行政院國家科學委員會專題研究計畫　成果報告

耦合渾沌系統的相位、延後及廣義同步化：部分變量穩定性理論方法

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計畫參與人員：陳炎生

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(一) 研究計畫中英文摘要

計畫中文摘要

關鍵詞：同步化、部分穩定性

本計畫是原申請為期二年之計畫，只核准為一年之計畫，故執行的是原計畫內容之一半，即有關廣義同步部份。

廣義渾沌同步化是近年來發現的重要研究主題。對於此主題，目前的研究以針對各別系統行為的數值模擬研究遠多於理論研究。因為有關廣義渾沌同步之普遍的精確理論迄今未建立，本計畫欲對各種渾沌耦合系統之廣義同步判據給出普遍的精確定理。由於許多研究是用李亞普諾夫指數的跨零來對照模擬系統的廣義同步化，但此準則有時會失敗。且依其定義，計算李亞普諾夫指數需無窮長時間的迭代，而模擬卻永遠不可能做到此點。另一方面，傳統的李亞普諾夫直接法也會無法應用，因誤差系統的向量場函數不僅為誤差的函數，而且為狀態變量的函數。本計畫利用部份變量穩定性的理論(如附件)，對於自治及非自治系統的各種耦合渾沌系統，藉由判別一個增廣系統(extended system)的部分變量穩定性，進而建立嶄新的廣義同步化的一般性精確準則，則上述問題得以解決，具有重大之永遠學術價值。［76-81］諸篇 SCI 論文皆在本計畫經費贊助下出版。

Abstract

key words: synchronization, partial stability

This project is a two year project originally, but only one year project was ratified by NSC. Therefore, only the half of the original project, content about generalized synchronization, was executed.

Generalized synchronization of chaos is an important research topic discovered in recent years. The results of most researches were much more focused on the simulation research of system phenomena than theoretical ones. Since general exact theory of generalized synchronization of chaos has not obtained up to now, general theoretical criterions of generalized synchronization of various chaotic systems are the main contents in this project. Since the zero-crossing of Lyapunov exponents is used as a criterion for chaos synchronization. But this rule does not always work. Furthermore, there is a drawback that we can only calculate finite evolution time in computer simulation but infinite evolution time is needed by its definition. The traditional Lyapunov direct method does not work since the vector functions of error systems are not only function of state error but also function of state. In this project, the partial stability theory (encolsed) is used as a tool to verify the generalized synchronization of autonomous and non-autonomous chaotic systems by verifying the partial stability of an extended system. General exact criteria are then established for generalized synchronizations of chaos, which have important and permanent academic interest. SCI papers [76-81] are sponsored by the funds of this project.

(二) 研究計畫內容

1. 前言

由於渾沌系統表現出對於初始條件極度敏感的特性,使得渾沌現象被認為難以同步化。自從關鍵性研究成果發表以來[1-3],我們了解到同步化是可能的,因而吸引了大量的研究興趣 [4-16],在秘密通訊方面、模式成型(pattern formation)以及資訊處理(information processing)也有重要的成果[18-36,44]。在這些研究工作中,絕大多數為探討單向耦合(unidirectional coupling)系統。針對各別系統的數值模擬行為研究遠多於理論研究。

截至目前為止,廣義同步(generalized synchronization)未有精確而又據普遍性的判準。因為除了少數又不易應用同步化理論外,許多研究是用李亞普諾夫指數的跨零來對照模擬系統的同步化,但此法有時會失效。另外,依其定義,計算李亞普諾夫指數需無窮長時間的迭代,而模擬卻永遠不可能做到此點;再者若要應用傳統的李亞普諾夫直接法也會遭遇困難,因誤差系統的向量場函數不僅為狀態誤差的函數且為狀態的函數。本計畫利用部份變量穩定性的理論,對於對自治及非自治的各種耦合系統,藉由將原系統改成為一個增廣系統(extended system),再來判別此系統的部分變量,即狀態誤差之穩定性,進而建立廣義渾沌同步化普遍的精確準則,則上述問題迎刃而解。

2. 研究目的

對於耦合系統的廣義渾沌同步判據,截至目前為止尚無令人滿意之理論及可實際應用的成果。本計畫將利用部份變量穩定性的理論,提出廣義同步化的普遍的精確準則,且此方法對自治及非自治系統的各種耦合系統皆可適用。其中的部份變量穩定性理論事實上是傳統穩定性理論的推廣,因為傳統的穩定性理論只能針對全部變量作判別,而部分變量穩定性能只對一部份的變量做穩定性判別,此法的應用範圍遠比傳統方法廣泛,它不僅能判別部分變量的穩定性,而且當變量數目取最大就變成全變量穩定性,是很有效的方便方法。

3. 文獻探討

渾沌現象是二十世紀學術界的重大進展,其基本行為及理論在各領域已有許多專書

[37-42]。在渾沌同步與控制也有專書 [43-44]。渾沌同步除了一些基礎研究之外 [1-17]，其應用於秘密通訊上亦有長足進展 [18-36]，利用控制方法達到同步化同樣有不錯的成果 [45-58]，對於廣義同步也有部分成果[59−63]，在國內也有林文偉、莊重及莊正等人有成果[64-71]。本計劃提出以部份變量穩定性理論應用於各種系統的廣義同步化判別，以解決耦合系統廣義同步化精確的普遍判據至今付之闕如此一極為重要的問題。

4. 研究方法及成果

Consider the following unidirectional coupled nonautonomous systems

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(t,\mathbf{x}) \\ \hat{\mathbf{z}} &= \mathbf{g}(t,\hat{\mathbf{z}}) + \mathbf{u}(t,\hat{\mathbf{z}},\mathbf{x}) \end{aligned} \tag{1}$$

where $\mathbf{x}, \hat{\mathbf{z}} \in \mathbf{R}^n$ and $\mathbf{f}, \mathbf{g} : \Omega_1 \subset \mathbf{R} \times \mathbf{R}^n \to \mathbf{R}^n$, $\mathbf{u} : \Omega_2 \subset \mathbf{R} \times \mathbf{R}^{2n} \to \mathbf{R}^n$ satisfy Lipschitz condition. Ω_1, Ω_2 are domains containing the origin. Assume that the solutions of Eqs. (1) have *a priori* bounds then they must exist for infinite time. That is, for given $(t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0) \in \Omega_1 \cap \Omega_2$ the solutions $\mathbf{x}(t, t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0)$, $\hat{\mathbf{z}}(t, t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0)$ of Eqs. (1) exist for $t \ge t_0$. At the first, we recall the definition of generalized synchronization.

Definition *The systems (1) are generalized synchronized if there is a continuous function* $H : \mathbf{R}^n \to \mathbf{R}^n$ *s.t.* $\lim_{t\to\infty} \left\| H[\mathbf{x}(t, t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0) - \hat{\mathbf{z}}(t, t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0) \right\| = 0$ *with* $(t_0, \mathbf{x}_0, \hat{\mathbf{z}}_0) \in \Omega_1 \cap \Omega_2$.

In Eq. (1) $\mathbf{u}$ is the coupling function. In order to discuss the transversal stability of synchronization manifold, define $\mathbf{z} = H(\mathbf{x})$ and $\mathbf{e} = \hat{\mathbf{z}} - \mathbf{z}$ to be the state error. Then the error equations can be written as

$$\dot{\mathbf{e}} = \dot{\hat{\mathbf{z}}} - \mathbf{z} = \mathbf{g}(t,\hat{\mathbf{z}}) + \mathbf{u}(t,\hat{\mathbf{z}},\mathbf{x}) - \dot{H}(\mathbf{x})$$

where

$$\dot{H}(\mathbf{x}) = \frac{\partial H}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{x}} \mathbf{f}(t,\mathbf{x}) .$$

So we have

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$$\dot{\mathbf{e}} = \mathbf{g}(t, \hat{\mathbf{z}}) - \frac{\partial H}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}) + \mathbf{u}(t, \hat{\mathbf{z}}, \mathbf{x}) . \qquad (2)$$

Notice that since the right hand side of Eq. (2) is not only a function of t and error $\mathbf{e}$, but also a function of $\mathbf{x}$, as a result, the traditional Lyapunov direct method can not be used. On the other hand, the variational equation or Lyapunov exponents can be used to clarify transversal stability. Josić [75] pointed out that synchronization manifolds will persist under perturbation if such manifolds possess a property of k-hyperbolicity. Herein, we add the upper half (lower half also works) of Eq. (1) with $\hat{\mathbf{z}}$ replaced by $\hat{\mathbf{z}} = \mathbf{e} + \mathbf{z}$ to Eq. (2), then extended equations are obtained as following

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \\ &\dot{\mathbf{e}} = \mathbf{g}(t, \hat{\mathbf{z}}) - \frac{\partial H}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x}) + \mathbf{u}(t, \hat{\mathbf{z}}, \mathbf{x}) . \end{aligned} \qquad (3)$$

If the partial variables $\mathbf{e}$ in Eq. (3) are asymptotically stable about $\mathbf{e} = \mathbf{0}$, the synchronization manifold is stable in transversal directions. This can be done via stability with respect to partial variables. The theory of partial stability can be found in Appendix.

In the following, we choose $\mathbf{u}(t, \hat{\mathbf{x}}, \mathbf{z}) = \Gamma(\mathbf{z} - \hat{\mathbf{z}})$ and $\mathbf{g}(t, \mathbf{z}) = \frac{\partial H}{\partial \mathbf{x}} \mathbf{f}(t, \mathbf{x})$, where $\Gamma \in M_{n \times n}$ is a constant matrix whose entries represent the coupling strength of the linear feedback term $(\mathbf{z} - \hat{\mathbf{z}})$. Then the Eqs. (3) becomes

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \\ &\dot{\mathbf{e}} = \mathbf{g}(t, \hat{\mathbf{z}}) - \mathbf{g}(t, \mathbf{z}) - \Gamma(\hat{\mathbf{z}} - \mathbf{z}). \end{aligned} \qquad (4)$$

Theorem The partial state $\mathbf{e}$ asymptotically approaches to $\mathbf{0}$ in Eq. (5) if $L\mathbf{I}_n - \Gamma$ is negative definite, i.e. the systems in Eq. (4) are in generalized synchronization if $L\mathbf{I}_n - \Gamma$ is negative definite.

Remark: From the matrix theory, $L\mathbf{I}_n - \Gamma$ is negative definite if and only if all its eigenvalues are negative. Moreover, the result is global if $\mathbf{f}$ is globally Lipschitz.

5. 結果與討論

A general scheme to achieve chaos generalized synchronization via partial stability was proposed. One theorem is proven to ensure generalized synchronization for a general kind of unidirectional coupled nonautonomous systems by linear feedback coupling term. The result works for both regular and chaotic systems. (本計畫申請之研究期限為二年，核准為一年期限。所完成者為原計畫之“廣義同步化”部份)

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Appendix

The content of this appendix follows [55-57]. Consider a differential system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \tag{A1}$$

where $\mathbf{f} : [t_0, \infty) \times \Omega \to \mathbf{R}^n$, $\mathbf{f}(t, \mathbf{0}) = \mathbf{0}$ $\forall t \in [t_0, \infty)$ and $\Omega \subset \mathbf{R}^n$ is a region containing the origin. Assume that $\mathbf{f}$ is smooth enough to ensure that the solution of (A1) exists uniquely. To shorten the notation, write $\mathbf{x} = (y_1, \cdots y_m, z_1, \cdots, z_{n-m})^T$, $\|\mathbf{y}\| = \left(\sum_{i=1}^{m} y_i^2 \right)^{1/2}$, $\|\mathbf{z}\| = \left(\sum_{i=1}^{n-m} z_i^2 \right)^{1/2}$ and $\|\mathbf{x}\| = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2} = \left(\|\mathbf{y}\|^2 + \|\mathbf{z}\|^2 \right)^{1/2}$ with $0 < m \le n$. We assume that the solution of (A1) is $\mathbf{z}$-extendable, i.e. any solution of (A1) exists for all $t \ge t_0$ and $\|\mathbf{y}(t)\| \le H$, H is a constant. Write $Q = \left\{ (t, \mathbf{x}) \middle| \, t \ge t_0, \|\mathbf{y}\| \le H, 0 \le \|\mathbf{z}\| < +\infty \right\}$ and $\tilde{Q} = \left\{ (t, \mathbf{x}) \middle| \, t \ge t_0, \|\mathbf{x}\| < \infty \right\}$.

Definition A1 The solution of (A1) is stable with respect to $\mathbf{y}$ ($\mathbf{y}$-stable) if $\forall \varepsilon > 0$, $\forall t_0 \in [0, \infty)$, $\exists \delta(t_0, \varepsilon) > 0$, $\forall \mathbf{x}_0 \in B_\delta := \left\{ \mathbf{x} \middle| \, \|\mathbf{x}\| < \delta(t_0, \varepsilon) \right\}$ such that $\|\mathbf{y}(t, t_0, \mathbf{x}_0)\| < \varepsilon$ $\forall t \ge t_0$. The solution of (A1) is uniformly $\mathbf{y}$-stable if $\delta(t_0, \varepsilon)$ is independent of t_0 in the definition of $\mathbf{y}$-stable.

The solution of (A1) is asymptotically stable with respect to $\mathbf{y}$ (asymptotically $\mathbf{y}$-stable) if it is (1) $\mathbf{y}$-stable and (2) $\mathbf{y}$-attractive, i.e. $\forall t_0 \in [0, \infty)$, $\exists \delta'(t_0) > 0$, $\forall \varepsilon' > 0$, $\forall \mathbf{x}_0 \in B_{\delta'} := \left\{ \mathbf{x} \middle| \, \|\mathbf{x}\| < \delta'(t_0) \right\}$, $\exists T(t_0, \mathbf{x}_0, \varepsilon')$ such that $\|\mathbf{y}(t, t_0, \mathbf{x}_0)\| < \varepsilon'$ $\forall t \ge t_0 + T$. The solution of (A1) is uniformly asymptotically $\mathbf{y}$-stable if it is (1) uniformly $\mathbf{y}$-stable and (2) uniformly $\mathbf{y}$-attractive, i.e. $\delta'(t_0)$ is independent of t_0 and $T(t_0, \mathbf{x}_0, \varepsilon')$ is independent of $t_0, \mathbf{x}_0$ in the definition of $\mathbf{y}$-attractive.

The solution of (A1) is globally $\mathbf{y}$-attractive if $B_\delta = \mathbf{R}^n$ in the definition of $\mathbf{y}$-attractive. Furthermore, if $B_\delta = \mathbf{R}^n$ and $\exists \delta'(t_0) > 0$ can be replaced by $\forall \delta'$ the solution of (A1) is globally uniformly $\mathbf{y}$-attractive. The solution of (A1) is globally asymptotically $\mathbf{y}$-stable if it is (1) $\mathbf{y}$-stable and (2) globally $\mathbf{y}$-attractive. The solution of (A1) is globally uniformly asymptotically **y**-stable if it is (1) uniformly **y**-stable and (2) globally uniformly **y**-attractive.

The next definition extends the notation of definite functions with respect to partial variables. Let $V(t,\mathbf{x}) \in C\left([t_0,\infty)\times\mathbf{R}^n,\mathbf{R}\right)$ with $V(t,\mathbf{0})=\mathbf{0}$ and V defined on Q.

Definition A2 A t implicit positive (negative) semi-definite function $V(\mathbf{x})$ is called positive (negative) definite with respect to **y** if $V(\mathbf{x})$ can vanish only when $\mathbf{y}=\mathbf{0}$.

A positive (negative) semi-definite function $V(t,\mathbf{x})$ is called positive (negative) definite with respect to **y** if there is a positive (negative) definite function $W(\mathbf{y})$ such that $V(t,\mathbf{x}) \ge W(\mathbf{y})$ ($V(t,\mathbf{x}) \le W(\mathbf{y})$).

Definition A3 A function $V(t,\mathbf{x})$ is called bounded if $\exists M>0$ such that $|V(t,\mathbf{x})| \le M$. A bounded function $V(t,\mathbf{x})$ possesses an infinitesimal upper bound if $\forall \tilde{\varepsilon}>0, \exists \tilde{\delta}(\tilde{\varepsilon})>0$, for $t \ge t_0$ and $\|\mathbf{x}\| < \tilde{\delta}(\tilde{\varepsilon})$ such that $|V(t,\mathbf{x})| \le \tilde{\varepsilon}$. A bounded function $V(t,\mathbf{x})$ possesses an infinitesimal upper bound with respect to $x_1,\cdots,x_k$ $(m \le k \le n)$ if $\forall \tilde{\varepsilon}>0, \exists \tilde{\delta}(\tilde{\varepsilon})>0$, for $t \ge t_0, \sum_{i=1}^{k} x_i^2 < \tilde{\delta}^2, -\infty < x_{k+1},\cdots,x_n < \infty$ such that $|V(t,\mathbf{x})| \le \tilde{\varepsilon}$.

The following four theorems still hold when the undisturbed motion has nonzero **z**.

Theorem A1 Suppose there exists a positive definite function $V(t,\mathbf{x})$ with respect to $x_1,\cdots,x_k$ $(k \le n)$ such that $\dot{V}(t,\mathbf{x})$ is negative semi-definite or vanishes, then the undisturbed motion is stable with respect to $x_1,\cdots,x_k$ $(k \le n)$.

Theorem A2 Suppose there exists a positive definite function $V(t,\mathbf{x})$ with respect to $x_1,\cdots,x_k$ $(k \le n)$ such that $V(t,\mathbf{x})$ possesses an infinitesimal upper bound and $\dot{V}(t,\mathbf{x})$ is negative definite with respect to $x_1,\cdots,x_k$, then the undisturbed motion is asymptotically stable with respect to $x_1,\cdots,x_k$.

Theorem A3 Suppose there exist a function $V:[0,\infty)\times\Omega\to\mathbf{R}$ such that for some functions $a,b,c\in\mathcal{K}$ and every $(t,\mathbf{x})\in Q$:

(i) $a\left(\|\mathbf{y}\|\right) \le V(t,\mathbf{x}), V(t,\mathbf{0})=\mathbf{0}$

(ii) $V(t,\mathbf{x}) \le b\left(\left(\sum_{i=1}^{k} x_i^2\right)^{1/2}\right), m \le k \le n$

(iii) $\dot{V}(t,\mathbf{x}) \le -c\left(\left(\sum_{i=1}^{k} x_i^2\right)^{1/2}\right)$,

then the origin is uniformly asymptotically **y**-stable.

Theorem A4 Suppose there exist a function $V:[0,\infty)\times\Omega\to\mathbf{R}$ such that for some functions $a,b,c\in\mathcal{K}$, $a:\mathbf{R}^+\to\mathbf{R}^+$ with $r\to+\infty\Rightarrow a(r)\to+\infty$ and every $(t,\mathbf{x})\in\tilde{Q}$:

(i) $a\left(\|\mathbf{y}\|\right)\le V(t,\mathbf{x}), V(t,\mathbf{0})=\mathbf{0}$

(ii) $V(t,\mathbf{x}) \le b\left(\left(\sum_{i=1}^{k} x_i^2\right)^{1/2}\right), m \le k \le n$

(iii) $\dot{V}(t,\mathbf{x}) \le -c\left(\left(\sum_{i=1}^{k} x_i^2\right)^{1/2}\right)$

(iv) $\sum_{i=1}^{n} x_i^2 \to +\infty \;\Rightarrow\; V(t,\mathbf{x})\to+\infty$,

then the origin is globally asymptotically **y**-stable.

（三）計畫成果自評

1. 本計畫利用了部份變量穩定性理論，針對單向耦合廣義同步化問題提出普遍而嚴格的解決之道，並依照此方法得到一定理。此結果對於各種動態系統，不論是自治或非自治、規律的或渾沌的系統，此定理皆能保證廣義同步化之發生，是有效且適用的理論判據，在學術上有重大意義。
2. 所得結果在動態系統領域(特別在動態系統同步化)具有重要學術價值。
3. 培養博碩士研究生科學研究能力。
4. 本計畫申請之研究期限為二年,核准為一年期限。故所完成者為原計畫之“廣義同步化”部份。