \begin{pmatrix} \mathbf{0} & 0 \\ \mathbf{0} & \left(\sum_{i=0}^{\infty} \mathbf{Q}^i \right) \mathbf{K} \end{pmatrix} \begin{pmatrix} \mathbf{0} \\ \mathbf{I}_{indep} \end{pmatrix}$$

where

$$\mathbf{Q}^{\infty} = \mathbf{0};$$

$$\mathbf{N} \equiv \sum_{i=0}^{\infty} \mathbf{Q}^i = (\mathbf{I} - \mathbf{Q})^{-1},$$

$$\mathbf{B}_{I\to b} \equiv \mathbf{B} = \left(\sum_{i=0}^{\infty} \mathbf{Q}^i \right) \mathbf{R} = \mathbf{NR},$$

$$\mathbf{B}_{I\to I\_last} \equiv \mathbf{NP_x}.$$

Finally, the solution of $\boldsymbol{V}_I(t)$ is

$$V_I(t) = \mathbf{B}_{I\to b} V_b(t) + \mathbf{B}_{I\to I\_last} V_I(t-h) - \mathbf{NKI}_{indep} \qquad \blacklozenge\blacklozenge\blacklozenge$$

Hence, since the items at right hand side of Equation (14) are constant at the current sampling time, the unknown vector of $\boldsymbol{V}_I(t)$ can be directly calculated.

- ***Hierarchical Based Markov-Chain Like Solver for RC Power Delivery Network***

Since the flat Markov chain solver needs to calculate the inverse or perform LU decomposition on a matrix, its time complexity is at least $O(N^2)$. We propose a ***Hierarchical based Markov chain solver*** to deal with this problem. This hierarchical analysis contains three phases

Phase 1. Partition the whole power grid into many sub-grids and view the nodes on cut lines as boundary nodes.

Phase 2. Construct the relation between boundary nodes and sub-grid nodes.

Phase 3. Solve the nodal voltage at each node.

Phase 1. **Partition the whole power grid into many sub-grids and view the nodes on cut lines as boundary nodes**

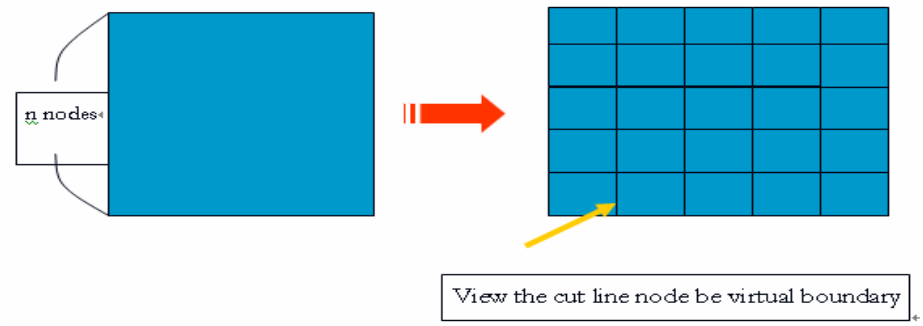


Figure 6. RC power delivery network partition

The suggested number of cut lines is $k = N^{0.5}$ to get an optimum efficiency for the hierarchical based algorithm.

Phase 2. **Construct the relation between virtual boundary nodes and sub-grid nodes**

There are two sub-phases in this Phase. We need to construct the relation between internal nodes and boundary nodes in SubPhase 2.1. Then, the relation between virtual boundary nodes and boundary nodes is constructed in SubPhase 2.2. The virtual boundary nodes are the nodes adjacent to the boundary nodes as illustrated in Figure 8.

SubPhase 2.1. ***Construct the relation between internal nodes and boundary nodes***

Those nodes on the cut lines are defined to be boundary nodes as illustrate in Figure 7, and the Markov chain equations for all sub-networks are constructed.

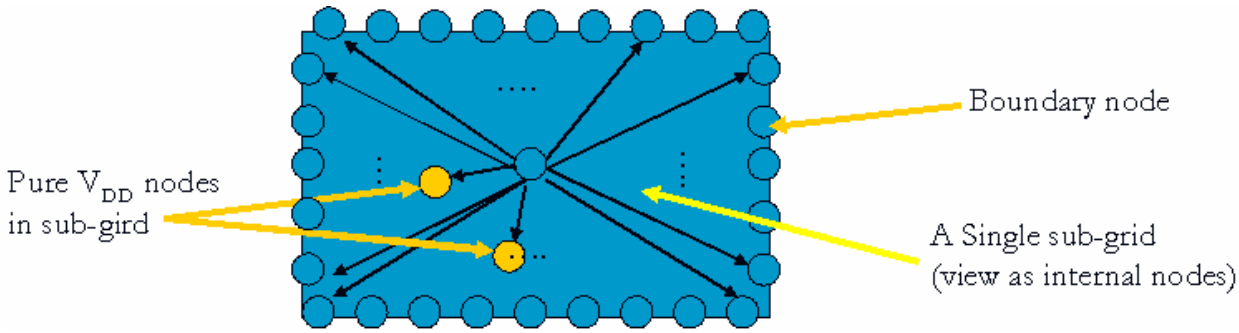


Figure 7. The relation between internal nodes and boundary nodes

After constructing the relation between sub-grids and boundary nodes, the sub-grids

nodal voltage vector can be expressed as

$$V_I(t) = B_{\mathrm{I}\to b}V_b(t) + B_{\mathrm{I}\to \mathrm{I\_last}}V_{\mathrm{I}}(t-h) + B_{\mathrm{I}\to k}V_k - N_{I\to I}K_{I\to I}I_{I\_indep} \tag{15}$$

where

$\boldsymbol{V}_b(t)$: the vector of boundary nodal voltages.

$\boldsymbol{V}_i(t\text{-}h)$: the vector of internal last sampling nodal voltage states.

$\boldsymbol{V}_k$: the vector of pure V_{DD} states in sub-girds.

$\boldsymbol{KI}_{I\_indep}$: the vector of internal independent current source coefficients.

<u>SubPhase 2.1.</u> Construct the relation between virtual boundary and boundary nodes

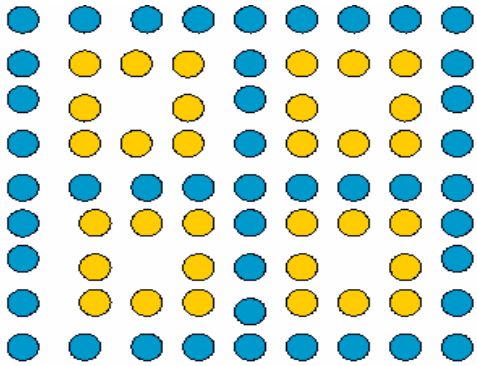
The relation between virtual boundary and boundary nodes can be built up by the follow steps.

Step A. The first-pass internal nodes are view as virtual boundary nodes, and the relation between virtual boundary and boundary nodes is

$$V_b(t) = B_{b\to f}V_f(t) + B_{b\to k}V_k + B_{b\to b\_last}V_b(t-h) - N_{b\to b}K_{b\to b}I_b \tag{16}$$

Step B. From the pervious discussion, the system equation of virtual boundary nodes to boundary nodes for each sub-grid network i is

$$V_{f_i}(t) = B_{f_i\to b_i}V_{b_i}(t) + B_{f_i\to I_{i\_last}}V_{I_{i\_last}}(t-h) + B_{f_i\to k_i}V_{k_i} - N_{f_i\to I_i}K_{I_i\to I_i}I_{I_i} \tag{17}$$



Where the yellow state is the virtual boundary nodes when a boundary state go into.

Figure 8. The virtual boundary nodes

Therefore, the hierarchical iterative system matrix is

$$\begin{pmatrix} V_b(t) \\ V_f(t) \end{pmatrix} = \begin{pmatrix} 0 & U \\ L & 0 \end{pmatrix}\begin{pmatrix} V_b(t) \\ V_f(t) \end{pmatrix} + \begin{pmatrix} B_{b\to b\_last} & 0 \\ 0 & B_{f\to I\_last} \end{pmatrix}\begin{pmatrix} V_b(t-h) \\ V_I(t-h) \end{pmatrix} + \begin{pmatrix} B_{b\to k} & 0 \\ 0 & B_{f\to k} \end{pmatrix}\begin{pmatrix} V_{b\to k} \\ V_{f\to k} \end{pmatrix} - \begin{pmatrix} N_{b\to b}K_b & 0 \\ 0 & N_{f\to I}K_s \end{pmatrix}\begin{pmatrix} I_{indep\_b} \\ I_{indep\_I} \end{pmatrix}$$

Let us rewrite the above hierarchical iterative matrix as

$$\begin{pmatrix} V_b(t) \\ V_f(t) \end{pmatrix}^{k+1} = \mathbf{G}\begin{pmatrix} V_b(t) \\ V_f(t) \end{pmatrix}^{k} + \mathbf{C} \tag{18}$$

where **C**: the last three terms of right hand side of the hierarchical iterative matrix

$$\mathbf{G} = \begin{pmatrix} 0 & U \\ L & 0 \end{pmatrix}$$

The matrix **I-G** is a strictly diagonal dominant matrix and the convergence of this iterative equation has been proven by [20]. Furthermore, the **G** matrix is *consistently ordered* [20], we can use SOR (successive over-relaxation method) to improve its convergent rate.

Phase 3. **Solve the nodal voltage at each node**

From the description of ***Phase 1*** and ***Phase 2***, the nodal voltages of all internal nodes are functions of boundary nodal voltages, and the boundary nodal voltages are functions of virtual boundary nodal voltages. Therefore, we first compute the nodal voltages at boundary nodes and virtual boundary nodes, and re-compute the nodal voltages at the internal nodes.

Figure 9 is the flow chart of the proposed hierarchical Markov chain method for RC power delivery network analysis.

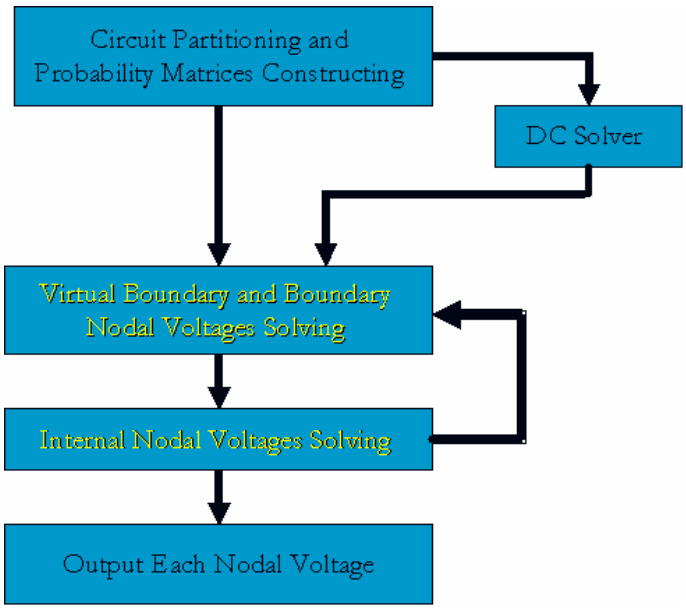
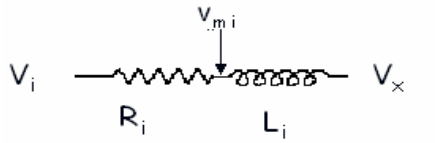


Figure 9. The flow chart of the proposed hierarchical Markov chain method for RC power delivery network analysis.

Markov-Chain Like Solver for RLKC Power Delivery Network

With including the effect of mutual inductance, the proposed Markov-chain like solver for RC power delivery network can not be directly applied to solve those additive variables, branch currents induced by the inductance components. An iterative procedure is used to overcome this problem. A single branch equation with including the inductance/mutual inductance effect can be represented as



$$v_i(t) - v_x(t) = I_i(t)R_i + \sum_{j \in L\_effect(i)} L_{ij} \frac{dI_i(t)}{dt}$$

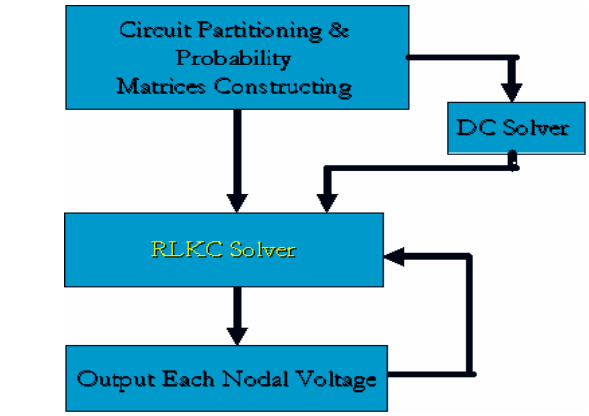
Let the matrix **K** be the inverse of inductance matrix **L**, and k_{ij} be the entry of **K** at row *i* and column *j*. After applying KCL, backward Euler and with some manipulations of the above branch equation, the nodal equation for a node and its iterative relation are

$$v_{m_i}^t = \frac{g_i}{g_i + hk_{ii}} v_i^t + \frac{hk_{ii}}{g_i + hk_{ii}} v_x^t - \frac{\sum_{j \in L\_effect(i); j \neq i} hk_{ij}}{g_i + hk_{ii}} v_{L_j}^t - \frac{I_i^{t-h}}{g_i + hk_{ii}} \tag{19}$$

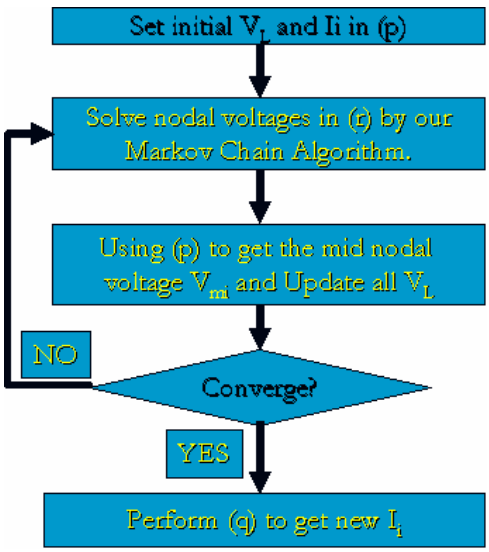
$$I_i^t = h \sum_{j \in L\_effect(i)} k_{ij} v_{L_j}^t + I_i^{t-h} \tag{20}$$

$$v_x^t = \sum_{i \in \deg ree(x)} \left(\frac{hg_i k_{ii}}{m_x \left(g_i + hk_{ii} \right)} \right) v_i^t + \frac{C_x}{m_x h} v_x^{t-h} + \sum_{i \in \deg ree(x)} \frac{g_i}{m_x} I_i^{t-h} + h \sum_{i \in \deg ree(x)} \left(\frac{\sum_{j \in L\_effect(i); j \neq i} hg_i k_{ij} v_{L_j}^t}{m_x \left(g_i + hk_{ii} \right)} \right) - \frac{I_x^t}{m_x} \tag{21}$$

Figure 10.(a) is the flow chart of the proposed hierarchical Markov chain method for RLKC power delivery network analysis, and Figure 10.(b) is the flow chart of the RLKC Solver.



(a)



(b)

Figure 10. (a) The flow chart of the proposed hierarchical Markov chain method for RLKC power delivery network; (b) The flow chart of RLKC Solver.

五、結論與討論

In this report, we have developed a multilevel method based on an aggregation based algebraic multigrid (AAMG) method to efficiently analyze the power delivery network. With this multilevel approach, the proposed method can speed up the convergence speed by avoiding the slow convergence in the low frequency errors which the basic iterative methods suffer.

We also proposed a semi-stochastic like method based on the Markov chain to simulate the power delivery network. This hierarchical based Markov-chain solver can efficiently analyze both the RC and RLKC power delivery networks.

六、成果

1. Yu-Min Lee, Yahong Cao, Tsung-Hao Chen, Janet Wang, and, and Charlie Chung-Ping Chen, "HiPRIME: Hierarchical and Passivity Preserved Interconnect Macro-modeling Engine for RLKC Power Delivery", to appear in *IEEE Transactions on Computer-Aided- Design of Integrated Circuits and Systems, June, 2005*.
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