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Blind Identification and Equalization of MIMO FIR Systems (II)
A Progress Report

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We describe briefly the research work in progress, the preliminary results obtained so far, and the expected final results at the end of the first two years's research. Topics of research we have undertaken include

- Blind identification of MIMO channel using periodic precoding.
- Reverse link power control in CDMA.
- Space-time block coding scheme

1 Blind identification of MIMO channel using periodic precoding

The basic idea is to induce cyclostationarity at the transmitter and to exploit the linear relation between the covariance of the received data. This is an extension of our pervious result on SISO channel. A preliminary version of the result had been presented at 2005 ISCAS (Kobe Japan). Quite a few people at the conference are interested in our result. One of the feature attract most attention: the ability of the proposed algorithm to identify the channel when the number of outputs is less than the number of inputs.

The complete result has been submitted to IEEE Transaction on Signal Processing (date: 2005/4/19). The manuscripts is attached at the end of this report.

Currently we are working on an algorithm for blind identification of MIMO channel without using periodic precoding. Preliminary simulation results are promising. We expect to finish the investigation and submit a paper at the end of this budget year.

2 Reverse link power control in CDMA.

In CDMA multi-user environment, the signal transmitted by a mobile will cause interference at the base station for other user. This is commonly called multiple access interference (MAI), which is due to the non-orthogonality of the codes that identify the individual user. The MAI makes power control important in maintaining a desired level of quality of service (QOS).

The current practice is to use a very conservation control strategy in order to guarantee stability at the expense of the achievable performance. In other words, if an more aggressive strategy is used it may be possible to have faster tracking of the intended signal-to-interference (SIR) level. Of course, in a feedback loop an aggressive control strategy runs the risk of causing instability if the controller is not properly designed. We believe feedback control theory should be used to design power control strategy so as to improve quality of service or to increase the network capacity.

The power control problem can be modelled as a decentralized control problem, with interconnected dynamics and distributed control. The interconnected dynamics has diagonal dominance structure: the correct correlations gives the intended strong links and the incorrect corrections can be modelled as weak links. There is stability theory based on diagonal dominance available. The purpose of our research is to investigate the possibility of using the theory do analyze the performance of power control algorithm. We will try to establish quantitative relation between the interference level and the achievable performance

We expected some preliminary results at the end of the current budget year.

3 Space-time block coding scheme

This is a topic we just started to investigate. In the MIMO setup (multiple transmit and receive antenna), the coding is important in using the channel diversity while maintain a reasonable transmission rate. We will focus on the topics of identification and equalization in a space-time coded transmission.

We are currently studying the fundamental aspect of space-time coding scheme and we do not expect any significant process at the end of this budget year.

4 Paper submitted to IEEE Trans. Signal Processing

Blind Identification of MIMO Channels Using Optimal Periodic Precoding*

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Abstract

We propose a method for blind identification of MIMO FIR channels that exploits cyclostationarity of the received data induced at the transmitters by periodic precoding. It is shown that, by properly choosing the precoding sequence, the MIMO FIR transfer functions, with M_t inputs and M_r outputs, can be identified up to a unitary matrix ambiguity. The transfer functions need not be irreducible or column reduced, and there can be more outputs ($M_r \geq M_t$) or more inputs ($M_r < M_t$). The method exploits the linear relation between the covariance matrix of the received data and the "channel product matrices". The method is shown to be robust with respect to channel order overestimation. The proposed algorithm requires solving linear equations and computing the nonzero eigenvalues and eigenvectors of a Hermitian positive semidefinite matrix. The performance of the algorithm, and indeed the identifiability, depend on the choice of the precoding sequence. We propose a method for optimal selection of the precoding sequence which takes into account the effect of additive channel noise and numerical error in covariance estimation. Simulation results are used to demonstrate the performance of the algorithm.

Key Words : MIMO channel, blind identification, transmitter induced cyclostationarity, periodic precoding

1 Introduction

Blind identification of SISO frequency selective channels exploiting transmitter induced cyclostationarity of the second-order statistics of the received data is first proposed in [1, 2]. Since then, various schemes have been proposed to induce cyclostationarity at the transmitter and to blindly identify SISO [3]-[8] and MIMO channels [9]-[13]. One way to induce cyclostationarity at the transmitter is by periodic precoding, i.e., multiplying the source symbols

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with a periodic sequence before transmission [2, 5, 6, 8],[9]-[11]. For SISO channels, blind identification methods based on periodic precoding are shown to be robust with respective to channel order overestimation and impose no restriction on the locations of channel zeros [2, 5, 6, 8].

In the MIMO context, Chevreuil and Loubaton [9] proposes a scheme that multiplies each input by a constant modulus complex exponential precoding sequence to induce conjugate cyclostationarity at the transmitter. The scheme is used to reduce the MIMO channel identification preblem to several SIMO ones, which are then solved by the subspace method [16]. Each SIMO channel is required to be free from common zeros and only real symbols can be used. Bölcskei et. al. [10] proposes a method that can identify each of scalar channels up to a phase ambiguity using non-constant modulus periodic precoding sequences. The method imposes no restriction on channel zeros and is insensitivity to channel order overestimation. However, no general procedure for the design of the precoding sequences is given. The method is extended to the multicarrier case [11].

In this paper, we propose a method for blind identification of MIMO FIR channels using periodic precoding as a means to induce cyclostationarity. It is shown that, by properly choosing the precoding sequence, the MIMO FIR transfer functions, with M_t inputs and M_r outputs, can be identified up to a unitary matrix ambiguity. The transfer functions need not be irreducible or column reduced [14, 15], and there can be more outputs ($M_r \geq M_t$) or more inputs ($M_r < M_t$). The method exploits the linear relation between the covariance matrix of the received data and the "channel product matrices". The method is shown to be robust with respect to channel order overestimation. The proposed algorithm requires solving linear equations and computing the nonzero eigenvalues and eigenvectors of a Hermitian positive semidefinite matrix. The performance of the algorithm, and indeed the identifiability, depend on the choice of the precoding sequence. We propose a method for optimal selection of the precoding sequence which takes into account the effect of additive channel noise and numerical error in covariance estimation. Simulation results are used to demonstrate the performance of the algorithm. The paper generalizes the results for the SISO case discussed in [8].

The paper is organized as follows. Section 2 is problem statement and formulation. In Section 3, we derive the identification method and propose the blind identification algorithm. In Section 4, we discuss optimal selection of the precoding sequence. Simulation results are given in Section 5. Section 6 concludes the paper.

Notations used in this paper are quite standard: Bold uppercase is used for matrices, and bold lowercase is used for vectors. $\mathbf{A}^T$ represents transpose of the matrix $\mathbf{A}$, and $\mathbf{A}^*$ represents conjugate transpose of the matrix $\mathbf{A}$. $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$. $\mathbf{0}_{M\times N}$ is the zero matrix of dimension $M \times N$, and $\mathbf{I}_M$ is the identity matrix of dimension $M \times M$.

2 Problem Statement and Formulation

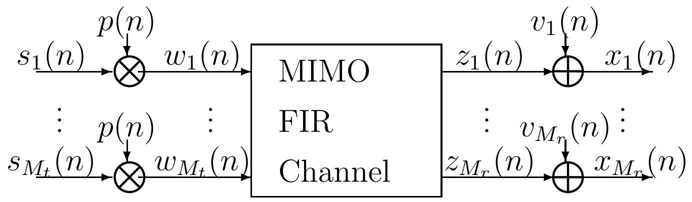

Figure 2.1: MIMO Channel Model

We consider the linear MIMO baseband model of a communication channel with M_t transmitters and M_r receivers shown in Figure 2.1, where each source symbol sequence is multiplied by an N-periodic sequence, $p(n)$, before transmission. The transmitted signal is

$$w_j(n) = p(n)s_j(n), \quad j = 1, 2, \cdots, M_t \tag{2.1}$$

where $p(n+N) = p(n)$, $\forall\, n$. The discrete time model describing the relation between the transmitted signal $w_j(n)$ and the received signal $x_i(n)$ has the form of an MIMO FIR filter with additive noise:

$$x_i(n) = \sum_{j=1}^{M_t} \sum_{l=0}^{L_{ij}} h_{ij}(l)w_j(n-l) + v_i(n), \quad i = 1, 2, ..., M_r \tag{2.2}$$

where $h_{ij}(0)$, $h_{ij}(1)$, $\cdots$, $h_{ij}(L_{ij})$, are the impulse responses of the channel between the jth transmitter and the ith receiver, and $v_i(n)$ is the channel noise seen at the input of the ith receiver. The equations (2.1) and (2.2) can be written more compactly as

$$\mathbf{w}(n) = p(n)\mathbf{s}(n), \quad \mathbf{x}(n) = \sum_{l=0}^{L} \mathbf{H}(l)\mathbf{w}(n-l) + \mathbf{v}(n) \tag{2.3}$$

where $\mathbf{w}(n)$, $\mathbf{s}(n) \in \mathsf{C}^{M_t}$, and $\mathbf{x}(n)$, $\mathbf{v}(n) \in \mathsf{C}^{M_r}$ are vector signals formed by stacking the respective scalar signals together, e.g., $\mathbf{x}(n) = [x_1(n) \;\; x_2(n) \;\; \cdots \;\; x_{M_r}(n)]^T$. The ijth element of $\mathbf{H}(l) \in \mathsf{C}^{M_r \times M_t}$ is $h_{ij}(l)$, and $L = \max_{i,j}\{L_{ij}\}$ is the order of the MIMO channel. Thus $\mathbf{H}(L) \neq \mathbf{0}_{M_r \times M_t}$. The following assumptions are made throughout.

(A1) $\mathbf{s}(n)$ and $\mathbf{v}(n)$ are white with zero mean vector sequences, and $\mathbf{s}(n)$ and $\mathbf{v}(n)$ are temporally and spatially uncorrelated. More precisely, $E[\mathbf{s}(k)\mathbf{s}(j)^*] = \delta(k-j)\mathbf{I}_{M_t} \in \mathsf{R}^{M_t \times M_t}$, $E[\mathbf{v}(k)\mathbf{v}(j)^*] = \delta(k-j)\sigma_v^2\mathbf{I}_{M_r} \in \mathsf{R}^{M_r \times M_r}$, $E[\mathbf{s}(k)\mathbf{v}(j)^*] = \mathbf{0}_{M_t \times M_r}$, $\forall\, k, j$, where $\delta(k-j)$ is the Kronecker delta function.

(A2) An upper bound $\hat{L}$ of the channel order L is known and the period $N > \hat{L} + 1$.

(A3) $\mathrm{rank}([\mathbf{H}(0)^T \ \mathbf{H}(1)^T \ \cdots \ \mathbf{H}(L)^T]^T) = M_t$.

Due to periodic precoding, the input-output relation between the source $\mathbf{s}(n)$ and the received signal $\mathbf{x}(n)$, described by (2.3), is periodically time-varying. In order to obtain a time-invariant representation, we consider input-output relation between block input and block output of size N [17]. Define block signal $\bar{\mathbf{x}}(n) = [\mathbf{x}(Nn)^T, \mathbf{x}(Nn+1)^T, \cdots, \mathbf{x}(Nn+N-1)^T]^T \in \mathsf{C}^{M_t N}$, and let $\bar{\mathbf{v}}(n), \bar{\mathbf{w}}(n), \bar{\mathbf{s}}(n)$ be similarly defined. Since $p(n)$ is periodic, $\bar{\mathbf{w}}(n) = \mathbf{G}\bar{\mathbf{s}}(n)$ for all n, where $\mathbf{G} = \mathrm{diag}[p(0)\mathbf{I}_{M_t}, p(1)\mathbf{I}_{M_t}, \cdots, p(N-1)\mathbf{I}_{M_t}] \in \mathsf{R}^{M_t N \times M_t N}$ is a diagonal matrix. In terms of block signals, (2.3) can be written as

$$\bar{\mathbf{x}}(n) = \mathbf{H_0}\bar{\mathbf{w}}(n) + \mathbf{H_1}\bar{\mathbf{w}}(n-1) + \bar{\mathbf{v}}(n) = \mathbf{H_0}\mathbf{G}\bar{\mathbf{s}}(n) + \mathbf{H_1}\mathbf{G}\bar{\mathbf{s}}(n-1) + \bar{\mathbf{v}}(n) \tag{2.4}$$

where $\mathbf{H_0}$ is an $M_r N \times M_t N$ block lower-triangular Toeplitz matrix with $[\mathbf{H}(0)^T \ \mathbf{H}(1)^T \ \cdots \ \mathbf{H}(L)^T \ \mathbf{0}^T_{M_r \times M_t} \ \cdots \ \mathbf{0}^T_{M_r \times M_t}]^T \in \mathsf{C}^{M_r N \times M_t}$ as its first block column (i.e., the first M_t columns), and $\mathbf{H_1}$ is an $M_r N \times M_t N$ block upper-triangular Toeplitz matrix with $[\mathbf{0}_{M_r \times M_t} \ \cdots \ \mathbf{0}_{M_r \times M_t} \ \mathbf{H}(L) \ \mathbf{H}(L-1) \ \cdots \ \mathbf{H}(1)] \in \mathsf{C}^{M_r \times M_t N}$ as its first block row (i.e., the first M_r rows).

The problem we study in this paper is blind identification of the MIMO channel matrix $\mathbf{H} = [\mathbf{H}(0)^T \ \mathbf{H}(1)^T$
$\cdots \ \mathbf{H}(L)^T]^T$ using second-order statistics of the received data. The proposed method exploits the cyclostationarity induced by periodic precoding at the transmitters. The performance of the proposed identification algorithm (Section 3.3) depends critically on the choice of the precoding sequence. We discuss the optimal selection of the sequence that yields the best performance. We define the following operations that will be used in the derivation of the main result. First, for any $m \times m$ matrix $\mathbf{A} = [a_{k,l}]_{0 \le k,l \le m-1}$, define $\Gamma_j(\mathbf{A}) = [a_{0,j} \ \ a_{1,j+1} \ \ \cdots \ \ a_{m-1-j,m-1}]^T$ for $0 \le j \le m-1$, i.e., $\Gamma_j(\mathbf{A})$ is the vector formed from the jth super-diagonal of $\mathbf{A}$. Second, for any $M_r n \times M_r n$ matrix $\mathbf{B} = [\mathbf{B}_{k,l}]_{0 \le k,l \le n-1}$, where $\mathbf{B}_{k,l}$ is a block matrix of dimension $M_r \times M_r$, define $\Upsilon_j(\mathbf{B}) = [\mathbf{B}^T_{0,j} \ \ \mathbf{B}^T_{1,j+1} \ \ \cdots \ \ \mathbf{B}^T_{n-1-j,n-1}]^T$ for $0 \le j \le n-1$, i.e., $\Upsilon_j(\mathbf{B})$ is the matrix formed from the jth block super-diagonal of $\mathbf{B}$.

3 Channel Identification

We study channel identification in this section. In Section 3.1, we derive the proposed method assuming the channel order is known and the noise is absent. We show that by appropriately selecting the periodic precoding sequence, any MIMO channel satisfying (**A3**) is identifiable up to an $M_t \times M_t$ unitary matrix ambiguity. In Section 3.2, we show that the proposed method is robust with respect to channel order overestimation and we propose an identification algorithm in Section 3.3. The effect of noise and optimal selection of the precoding sequence are discussed in Section 4.

3.1 The Identification Method

We consider the noise free case and assume that the channel order L is known. Equation (2.4) now becomes

$$\bar{\mathbf{x}}(n) = \mathbf{H_0}\mathbf{G}\bar{\mathbf{s}}(n) + \mathbf{H_1}\mathbf{G}\bar{\mathbf{s}}(n-1) \tag{3.1}$$

With assumption (**A1**), the covariance matrix of $\bar{\mathbf{x}}(n)$ can be written as

$$\mathbf{R}_{\bar{\mathbf{x}}}(0) = E[\bar{\mathbf{x}}(n)\bar{\mathbf{x}}(n)^*] = \sigma_s^2 \left(\mathbf{H_0}\mathbf{G}^2\mathbf{H_0^*} + \mathbf{H_1}\mathbf{G}^2\mathbf{H_1^*}\right) \tag{3.2}$$

Let $\mathbf{J} \in \mathsf{R}^{N\times N}$ be the matrix whose first sub-diagonal are all one, i.e., $\Gamma_1(\mathbf{J}^T) = [1\ 1\ \cdots\ 1]^T \in \mathsf{R}^{(N-1)}$, and all remaining entries are zero. The block Toeplitz structures of $\mathbf{H_0}$ and $\mathbf{H_1}$ allow us to write $\mathbf{H_0} = \sum_{k=0}^{L} \mathbf{J}^k \otimes \mathbf{H}(k)$ and $\mathbf{H_1} = \sum_{k=0}^{L} (\mathbf{J}^T)^{N-k} \otimes \mathbf{H}(k)$, respectively. Besides, we define $\mathbf{G_p} = \mathrm{diag}[p(0), p(1), \cdots, p(N-1)] \in \mathsf{R}^{N\times N}$. Hence $\mathbf{H_0}\mathbf{G}^2\mathbf{H_0^*}$ can be written as

$$\begin{aligned}
\mathbf{H_0}\mathbf{G}^2\mathbf{H_0^*} &= \sum_{k=0}^{L} \mathbf{J}^k \otimes \mathbf{H}(k) \left(\mathbf{G_p^2} \otimes \mathbf{I}_{M_t}\right) \sum_{l=0}^{L} \left(\mathbf{J}^l \otimes \mathbf{H}(l)\right)^* \\
&= \sum_{k=0}^{L}\sum_{l=0}^{L} \left(\mathbf{J}^k \otimes \mathbf{H}(k)\right)\left(\mathbf{G_p^2} \otimes \mathbf{I}_{M_t}\right)\left((\mathbf{J}^T)^l \otimes \mathbf{H}(l)^*\right) \\
&= \sum_{k=0}^{L}\sum_{l=0}^{L} \left(\mathbf{J}^k\mathbf{G_p^2}(\mathbf{J}^T)^l\right) \otimes \left(\mathbf{H}(k)\mathbf{H}(l)^*\right)
\end{aligned} \tag{3.3}$$

where we have used the identies $(\mathbf{A}\otimes\mathbf{B})^* = \mathbf{A}^*\otimes\mathbf{B}^*$ and $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC})\otimes(\mathbf{BD})$ [20, p.190]. Similarly, $\mathbf{H_1}\mathbf{G}^2\mathbf{H_1^*}$ can be written as

$$\mathbf{H_1}\mathbf{G}\mathbf{H_1^*} = \sum_{k=0}^{L}\sum_{l=0}^{L} \left((\mathbf{J}^T)^{N-k}\mathbf{G_p^2}\mathbf{J}^{N-l}\right) \otimes \left(\mathbf{H}(k)\mathbf{H}(l)^*\right) \tag{3.4}$$

The following proposition shows that the matrices $\mathbf{J}^k\mathbf{G_p^2}(\mathbf{J}^T)^l$ and $(\mathbf{J}^T)^{N-k}\mathbf{G_p^2}\mathbf{J}^{N-l}$ have special structures that allow decomposition of (3.2) into a group of decoupled equations. Roughly speaking, the jth block super-diagonal part of (3.2) involves only the unknown "channel product matrices", $\mathbf{H}(k)\mathbf{H}(k+j)^*$, $k = 0, 1, \cdots, L-j$. For example, the equations corresponding to the diagonal blocks ($j = 0$) involve only $\mathbf{H}(k)\mathbf{H}(k)^*$, $k = 0, 1, \cdots, L$. In the proposed identification algorithm, these "channel product matrices" are computed first by solving linear equations, and then the channel impulse response matrices $\mathbf{H}(k)$ are computed via eigenvalue-eigenvector decomposition.

Proposition 3.1 : Let $0 \le k, l \le L$ be two non-negative integers. Then

(a) For $l = k + j$, where $0 \leq j \leq L - k$, both $\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l$ and $(\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}$ are upper triangular matrices with only the respective jth upper diagonals nonzero, and

$$\Gamma_j\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) = [\underbrace{0 \ \cdots \ 0}_{k\ entries} \ \underbrace{p(0)^2 \ \ p(1)^2 \ \ \cdots \ \ p(N-1-k-j)^2}_{N-k-j\ entries}]^T \tag{3.5}$$

$$\Gamma_j\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right) = [\underbrace{p(N-k)^2 \ \ p(N-k+1)^2 \ \ \cdots \ \ p(N-1)^2}_{k\ entries} \ \underbrace{0 \ \cdots \ 0}_{N-k-j\ entries} \ ]^T \tag{3.6}$$

(b) For $l < k$, both $\Gamma_j\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right)$ and $\Gamma_j\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right)$ are lower triangular with zero diagonal matrices.

Proof : See [8].

It follows from (3.5) and (3.6) that

$$\begin{aligned}
&\Gamma_j\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) + \Gamma_j\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right) \\
&= \begin{cases} [\underbrace{p(N-k)^2 \ \cdots \ p(N-1)^2}_{k\ entries} \ \underbrace{p(0)^2 \ \cdots \ p(N-1-k-j)^2}_{N-k-j\ entries}]^T & \text{if} \quad j = l-k \geq 0 \\ \mathbf{0}_{(N-j)\times 1} \qquad \text{if} \quad j \neq l-k \end{cases}
\end{aligned} \tag{3.7}$$

Since

$$\Upsilon_j\left(\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^*\right) = \Gamma_j\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^* \tag{3.8}$$

and

$$\Upsilon_j\left(\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^*\right) = \Gamma_j\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right) \otimes \mathbf{H}(k)\mathbf{H}(l)^* \tag{3.9}$$

it follows from (3.2)-(3.4) and (3.7)-(3.9) that $\Upsilon_j(\mathbf{R}_{\bar{\mathbf{x}}}(0))$ can be derived as follows.

$$\begin{aligned}
\Upsilon_j(\mathbf{R}_{\bar{\mathbf{x}}}(0)) &= \Upsilon_j\left(\mathbf{H_0}\mathbf{G}^2\mathbf{H_0^*} + \mathbf{H_1}\mathbf{G}^2\mathbf{H_1^*}\right) \\
&= \sum_{k=0}^{L}\sum_{l=0}^{L}\Upsilon_j\left(\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*)\right) + \Upsilon_j\left(\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right) \otimes (\mathbf{H}(k)\mathbf{H}(l)^*)\right) \\
&= \sum_{k=0}^{L}\sum_{l=0}^{L}\{\Gamma_j\left(\mathbf{J}^k\mathbf{G}_\mathbf{p}^2(\mathbf{J}^T)^l\right) + \Gamma_j\left((\mathbf{J}^T)^{N-k}\mathbf{G}_\mathbf{p}^2\mathbf{J}^{N-l}\right)\} \otimes \mathbf{H}(k)\mathbf{H}(l)^* \\
&= \sum_{k=0}^{L-j}[p(N-k)^2 \ \ \cdots \ \ p(N-1)^2 \ \ p(0)^2 \ \ \cdots \ \ p(N-1-k-j)^2]^T \otimes \mathbf{H}(k)\mathbf{H}(k+j)^* \\
&= \sum_{k=0}^{L-j}[p(N-k)^2\mathbf{I}_{M_r} \ \ \cdots \ \ p(N-1)^2\mathbf{I}_{M_r} \ \ p(0)^2\mathbf{I}_{M_r} \ \ \cdots \ \ p(N-1-k-j)^2\mathbf{I}_{M_r}]^T\mathbf{H}(k)\mathbf{H}(k+j
\end{aligned} \tag{3.10}$$

The right hand side of (3.10) is a linear combination of block columns with the channel product matrices, $\mathbf{H}(k)\mathbf{H}(k+j)^*$, as coefficients. If we define, for $0 \leq j \leq L$,

$$\mathbf{F}_j = [(\mathbf{H}(0)\mathbf{H}(j)^*)^T \ \ (\mathbf{H}(1)\mathbf{H}(j+1)^*)^T \ \ \cdots \ \ (\mathbf{H}(L-j)\mathbf{H}(L)^*)^T]^T \in \mathbb{C}^{M_r(L-j+1)\times M_r} \tag{3.11}$$

then (3.10) can be written in a more compact form (3.12).

$$\Upsilon_j\left(\mathbf{R}_{\bar{\mathbf{x}}}(0)\right) = \mathbf{M}_j \mathbf{F}_j \qquad \forall\ 0 \le j \le L \tag{3.12}$$

where $\mathbf{M}_j \in \mathsf{R}^{M_r(N-j)\times M_r(L-j+1)}$ is defined as

$$\mathbf{M}_j = \begin{bmatrix} p(0)^2 & p(N-1)^2 & \cdots & p(N-L+j)^2 \\ p(1)^2 & p(0)^2 & \cdots & p(N-L+j+1)^2 \\ \vdots & \vdots & \vdots & \vdots \\ p(N-2-j)^2 & p(N-3-j)^2 & \cdots & p(N-L-2)^2 \\ p(N-1-j)^2 & p(N-2-j)^2 & \cdots & p(N-L-1)^2 \end{bmatrix} \otimes \mathbf{I}_{M_r} \tag{3.13}$$

We note that $\mathbf{M}_j$, $1 \le j \le L$, is obtained from $\mathbf{M}_0$ by deleting its last jM_r rows and last jM_r columns.

Since $N > L+1$, the $(L+1)$ equations in (3.12) are overdetermined and the equations are also consistent. If $\mathbf{M}_j$ is full column rank, then the solution can be obtained as

$$\mathbf{F}_j = (\mathbf{M}_j^T \mathbf{M}_j)^{-1} \mathbf{M}_j^T \Upsilon_j\left(\mathbf{R}_{\bar{\mathbf{x}}}(0)\right) \tag{3.14}$$

If $\mathbf{F}_j$, $0 \le j \le L$, are computed from (3.14), then we have the channel product matrices $\mathbf{H}(k)\mathbf{H}(l)^*$ for $0 \le k \le l \le L$. We now consider the computation required to determine the channel impulse response matrices $\mathbf{H}(0)$, $\mathbf{H}(1)$, $\cdots, \mathbf{H}(L)$ from $\mathbf{F}_j$.

Let $\mathbf{Q}$ be the Hermitian matrix defined by $\Upsilon_j(\mathbf{Q}) = \mathbf{F}_j$ for $j = 0, 1, \cdots, L$, and let the channel matrix $\mathbf{H} = [\mathbf{H}(0)^T\ \ \mathbf{H}(1)^T\ \ \cdots\ \ \mathbf{H}(L)^T]^T$. Clearly we have

$$\mathbf{Q} = \mathbf{H}\mathbf{H}^* \tag{3.15}$$

Since $\text{rank}(\mathbf{H}) = M_t$ by assumption (**A3**), $\mathbf{Q}$ has rank M_t. Since $\mathbf{Q}$ is Hermitian and positive semidefinite, $\mathbf{Q}$ has M_t positive eigenvalues, say, $\lambda_1, \cdots, \lambda_{M_t}$. We can expand $\mathbf{Q}$ as

$$\mathbf{Q} = \sum_{j=1}^{M_t} (\sqrt{\lambda_j}\mathbf{d}_j)(\sqrt{\lambda_j}\mathbf{d}_j)^* \tag{3.16}$$

where $\mathbf{d}_j$ is a unit norm eigenvector of $\mathbf{Q}$ associated with $\lambda_j > 0$. We can thus choose the channel matrix to be

$$\widehat{\mathbf{H}} = [\sqrt{\lambda_1}\mathbf{d}_1\ \ \sqrt{\lambda_2}\mathbf{d}_2\ \ \cdots\ \ \sqrt{\lambda_M}\mathbf{d}_{M_t}] \in \mathsf{C}^{M_r(L+1)\times M_t} \tag{3.17}$$

We note $\mathbf{H}$ can only be identified up to a unitary matrix ambiguity $\mathbf{U} \in \mathsf{C}^{M_t \times M_t}$, i.e., $\widehat{\mathbf{H}} = \mathbf{H}\mathbf{U}$, since $\widehat{\mathbf{H}}\widehat{\mathbf{H}}^* = \mathbf{H}\mathbf{H}^* = \mathbf{Q}$.

We note that the matrix $\mathbf{M}_j$, $j = 0, 1, \cdots, L$, is determined by the precoding sequence. By appropriately selecting the precoding sequence, we can make each $\mathbf{M}_j$ full column rank.

We summarize what we have so far:

(a) If the MIMO channel described by (2.3) satisfies (**A1**) and (**A3**) and the channel order L is known, then the channel matrix $\mathbf{H}$ can be identified up to a unitary matrix ambiguity.

(b) The proposed identification method use the covariance matrix of the received signal $\mathbf{R}_{\bar{\mathbf{x}}}(0)$ as data, and the computations involved are solving linear equations (3.12) and performing eigenvalue-eigenvector decomposition of the Hermitian matrix $\mathbf{Q}$ in (3.16).

We note that in the proposed method, the channel condition is assumption (**A3**), i.e., the channel matrix $\mathbf{H}$ is full column rank. Hence the channel needs not be irreducible or column reduced. If $M_r \geq M_t$ (more outputs), then (**A3**) is generically satisfied [18, ch.7]. If $M_t > M_r$ (more inputs), then (**A3**) is generically satisfied provided $(L+1)M_r \geq M_t$. Besides, in (**A1**), for simplicity, we have assumed that the covariance matrix of the source is the identity matrix. The method still applies if the covariance matrix $\mathbf{\Sigma}_{\mathbf{s}}^2$ is diagonal. The equation (3.15) becomes $\mathbf{Q} = \mathbf{H}\mathbf{\Sigma}_{\mathbf{s}}^2\mathbf{H}^*$ and the channel is identifiable up to a unitary matrix ambiguity.

3.2 Channel Order Overestimation

So far we have assumed that the channel order L is known. If only an upper bound $\hat{L} \geq L$ is available with $N > \hat{L}+1$, then following the same process given in Section 3.1, the corresponding $M_r(\hat{L}+1) \times M_r(\hat{L}+1)$ matrix $\mathbf{Q}$ can be similarly constructed as in (3.15). The last $(\hat{L}-L)$ block columns (i.e., $(\hat{L}-L)M_r$ columns) of $\mathbf{Q}$ are zero, so are its last $(\hat{L}-L)$ block rows. Hence again, $\mathbf{Q}$ is of rank M_t and has M_t positive eigenvalues with the associated eigenvectors all of the form $\hat{\mathbf{d}} = [\mathbf{d}^T \ 0 \ \cdots \ 0]^T \in \mathsf{C}^{M_r(\hat{L}+1)}$ where $\mathbf{d} \in \mathsf{C}^{M_r(L+1)}$. Thus, we can determine the actual channel order and impulse response matrices, up to a unitary matrix ambiguity, from the M_t eigenvectors associated with the M_t positive eigenvalues of $\mathbf{Q}$.

3.3 Identification Algorithm

We summarize the proposed method as the following algorithm.

1) Select the precoding sequence $p(n)$ such that each matrix $\mathbf{M}_j$ defined in (3.13) is full column rank.

2) Estimate the autocovariance matrix $\mathbf{R}_{\bar{\mathbf{x}}}(0)$ via the time average

$$\hat{\mathbf{R}}_{\bar{\mathbf{x}}}(0) = \frac{1}{K}\sum_{i=1}^{K} \bar{\mathbf{x}}(i)\bar{\mathbf{x}}(i)^* \tag{3.18}$$

where K is the number of data block (i.e., KN is the number of samples for each transmitter).

3) Compute $\mathbf{F}_j$, formed by the channel product matrices, for $j = 0, 1, \cdots, L$, using (3.14).

4) Form the matrix $\mathbf{Q}$ as in (3.15), and obtain the channel impulse response (3.17) by computing the M_t largest eigenvalues and the associated eigenvectors of $\mathbf{Q}$.

4 Optimal Selection of the Precoding Sequence

In Section 3, we see that in order to identify the channel, the precoding sequence must be selected so that the resulting matrix $\mathbf{M}_j$ is full column rank such that $\mathbf{F}_j$ can be exactly solved as (3.14). However, when noise is present, the covariance matrix $\hat{\mathbf{R}}_{\bar{\mathbf{x}}}(0)$ contains the contribution of noise and numerical error is present in the estimation of $\hat{\mathbf{R}}_{\bar{\mathbf{x}}}(0)$ by (3.18). This implies that (3.12) usually has no solution and (3.14) becomes a least squares approximate solution. The choice of $\mathbf{M}_j$ will affect error in the computation of $\mathbf{F}_j$ since different $\mathbf{M}_j^T\mathbf{M}_j$ in (3.14) may have different condition numbers. In this section, we discuss the optimal selection of the precoding sequence, which takes into account the effect of noise and numerical error in estimating $\hat{\mathbf{R}}_{\bar{\mathbf{x}}}(0)$, so as to increase the accuracy of $\mathbf{F}_j$ and thus reduce the channel estimation error.

4.1 Optimality Criterion

Now we consider the general case that the noise is present and discuss the design of the precoding sequence $p(n)$. From (2.4) and assumption (**A1**), the covariance matrix of the received signal is

$$\mathbf{R}_{\bar{\mathbf{x}}}(0) = \mathbf{H}_0\mathbf{G}^2\mathbf{H}_0^* + \mathbf{H}_1\mathbf{G}^2\mathbf{H}_1^* + \sigma_v^2\mathbf{I}_{M_r} \otimes \mathbf{I}_N \tag{4.1}$$

From (4.1) and (3.2), we see that noise has only contribution to the diagonal entries of $\mathbf{R}_{\bar{\mathbf{x}}}(0)$. Therefore the $(L+1)$ decoupled groups of equations in (3.12) remain unchanged, except for the $j = 0$ group, which becomes

$$\begin{aligned} \Upsilon_0\left(\mathbf{R}_{\bar{\mathbf{x}}}(0)\right) &= \Upsilon_0\left(\mathbf{H}_0\mathbf{G}^2\mathbf{H}_0^* + \mathbf{H}_1\mathbf{G}^2\mathbf{H}_1^*\right) + \sigma_v^2\Upsilon_0\left(\mathbf{I}_{M_r} \otimes \mathbf{I}_N\right) \\ &= \mathbf{M}_0\mathbf{F}_0 + \mathbf{Y} \end{aligned} \tag{4.2}$$

where $\mathbf{Y} = \sigma_v^2[\mathbf{I}_{M_r}\ \mathbf{I}_{M_r}\ \cdots\ \mathbf{I}_{M_r}]^T \in \mathsf{R}^{M_rN\times M_r}$ with unknown σ_v^2. Thus from (3.14), $\hat{\mathbf{F}}_0$, the least squares approximation of $\mathbf{F}_0$, can be written by

$$\hat{\mathbf{F}}_0 = (\mathbf{M}_0^T\mathbf{M}_0)^{-1}\mathbf{M}_0^T\underbrace{(\mathbf{M}_0\mathbf{F}_0 + \mathbf{Y})}_{\Upsilon_0(\mathbf{R}_{\bar{\mathbf{x}}}(0))} = \mathbf{F}_0 + (\mathbf{M}_0^T\mathbf{M}_0)^{-1}\mathbf{M}_0^T\mathbf{Y} = \mathbf{F}_0 + \mathbf{Z} \tag{4.3}$$

which is $\mathbf{F}_0$ plus a perturbation term due to noise. The perturbation term $\mathbf{Z}$ is the least squares solution of the equation $\mathbf{M}_0\mathbf{Z} = \mathbf{Y}$. We note that if every column of $\mathbf{Y}$ is orthogonal to every column of $\mathbf{M}_0$, then $\mathbf{Z} = \mathbf{0}$, which implies $\hat{\mathbf{F}}_0 = \mathbf{F}_0$. But that is impossible since the entries of $\mathbf{M}_0$ are positive and those of $\mathbf{Y}$ are nonnegative. Therefore, we seek to appropriately choose the precoding sequence $p(n)$ such that every column of $\mathbf{Y}$ is as close to being orthogonal to that of $\mathbf{M}_0$ as possible. To this end, we first define $\mathbf{q}_{ki}$ and $\mathbf{y}_i$ shown below as the columns of $\mathbf{M}_0$ and $\mathbf{Y}$, respectively:

$$\mathbf{M}_0 = \left[\underbrace{\mathbf{q}_{01}\ \mathbf{q}_{02}\ \cdots\ \mathbf{q}_{0M_r}}_{\mathbf{M}_0(:,1:M_r)}\ \underbrace{\mathbf{q}_{11}\ \mathbf{q}_{12}\ \cdots\ \mathbf{q}_{1M_r}}_{\mathbf{M}_0(:,M_r+1:2M_r)}\ \cdots\ \underbrace{\mathbf{q}_{L1}\ \mathbf{q}_{L2}\ \cdots\ \mathbf{q}_{LM_r}}_{\mathbf{M}_0(:,LM_r+1:(L+1)M_r)} \right] \tag{4.4}$$

$$\mathbf{Y} = \sigma_v^2[\mathbf{I}_{M_r}\ \mathbf{I}_{M_r}\ \cdots\ \mathbf{I}_{M_r}]^T = [\mathbf{y}_1\ \mathbf{y}_2\ \cdots\ \mathbf{y}_{M_r}] \tag{4.5}$$

Then, due to the special structure of the block matrix $\mathbf{M}_0$ and $\mathbf{Y}$, it is easy to check that $\mathbf{q}_{ki}$ is orthogonal to $\mathbf{y}_j$, i.e., $\mathbf{q}_{ki}^T\mathbf{y}_j = 0$ for $j \neq i$, e.g.,

$$\mathbf{q}_{01}^T\mathbf{y}_2 = [\underbrace{p(0)^2\ 0\ \cdots\ 0}_{M_r\ entries}\ \cdots\ \underbrace{p(N-1)^2\ 0\ \cdots\ 0}_{M_r\ entries}][\underbrace{0\ \sigma_v^2\ 0\ \cdots 0}_{M_r\ entries}\ \cdots\ \underbrace{0\ \sigma_v^2\ 0\ \cdots 0}_{M_r\ entries}]^T = 0$$

and each $\mathbf{q}_{ki}^T\mathbf{y}_i$ assumes the same value, $\sigma_v^2\sum_{n=0}^{N-1}p(n)^2$, for $k = 0, 1, \cdots, L$, $i = 1, 2, \cdots, M_r$, e.g.,

$$\mathbf{q}_{01}^T\mathbf{y}_1 = [\underbrace{p(0)^2\ 0\ \cdots\ 0}_{M_r\ entries}\ \cdots\ \underbrace{p(N-1)^2\ 0\ \cdots\ 0}_{M_r\ entries}][\underbrace{\sigma_v^2\ 0\ \cdots 0}_{M_r\ entries}\ \cdots\ \underbrace{\sigma_v^2\ 0\ \cdots 0}_{M_r\ entries}]^T = \sigma_v^2\sum_{n=0}^{N-1}p(n)^2$$

Thus we only need to consider the relation between columns of $\mathbf{q}_{01}$ and $\mathbf{y}_1$ (the case of $k = 0$ and $i = 1$). Define the correlation coefficient

$$\gamma = \frac{\mathbf{q}_{01}^T\mathbf{y}_1}{\|\mathbf{q}_{01}\|_2\|\mathbf{y}_1\|_2} \tag{4.6}$$

Since γ is nonnegative and by Cauchy-Schwarz inequality, $0 \leq \gamma \leq 1$. In order to make the perturbation term $\mathbf{Z}$ small, we choose $\mathbf{q}_{01}$ so that the correlation coefficient γ is as small as possible. Based on this point of view, we formulate the optimal selection problem as minimizing γ subject to

$$\frac{1}{N}\sum_{n=0}^{N-1}|p(n)|^2 = 1 \tag{4.7}$$

$$|p(n)|^2 \geq \tau > 0, \qquad \forall\, 0 \leq n \leq N-1 \tag{4.8}$$

Roughly, constraint (4.7) normalizes the power gain of the precoding sequence of each transmitter to 1; constraint (4.8) requires that at each instant, the power gain is no less than τ. Note that the problem of selecting the precoding sequence is identical to the SISO case considered in [8]. Thus the optimal precoding sequence $p(n)$ is a two-level sequence with a single peak in one period [8]. More specifically, for each m, $0 \leq m \leq N-1$,

$$p(n) = \begin{cases} \sqrt{N(1-\tau)+\tau}\,, & n = m \\ \sqrt{\tau}\,, & n \neq m,\ 0 \leq n \leq N-1 \end{cases} \tag{4.9}$$

is an optimal precoding sequence. Because the precoding sequence is periodic with period N, the single peak can be placed at any one of the N positions which yield the same γ. However the peak location m does significantly affect the numerical condition of the linear equation (3.12) as we discuss next.

4.2 On Selection of m

We now consider the selection of m. We know that different choices of m result in different matrix $\mathbf{M}_j$ and affect the numerical computation of $\mathbf{F}_j, j = 1, 2, \cdots, L$, in (3.14) and $\hat{\mathbf{F}}_0$ in (4.3), since different $\mathbf{M}_j^T\mathbf{M}_j$ may have different condition number. If the condition number is large, then the matrix $\mathbf{M}_j^T\mathbf{M}_j$ is ill-conditioned and the computation in (3.14) and (4.3) are sensitive to data error. Let

$$\mu = \max_{0 \le j \le L} \kappa(\mathbf{M}_j^T\mathbf{M}_j) \tag{4.10}$$

where $\kappa(\mathbf{A})$ is the condition number of $\mathbf{A}$. Our goal is to choose m so as to minimize the largest condition number of the corresponding matrices $\mathbf{M}_j^T\mathbf{M}_j$, $j = 0, 1, \cdots, L$. Since the peak appears at one of the N possible positions in the periodic precoding sequence, there are N precoding sequences which may result in N different μ. The following result shows that some choices of m are to be avoided since they result in some $\mathbf{M}_j$ being rank deficient and thus $\mu = \infty$.

Proposition 4.1 : At least one $\mathbf{M}_j$, $0 \le j \le L$, is not full column rank if and only if $N - L + 1 \le m \le N - 2$.

Proof : See Appendix A.

Hence if we choose, either $0 \le m \le N - L$ or $m = N - 1$, then each $\mathbf{M}_j$ is full column rank and the channel is identifiable. The following result shows that we can classify the remaining choices into 2 groups that are relevant to the optimal choice of m.

Proposition 4.2 :

(a) Each of the $(N - L)$ choices, $m = 0$, $m = 1$, $\cdots$, $m = N - L - 1$, results in the same μ denoted by μ_1.

(b) The two choices $m = N - L$ and $m = N - 1$ result in the same μ denoted by μ_2. Also $\mu_2 \ge \mu_1$.

Proof : See Appendix A.

From Proposition 4.2, we know if $\mu_2 > \mu_1$, then we choose case (a); if $\mu_2 = \mu_1$, we proceed to compare the second largest condition numbers of the set of matrices $\{\mathbf{M}_j^T\mathbf{M}_j\}_{j=0}^L$ for these two cases and choose the case whose value is smaller. If they are again equal, the same procedure can be done by comparing the third largest condition numbers and so on. Moreover, for $0 \le m \le N - L - 1$ (case (a)), since the condition numbers of $\mathbf{M}_j^T\mathbf{M}_j$ are the same for each fixed j, $j = 0, 1, \cdots, L$, (see Appendix A), we can use $m = 0$ to represent case (a). Similarly, $m = N - 1$ can be used to represent case (b). Hence the optimal selection of m reduces to one of two cases: $m = 0$ or $m = N - 1$. In other words, the optimal precoding sequence has a peak either at the beginning or at the end.

5 Simulation Results

In this section, we use several examples to demonstrate the performance of the proposed method. The channel normalized root-mean-square error (NRMSE) is defined as

$$\text{NRMSE} = \frac{1}{\|\mathbf{H}\|_F} \sqrt{\frac{1}{I} \sum_{i=1}^{I} \|\widehat{\mathbf{H}}^{(i)} \mathbf{U}^{(i)} - \mathbf{H}\|_F^2} \tag{5.1}$$

where the unitary matrix ambiguity $\mathbf{U}^{(i)}$ is computed by the least squares method [15], solving

$$\min_{\mathbf{U}^{(i)}} \|\widehat{\mathbf{H}}^{(i)} \mathbf{U}^{(i)} - \mathbf{H}\|^2 \tag{5.2}$$

I is the number of Monte Carlo runs, and $\widehat{\mathbf{H}}^{(i)} = [\widehat{\mathbf{H}}^{(i)}(0)^T \;\; \widehat{\mathbf{H}}^{(i)}(1)^T \;\; \cdots \;\; \widehat{\mathbf{H}}^{(i)}(L)^T]^T$ is the estimate of channel impulse response matrix $\mathbf{H}$. The input source symbols are i.i.d. QPSK signals. The channel noises are white Gaussian. The signal-to-noise ratio (SNR) at the output is defined as

$$\text{SNR} = \frac{\frac{1}{N} \sum_{n=0}^{N-1} E[\|\mathbf{z}(n)\|_2^2]}{E[\|\mathbf{v}(n)\|_2^2]} \tag{5.3}$$

where $\mathbf{z}(n) = [z_1(n) \;\; \cdots \;\; z_{M_r}(n)]^T$ is the signal component of the received signal (see Figure 2.1). For the simulations below, the number of Monte Carlo runs is fixed at $I = 100$.

1) Simulation 1 – optimal selection of precoding sequences

In this simulation, we use the following model

$$\mathbf{H}(z) = \underbrace{\begin{bmatrix} 0.4851 & 0.3200 \\ -0.3676 & 0.2182 \end{bmatrix}}_{\mathbf{H}(0)} + \underbrace{\begin{bmatrix} -0.4851 & 0.9387 \\ 0.8823 & 0.8729 \end{bmatrix}}_{\mathbf{H}(1)} z^{-1} + \underbrace{\begin{bmatrix} 0.7276 & -0.1280 \\ 0.2941 & -0.4364 \end{bmatrix}}_{\mathbf{H}(2)} z^{-2} \quad (5.4)$$

given in [11] to demonstrate the effect of different precoding sequences on the performance of the proposed method. For comparsion, the first sequence is chosen as {0.767 1.07 1.07 1.07}, which satisfies (4.7) and (4.8). The second and third sequences are chosen based on (4.9) for $N = 4$ and $\tau = 0.5878$ with the two possible peak positions: $m = 0$ and $m = 3$. By computation, the corresponding μ for the three cases are 40.0, 4.66 and 22.1, respectively. Thus $m = 0$ is the optimal selection. Figure 5.1 shows that for SNR=10 dB, there are about 5∼6 dB and 6∼7 dB difference in NRMSE between the optimal one and two others. Figure 5.2 shows the NRMSE versus SNR when the number of samples (for each transmitter) is fixed at 1000. For each sequence, the NRMSE decreases as SNR increases and is roughly constant for SNR $\geq$ 20 dB.

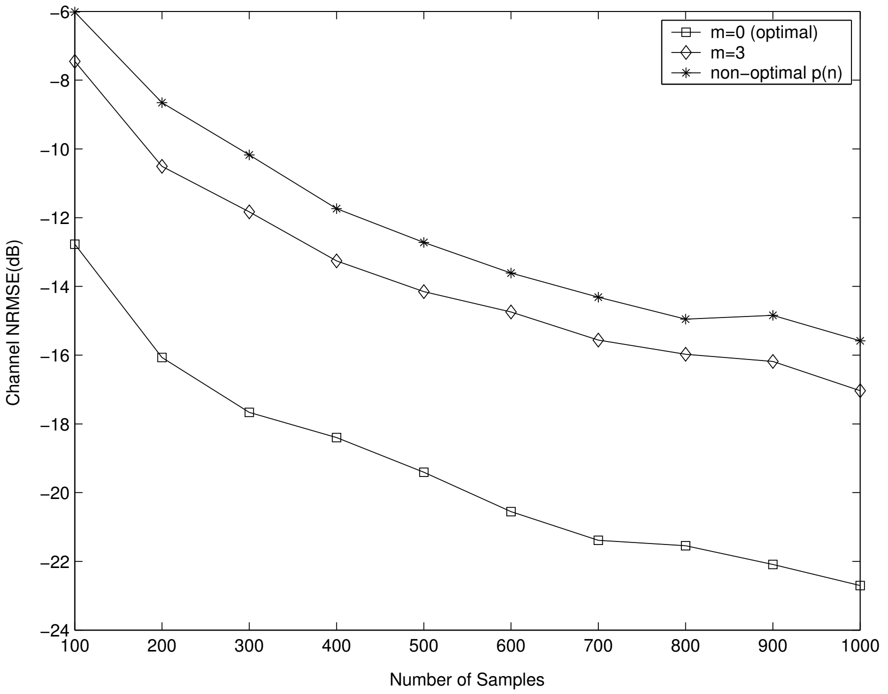

Figure 5.1: Channel NRMSE versus Number of Samples

2) Simulation 2 – robustness to channel order overestimation

In this simulation, we use the same channel model (5.4). For each upper bound $\hat{L}$, $0 \leq (\hat{L} - L) \leq 6$, we choose $N = \hat{L} + 2$, SNR=10 dB, and 1000 samples (for each transmitter)

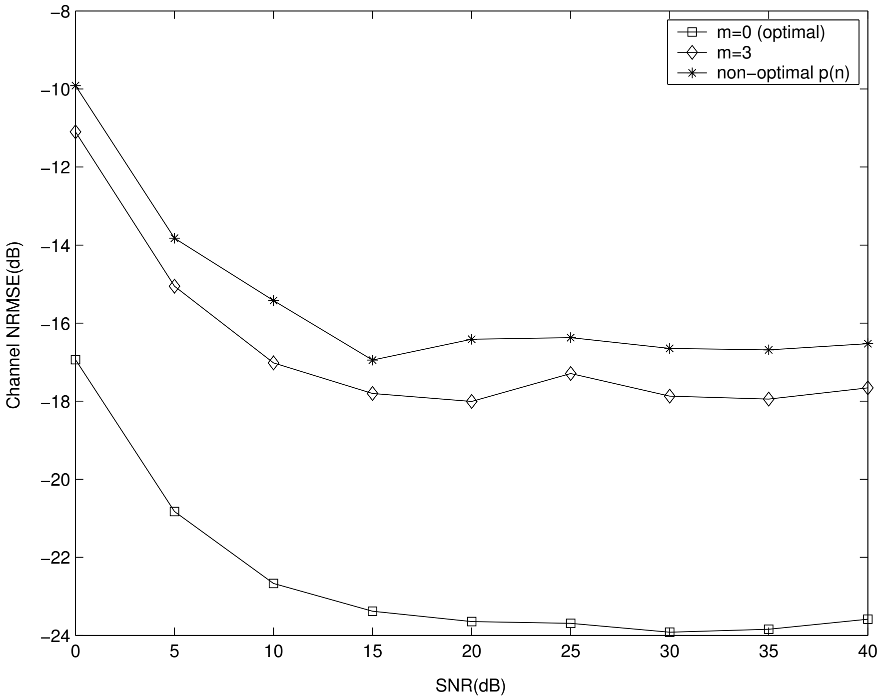

Figure 5.2: Channel NRMSE versus Output SNR

for simulation. The precoding sequences are chosen as (4.9) with $m = 0$ and $\tau = 0.5878$. Figure 5.3 shows the NRMSE increases with increasing channel order overestimation. With the channel order fixed at $L = 2$, the NRMSE increases from -22.5dB to -16dB as the $(\hat{L} - L)$ increases from 0 to 6. The proposed method is quite robust to channel order overestimation since the NRMSE still maintains a low value (about -16dB) when $(\hat{L} - L) = 6$.

3) Simulation 3 – a 3-input 2-output channel

In this simulation, we use the 3-input 2-output model

$$\mathbf{H}(z) = \underbrace{\begin{bmatrix} 1.6 & 0.88 & 0.66 \\ 0.8 & 0.44 & 0.33 \end{bmatrix}}_{\mathbf{H}(0)} + \underbrace{\begin{bmatrix} -0.44 & 0.35 & 0.14 \\ -0.14 & 0.37 & 0.23 \end{bmatrix}}_{\mathbf{H}(1)} z^{-1} + \underbrace{\begin{bmatrix} 0.13 & 0.01 & 0.08 \\ 0.26 & 0.02 & 0.16 \end{bmatrix}}_{\mathbf{H}(2)} z^{-2} \quad (5.5)$$

to illustrate the performance of the proposed method for channel with more inputs than outputs. Note that this model is not irreducible [15] because $\mathbf{H}(0)$ is not full rank, and it is not column reduced [15] either because $\mathbf{H}(2)$ is not full rank. It satisfies the condition $(L + 1)M_r \geq M_t$ given at the end in Section 3.1 and $\mathbf{H}$ is full column rank. The precoding sequences ($N = 4$) are given as in Simulation 1 for $m = 0$ and $m = 3$. Figures 5.4 shows NRMSE versus the number of data samples with SNR=10 dB. Figure 5.5 shows NRMSE versus SNR with the number of data samples (for each transmitter) fixed at 1000.

4) Simulation 4 – a 2-input 3-output channel

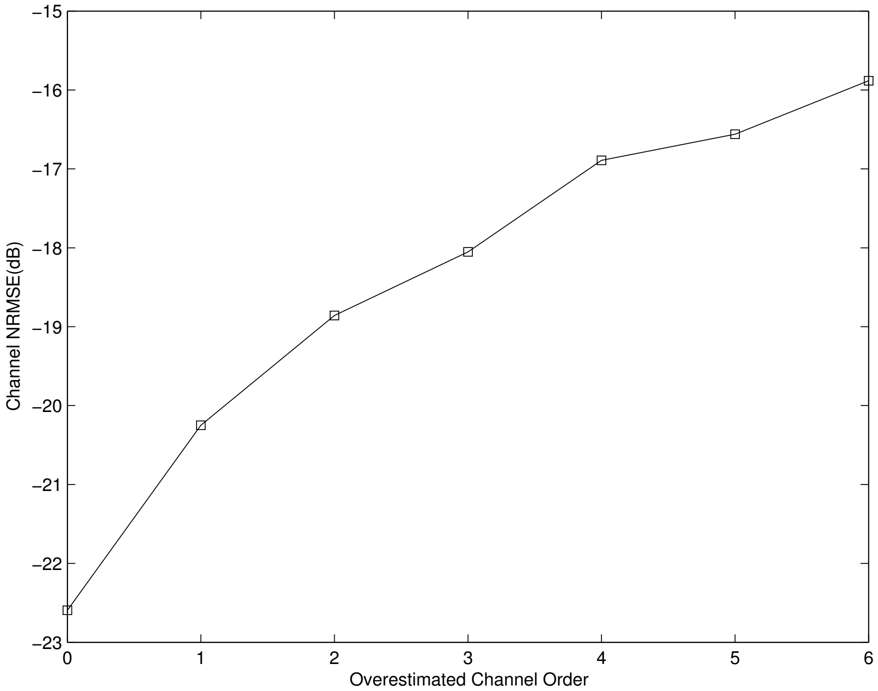

Figure 5.3: Channel NRMSE versus $(\hat{L} - L)$

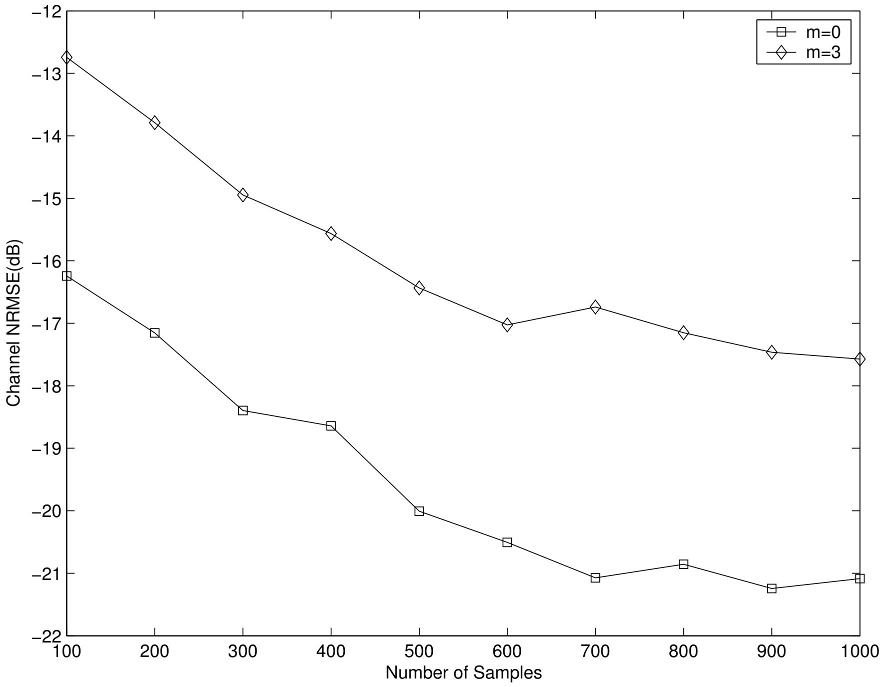

Figure 5.4: 3-input 2-output Model: Channel NRMSE versus Number of Samples

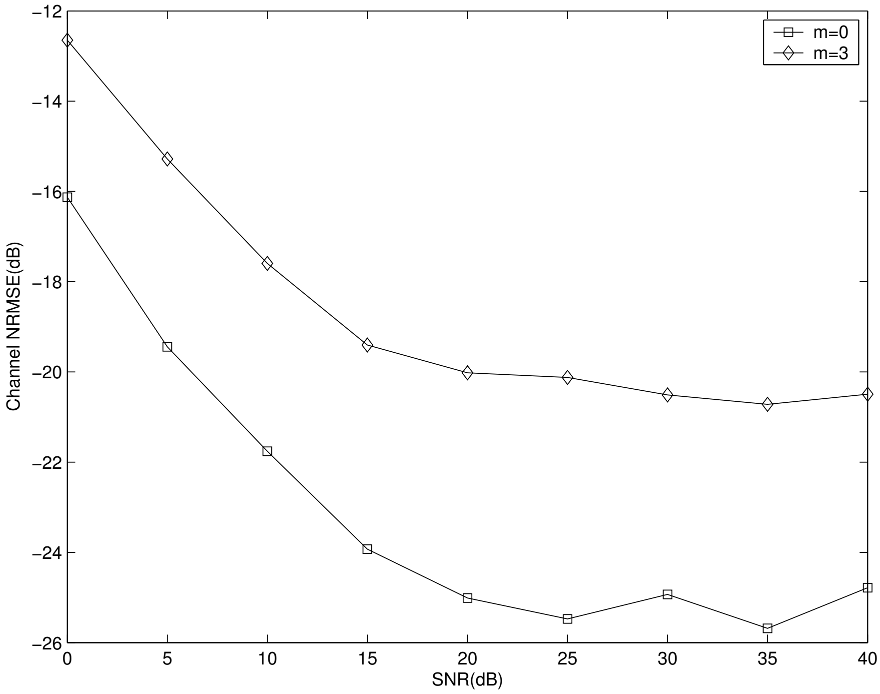

Figure 5.5: 3-input 2-output Model: Channel NRMSE versus Output SNR

In this simulation, we use the 2-input 3-output model

$$\mathbf{H}(z) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}}_{\mathbf{H}(0)} + \underbrace{\begin{bmatrix} -0.6 & -0.5 \\ 0 & 0 \\ -1.2 & -1 \end{bmatrix}}_{\mathbf{H}(1)} z^{-1} \tag{5.6}$$

given in [15]. We compare the performance of the proposed method with the outer-product-decomposition-algorithm (OPDA) method [14, 15]. The SNR is fixed at 10 dB. Note that this model is irreducible because $\mathbf{H}(0)$ is full rank, but not column reduced because $\mathbf{H}(1)$ is not full rank. The precoding sequences ($N = 4$) are given as in Simulation 1 for $m = 0$ and $m = 3$. As seen in Figure 5.6, with the optimal precoding sequence, the proposed method yields lower NRMSE than that of OPDA method, although with the selection of $m = 3$ (peak at the end), the resulting NRMSE is about 3dB higher than that of OPDA method.

5) Simulation 5 – comparison with the method in [10]

In this simulation, the performance of the proposed method is compared with that of [10], which also use periodic precoding to induce cyclostationarity at the transmitter. We use the

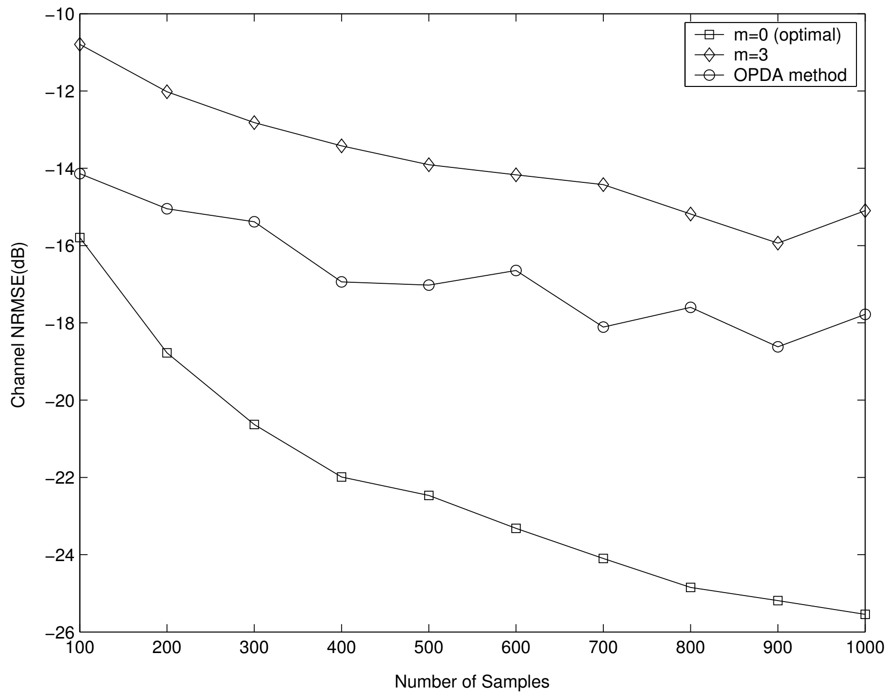

Figure 5.6: 2-input 3-output Model: Channel NRMSE versus Output SNR

channel (after normalized)

$$\mathbf{H}(z) = \underbrace{\begin{bmatrix} 0.4082 & 0.5392 \\ 0.6396 & 0.4264 \end{bmatrix}}_{\mathbf{H}(0)} + \underbrace{\begin{bmatrix} -0.4082 & 0.5392 \\ -0.4264 & -0.6396 \end{bmatrix}}_{\mathbf{H}(1)} z^{-1} + \underbrace{\begin{bmatrix} 0.8164 & 0.6470 \\ 0.6396 & 0.6396 \end{bmatrix}}_{\mathbf{H}(2)} z^{-2} \quad (5.7)$$

given in [10]. For the proposed method, we use three precoding sequences. Precoding sequences 1 and 2 are those given in Simulation 1 for $m = 0$ and $m = 3$ ($N = 4$), respectively. Precoding sequence 3 is $\{1.1\ 1.1\ 1.1\ 1\ 1\ 1\}$, which is the same given in [10]. We use 1200 i.i.d. 4-QAM symbols, 2000 Monte Carlo trials, and the same definitions of SNR and MSE given in [10, 11]. Figure 5.7 shows that the proposed method performs better than the method in [10].

6 Conclusions

We propose a method for blind identification of FIR MIMO channels using periodic precoding sequence. Since the cyclostationarity is induced at the transmitter, the identifiability condition imposed on the channel is minimum: it only requires that channel impulse response matrix $\mathbf{H} = [\mathbf{H}(0)^T\ \mathbf{H}(1)^T\ \cdots\ \mathbf{H}(L)^T]^T$ is full column rank. The channel transfer matrix is not required to be irreducible or column reduced. The channel can have more outputs or more

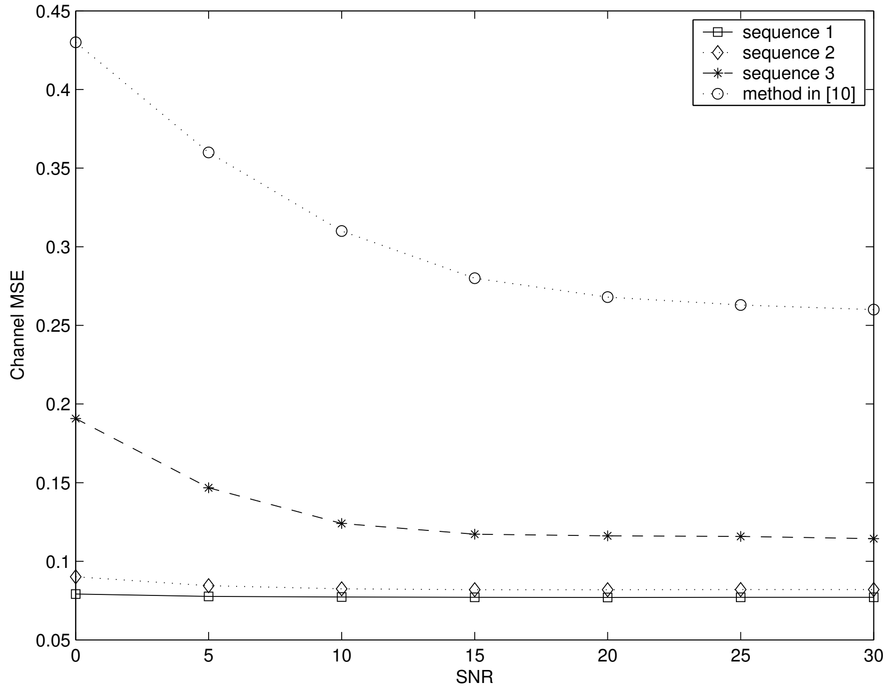

Figure 5.7: Proposed Method versus the method in [10]

inputs. The method is shown to be robust with respect to channel order overestimation. The performance of the algorithm depends on the precoding sequence which is optimally designed to reduce the effect of noise and error in estimating the covariance matrix of the received data. Simulation results show that the method yields good performance.

Appendix

A Proof of Proposition 4.1 and 4.2

Preliminary :

For each j, let $\mathbf{N}_j \in \mathsf{R}^{(N-j)\times(L-j+1)}$ be similarly defined as (3.13), except that $\mathbf{I}_{M_r}$ is replaced by 1. It can be easily check that there exists permutation matrices $\mathbf{P}_{\mathbf{l_j}} \in \mathsf{R}^{M_r(N-j)\times M_r(N-j)}$ and $\mathbf{P}_{\mathbf{r_j}} \in \mathsf{R}^{M_r(L-j+1)\times M_r(L-j+1)}$ such that $\mathbf{P}_{\mathbf{l_j}}\mathbf{M}_j\mathbf{P}_{\mathbf{r_j}} = \text{diag}[\mathbf{N}_j, \mathbf{N}_j, \cdots, \mathbf{N}_j] = \mathbf{D}_j \in \mathsf{R}^{M_r(N-j)\times M_r(L-j+1)}$ is a block diagonal matrix with each block of dimension $(N-j)\times(L-j+1)$. Since ${\mathbf{P}_{\mathbf{l_j}}}^T = {\mathbf{P}_{\mathbf{l_j}}}^{-1}$ and ${\mathbf{P}_{\mathbf{r_j}}}^T = {\mathbf{P}_{\mathbf{r_j}}}^{-1}$ [19, p.110], we have $\mathbf{M}_j = {\mathbf{P}_{\mathbf{l_j}}}^T\mathbf{D}_j{\mathbf{P}_{\mathbf{r_j}}}^T$. Hence $\mathbf{M}_j$ is full column rank if and only if $\mathbf{N}_j$ is full column rank for $j = 0, 1, \cdots, L$.

Also, $\mathbf{M}_j^T\mathbf{M}_j = (\mathbf{P}_{\mathbf{r_j}}\mathbf{D}_j^T\mathbf{P}_{\mathbf{l_j}})({\mathbf{P}_{\mathbf{l_j}}}^T\mathbf{D}_j{\mathbf{P}_{\mathbf{r_j}}}^T) = \mathbf{P}_{\mathbf{r_j}}\mathbf{D}_j^T\mathbf{D}_j{\mathbf{P}_{\mathbf{r_j}}}^T = \mathbf{P}_{\mathbf{r_j}}\text{diag}[\mathbf{N}_j^T\mathbf{N}_j, \cdots, \mathbf{N}_j^T\mathbf{N}_j]{\mathbf{P}_{\mathbf{r_j}}}^T$. Let $\lambda(\mathbf{A})$ denote the spectrum of $\mathbf{A}$ [19, p.310], that is, the set of eigenvalues of $\mathbf{A}$. Then $\lambda(\mathbf{M}_j^T\mathbf{M}_j) = \lambda(\mathbf{N}_j^T\mathbf{N}_j)$.

Proof of Proposition 4.1 :

If at $N-L+1 \leq m \leq N-2$, it can be checked that $\mathbf{N}_j$, $j = 2, 3, \cdots, L-1$ is not of full column rank since it has two columns both equal to $[\tau\, \tau \cdots \tau]^T$ which implies that at least one $\mathbf{M}_j$ is rank deficient and vice versa.

Proof of Proposition 4.2 :

From the **Preliminary**, since $\lambda(\mathbf{M}_j^T\mathbf{M}_j) = \lambda(\mathbf{N}_j^T\mathbf{N}_j)$, the condition number of $\mathbf{M}_j^T\mathbf{M}_j$ is identical to that of $\mathbf{N}_j^T\mathbf{N}_j$, i.e., $\kappa(\mathbf{M}_j^T\mathbf{M}_j) = \kappa(\mathbf{N}_j^T\mathbf{N}_j)$. Thus we need only compute the condition number of $\mathbf{N}_j^T\mathbf{N}_j$.

Case (a): For $m = 0, m = 1, \cdots$, and $m = N-L-1$, we know

$$\mathbf{N}_j^T\mathbf{N}_j = a \cdot \mathbf{I}_{L-j+1} + (2b + c_j) \cdot [1 \cdots 1]^T[1 \cdots 1] \tag{A.1}$$

where $a = N^2(1-\tau)^2$, $b = N\tau(1-\tau)$, $c_j = (N-j)\tau^2$. Hence the maximum and minimum eigenvalues are $a+(L-j+1)(2b+c_j)$ and a respectively. Thus the condition number of $\mathbf{M}_j^T\mathbf{M}_j$ is $1+[(L-j+1)(2b+c_j)/a]$ which is a decreasing function of j. Therefore the corresponding μ is equal to $\mu_1 = 1 + [(L+1)(2b+c_0)/a]$.

Case (b): For $m = N-L$ and $m = N-1$, we consider the $j = 0$ case and $j \neq 0$ case for N_j separately. For $j = 0$ with $m = N-L$ or $m = N-1$, direct multiplication of $\mathbf{N}_0^T\mathbf{N}_0$ gives the same matrix as (A.1), and the condition number of $\mathbf{M}_0^T\mathbf{M}_0$ is μ_1. For $j \neq 0$ with $m = N-L$, direct multiplication of $\mathbf{N}_j^T\mathbf{N}_j$ yields

$$\mathbf{N}_j^T\mathbf{N}_j = \begin{bmatrix} a+2b+c_j & 2b+c_j & 2b+c_j & \cdots & 2b+c_j & b+c_j \\ 2b+c_j & a+2b+c_j & 2b+c_j & \cdots & 2b+c_j & b+c_j \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 2b+c_j & 2b+c_j & 2b+c_j & \cdots & a+2b+c_j & b+c_j \\ b+c_j & b+c_j & b+c_j & \cdots & b+c_j & c_j \end{bmatrix} \in \mathsf{R}^{(L-j+1)\times(L-j+1)} \tag{A.2}$$

The eigenvalues of $\mathbf{N}_j^T\mathbf{N}_j$ in ascending order, are α_j, a, β_j, where a has a multiplicity $L-j-1$, and $\beta_j = \frac{1}{2}\{(L-j)(2b+c_j) + (a+c_j) + \sqrt{[(L-j)(2b+c_j)+(a-c_j)]^2 + 4(L-j)(b+c_j)^2}\}$, $\alpha_j = \frac{1}{2}\{(L-j)(2b+c_j) + (a+c_j) - \sqrt{[(L-j)(2b+c_j)+(a-c_j)]^2 + 4(L-j)(b+c_j)^2}\}$. All of the eigenvalues are positive and real. (A proof is given in Appendix B). It can be similarly shown that for $j \neq 0$ with $m = N-1$, $\mathbf{N}_j^T\mathbf{N}_j$ has the same eigenvalues α_j, a, β_j. Hence for $j = 1, 2, \cdots, L$, $\lambda(\mathbf{M}_j^T\mathbf{M}_j) = \{\alpha_j,\, a,\, \beta_j\}$ and the condition number is

$$\kappa(\mathbf{M}_j^T\mathbf{M}_j) = \frac{\beta_j}{\alpha_j} = 1 + \frac{\chi_j^2 - 4(N-L)b^2 + \chi_j\sqrt{\chi_j^2 - 4(N-L)b^2}}{2(N-L)b^2} \tag{A.3}$$

where $\chi_j = (L-j)(2b+c_j) + a + c_j$. Since β_j/α_j is also a decreasing function of j, then the maximum value is β_1/α_1. Therefore, combining the two cases ($j = 0, j \neq 0$), the corresponding μ is $\mu_2 = \max\{\mu_1, \beta_1/\alpha_1\} \geq \mu_1$.

B The Eigenvalues of $\mathbf{N}_j^T \mathbf{N}_j$ for $m = N - L$

Proof :

Let $\mathbf{A}_j = \mathbf{N}_j^T \mathbf{N}_j$ defined in (A.2), then $\mathbf{A}_j$ is positive definite since $\mathbf{N}_j$ is full column rank. It can be checked that the eigenvectors corresponding to $(L-j-1)$ multiple eigenvalue a are: $[1, -1, 0, 0, \cdots, 0]^T$, $[1, 1, -2, 0, \cdots, 0]^T$, $\cdots$, $[1, 1, \cdots, 1, -(L-j-1), 0]^T$. The remaining eigenvectors are $[1, 1, \cdots, 1, x]^T \in \mathsf{R}^{L-j+1}$. Hence

$$\mathbf{A}_j \begin{bmatrix} 1 \\ \vdots \\ 1 \\ x \end{bmatrix} = \begin{bmatrix} a + (L-j)(2b + c_j) + (b + c_j)x \\ \vdots \\ a + (L-j)(2b + c_j) + (b + c_j)x \\ (L-j)(b + c_j) + c_j x \end{bmatrix} = \lambda_j \begin{bmatrix} 1 \\ \vdots \\ 1 \\ x \end{bmatrix} \tag{B.1}$$

which implies the following two equations

$$a + (L-j)(2b + c_j) + (b + c_j)x = \lambda_j \tag{B.2}$$

$$(L-j)(b + c_j) + c_j x = \lambda_j x \tag{B.3}$$

Substitute (B.2) into (B.3), we can get an second order equation of x. Solving this equation can lead to two solutions of x. Bring these two x into (B.2) and we can obtain the two eigenvalues β_j, α_j. In addition, $\beta_j \geq a$ because of (B.4)

$$\begin{aligned} \beta_j &= \frac{1}{2}\{(L-j)(2b + c_j) + (a + c_j) + \sqrt{[(L-j)(2b + c_j) + a - c_j]^2 + 4(L-j)(b + c_j)^2}\} \\ &\geq \frac{1}{2}\{(L-j)(2b + c_j) + (a + c_j) + \sqrt{[(L-j)(2b + c_j) + a - c_j]^2}\} \\ &= \frac{1}{2}\{[(L-j)(2b + c_j) + (a + c_j) + [(L-j)(2b + c_j) + a - c_j]\} \\ &= a + (L-j)(2b + c_j) \\ &\geq a \end{aligned} \tag{B.4}$$

and $\alpha_j \leq a$ because of the interlacing property [19, p.396].

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