行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□期中進度報告

發展基於奇異值分解之前置狀態調節器以解決電磁相容及電磁干擾分析中近共振結構的幅射

計畫類別：■ 個別型計畫 □ 整合型計畫
計畫編號：NSC 93－2218－E－009－050－
執行期間：93 年 8月 1日至 94 年 7 月 31 日

計畫主持人：趙學永 國立交通大學電信工程系所
計畫參與人員：林旻靜、郭益廷、王義志 國立交通大學電信工程系所

成果報告類型(依經費核定清單規定繳交)：■精簡報告 □完整報告

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中 華 民 國 九 十 四 年 十 月 二 十 日

行政院國家科學委員會專題研究計畫成果報告

發展基於奇異值分解之前置狀態調節器以解決電磁相容及電磁干擾分析中近共振結構的幅射

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中文摘要

近年來基於積分方程式之解答器被廣泛用於電磁相容(EMC)及電磁干擾(EMI)分析之中。為了要降低計算複雜度，基於積分方程式之解答器所形成的矩陣方程式通常由快速演算法強化之迭代解答器求解。然而在近共振結構操作於共振頻率附近時，基於積分方程式之解答器極受限於病態的阻抗矩陣。此結構經常出現於 EMC/EMI 分析之中，譬如幅射源通常是由電路封裝或金屬盒屏蔽。如果直接以迭代解答器解病態的矩陣方程式，其解會不穩定且甚至不收斂。 此計畫利用迭代性正則化找出病態矩陣方程式最可靠之解。若以奇異值分解(SVD)分析，則該正則化可視為動態的前置狀態調節器。在迭代過程中電流輻射分量可事先算出，與近共振(近非輻射)分量相關的誤差則被抑制。藉由 L 曲線可進一步決定迭代解答器停止點，並可在迭代次數遠小於未知數數量的情況下求得正確的輻射遠場及輸入阻抗。此技術可應用於解決有關電腦系統、載具天線、及晶片連線相關之EMC/EMI 問題。此外，我們也找出了即使使用前置狀態調節器，迭代次數還是相當高的原因。

關鍵詞：積分方程式、迭代解答器、正則化、奇異值分解、近共振結構、電磁相容、電磁干擾

Abstract

Integral equation (IE) based solvers has been widely used for solving electromagnetic compatibility (EMC) and electromagnetic interference (EMI) problems in recent years. To reduce the computational complexity, the matrix equations formed by IE-based solvers are usually solved by iterative solvers enhanced by fast algorithms. However, IE based solvers are greatly limited by the ill-conditioned impedance matrices of nearly resonant structures operating around resonant frequencies. Such settings occur quite often in EMC/EMI analysis where radiation sources are shielded by packages or metal enclosures. If the ill-conditioned matrix equations are directly solved by iterative solvers, solutions will be unstable and even diverge. In this project, the iterative regularization is applied to obtain reliable solutions from

ill-conditioned matrix equations. If analyzed by the singular value decomposition (SVD), the regularization dynamically preconditions the matrix equations during each iteration, where the radiating components of the current are extracted first and the round-off errors associated with nearly resonant (almost non-radiating) modes are suppressed. Through the help of L-curves, the iterative solver can be terminated at the right point. Even if the iteration count is much less than the number of unknowns, we are still able to obtain highly accurate far fields and input impedances. The technique can be applied to solve EMC/EMI problems related to computer systems, vehicle antennas, and on-chip interconnects. Moreover, we have identified the root causes of the slow convergence of iterative solvers even if preconditioners are applied.

Keywords: integral equation, iterative solver, regularization, singular value decomposition, nearly resonant structure, electromagnetic compatibility, electromagnetic interference

I. Introduction

EMC/EMI designs often involve metal enclosures for shielding electromagnetics radiation and interferences [1-3]. However, such enclosures pose serious problems for electromagnetics solvers based on electric field integral equations (EFIEs) which are ill-conditioned at resonant frequencies. When the ill-conditioned EFIEs are solved by the frequency-domain method of moments (MoM) with iterative solvers, the resonances inside the shielding enclosures are reflected as high iteration counts. Or the iterative solvers converge very slowly due to extremely high condition numbers of the impedance matrices [3]. The iterative solvers may even diverge due to the round-off errors accumulated during iterations. On the other hand, the resonances are reflected as oscillations in the time-domain solvers and extrapolation techniques must be applied to extrapolate the long oscillatory tails [2].

Numerous techniques have been developed for obtaining stable solutions from the EFIE near interior resonances. The first category of methods requires a significant change of the EFIE or extra computational costs, such as the involvement of supplementary integral equations [4] and conditions [5], the needs to evaluate extra unknowns [5] and to solve EFIE repeatedly [6, 7]. Some of the techniques assume the structures must have specific geometries, such as closed surfaces for combined field integral equations (CFIE) [4] and small apertures for near resonance decoupling approach (NRDA) [8], and therefore are unsuitable to be adopted in general-purpose electromagnetics solvers. However, the solutions, i.e. surface currents on radiators and scatterers, obtained from the above methods are stable even right at resonant frequencies.

The second category of methods [9-13] is based on the exclusion of resonant modes from the erroneous EFIE solutions. Since the resonant or nonradiating components of surface currents are associated with the smallest singular values, these modes can be calculated by power iteration and then being excluded from the erroneous EFIE solution by orthogonalization. Only the radar cross section (RCS) and far fields calculated by the methods are exact instead of the surface currents. With extra efforts, the correct surface current can be solved for as a linear superposition of radiating and nonradiating currents [12, 13]. However, the method becomes less efficient when there are multiple resonant modes. It is also not theoretically sound, because the smallest eigenvalues may not all associate with resonant modes for arbitrary structures [14].

The methods in [9-13] can be considered as special cases of numerical regularization in [9, 15, 16], where current modes associated with small singular values are filtered out. With the help of L-curves [16], one can obtain the exact far fields from the minimal normal solutions. Since the direct regularization requires expensive SVD prior to the filtering of solutions, it is impractical for problems with a large number of unknowns. The iterative regularization builds up an uncorrupted solution during iteration and filters out undesired components on the fly. Therefore, it can be easily adopted in fast integral-equation-based solvers. In this report, we exploit the inherent regularization feature of the conjugate gradient least squares (CGLS) solver to regularized erroneous EFIE solutions. By identifying proper regularization parameters around the knees of L-curves, the CGLS solver can be terminated early as long as one can obtain highly accurate far fields and input impedances of antennas. The iterative regularization is applied to both 2D and 3D ill-conditioned EFIEs in Section II.

In addition to regularization, the high iteration counts for nearly resonant problems can be significantly reduced by applying full re-orthogonalization and double-precision arithmetic for iterative solvers. Furthermore, we also figure out the extreme long iterations reported in [3] can be cured by appropriate combinations of iterative solvers and preconditioners. The preconidtioners in [3] applied diagonal scaling by the absolute values of the diagonal entries of the impedance matrix and tends to redistribute the eigenvalues around a unit circle centered at the origin. Although the condition number of the linear system is greatly reduced because the ratio of the maximum and minimum singular values is close to one, the generalized minimal residuals (GMRES) solver with partial reorthogonalization performs the worst for such linear system [17, 18]. In the contrast, such system can be easily solved by a CG solver. Alternatively, one can perform diagonal scaling directly by diagonal elements. Such operation tends to redistribute the eigenvalues around a single point and results in fast convergence of GMRES but slow convergence of CG..

II. Results

The first example considers a uniform plane wave incident upon an infinite rectangular cylinder. The structure appears frequently in 2D interconnect problems. The cylinder is resonant when its edge length is $\lambda/\sqrt{2} = 0.70710678\lambda$. Due to a finite discretization, the structure is resonant when edge length is 0.7080204485λ and each side is discretized into 36 segments.

When the side length is 0.71λ, or when the structure is nearly resonant, the currents calculated by 2D CFIE and unregularized EFIE are slightly different (Fig. 1). By analyzing the singular values of the impedance matrix, we immediately find that the resonant mode associated with the smallest singular value causes a violation of the discrete Picard condition (Fig. 2). By identifying suitable regularization parameters from the L-curve, or terminating the CGLS solver at the 35^{th} iteration, the EFIE current can be corrected to be the same as the CFIE current (Fig. 3).

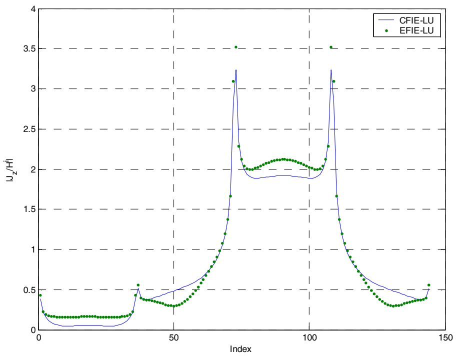

Fig. 1. Surface currents calculated by EFIE (dotted) and CFIE (solid) near the first resonance of a rectangular cylinder.

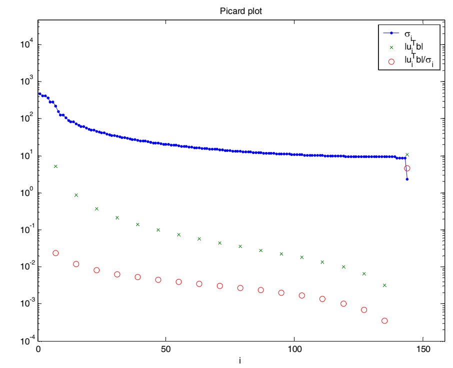

Fig. 2. Picard plot [16] for an infinite rectangular cylinder excited by a uniform plane wave.

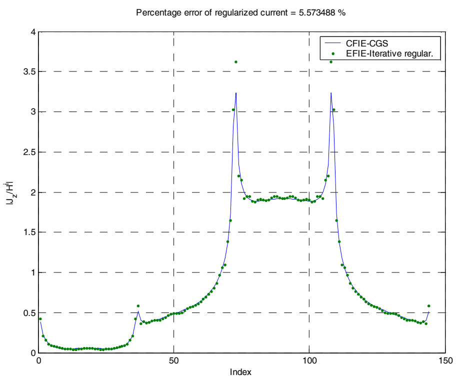

Fig. 3. Current corrected by the iterative regularization (dotted) is very close to the exact solution (solid) near the first resonance.

When the frequency is right at the interior resonance, the unregularized EFIE current is entirely different from the CFIE current (Fig. 4) and the RCS calculated from the unregularized EFIE current is also totally wrong (Fig. 5). However, the resonant components can be filtered out from the solution when the CGLS solver is terminated at the 68th iteration. Although the regularized current right at the interior resonance (Fig. 6) is not as close to the exact solution as the one near interior resonance (Fig. 3), it still enables a correct calculation of RCS (Fig. 7).

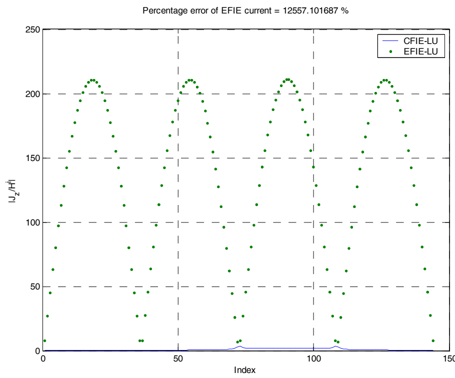

Fig. 4. Surface currents calculated by EFIE and CFIE right at the first resonance of a rectangular cylinder.

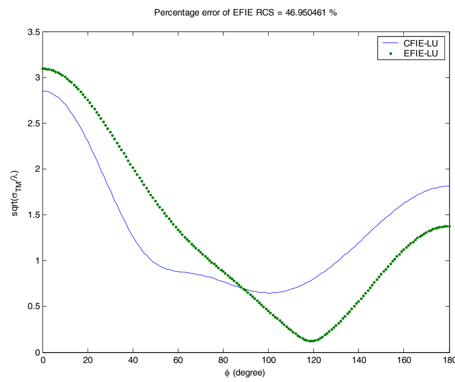

Fig. 5. The RCS calculated from the unregularized EFIE current is totally wrong mainly due to the error contributed from the smallest singular value.

The second example demonstrates the iterative regularization for the surface current and input impedance calculation of a 3D nearly resonant structure. The structure considered is a $\lambda \times \lambda \times \frac{2}{3}\lambda$ PEC chassis with an $0.8\lambda \times 0.1\lambda$

aperture on one facet of the box (Fig. 8) with a center-fed dipole antenna. The chassis and the wire antenna are

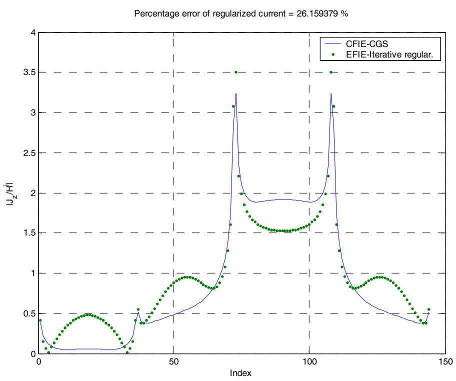

Fig. 6. Current corrected by the iterative regularization (dotted) is still different from the exact solution (solid) right at the first resonance.

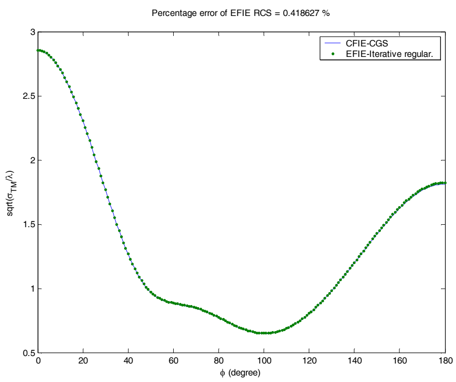

Fig. 7. RCS corrected by iterative regularization (dotted) is the same as the exact solution (solid) right at the first resonance.

discretized into 2280 unknowns. Since the 3D structure is not entirely closed, the ill-condition problem is mainly categorized as discrete ill-posed instead of rank-deficient. The singular values gradually drop to zero and there is no sudden violation of the discrete Picard condition (Fig. 9). Hence, the methods in [10-13] cannot be applied to correct such kind of ill-condition problems due to the absence of clear resonant modes related to interior resonances of closed PEC surfaces.

By identifying proper regularization parameters from the L-curve, the CGLS solver can be terminated long before the entire surface current can be calculated correctly. Although the regularized current is 97%

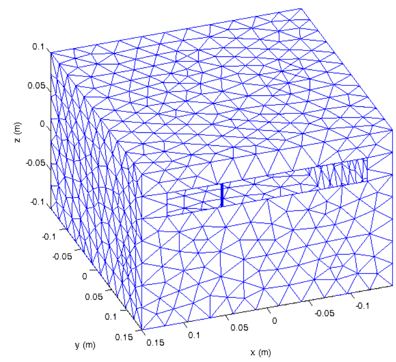

Fig. 8. The geometry and surface mesh for a center-fed dipole antenna inside a chassis with a ventilation slot.

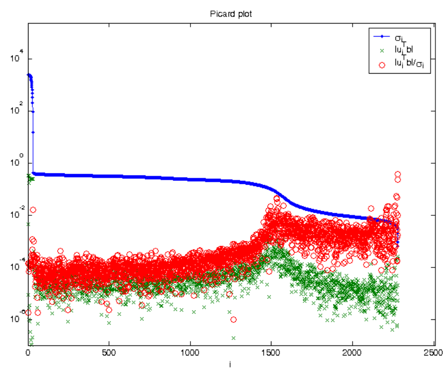

Fig. 9. Picard plot for the structure in Fig. 8 excited by a delta-gap voltage source.

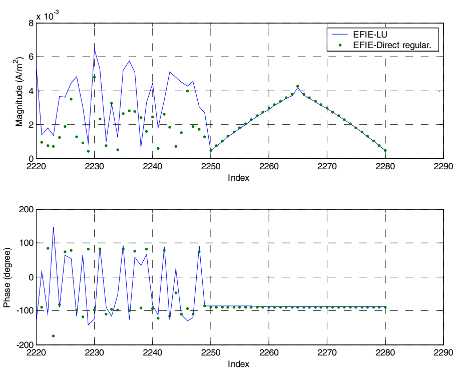

Fig. 10. Magnitudes and phases of the surface currents with and without iterative regularization for the structure in Fig. 8.

wrong when the CGLS solver is terminated at the 150^{th} iteration, the portion of current that is important for predicting the input impedance is correctly captured (Fig. 10). The regularized input impedance (Z_{in}) is $1.874 - j235.123\,\Omega$, which is only 2.81% different from the correct Z_{in}, $7.934 - j237.953\,\Omega$.

III. Conclusions

We have identified the root causes of extremely long iterations for solving nearly resonant problems based on 2D and 3D EFIEs. For structures consisting of closed PEC surfaces, the resonant modes of currents must be filtered out in order to have a minimal norm solution. The regularized current is then free of round-off errors amplified by small singular values associated with interior resonances. Although the regularized currents may be away from the exact solutions right at resonant frequencies, the minimal norm solution still leads to a correct far field.

For nearly resonant structures, the iteration counts can be significantly reduced by double-precision, fully reorthogonalized iterative solvers. The diagonal-scaling preconditioners must also be properly designed accordingly to the type of iterative solvers. With iterative regularization, the iterative solver can be terminated long before the entire current is calculated correctly. By terminating the solver once the Picard condition is violated, the most important portion of the current, or the current at the vicinity of the radiation source, can be captured accurately. Hence, the regularized current, albeit different from the exact solution, still leads to a highly accurate input impedance of the radiation source.

IV. Self-Evaluation

Under the sponsorship from NSC, we have developed two EFIE-based codes with iterative regularization for solving both 2D and 3D nearly resonant problems. However, due to the cut of budget for acquiring SolidWorks, we are unable to generate solid models. Therefore, we have developed geometry modeling and mesh generation tools to generate adaptive meshes for our in-house electromagnetics solvers [19, 20]. Results related to the project have been presented at the 2005 USNC/URSI National Radio Science Meeting [20, 21] and have received widespread interests from the audience.

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附件一

可供推廣之研發成果資料表

□ 可申請專利　■ 可技術移轉　　日期：94 年 10 月 20 日

國科會補助計畫	計畫名稱：發展基於奇異值分解之前置狀態調節器以解決電磁相容及電磁干擾分析中近共振結構的幅射 計畫主持人：趙學永 計畫編號：NSC 93－2218－E－009－050－ 學門領域：電信
技術/創作名稱	利用迭代性正則化修正電場積分方程式在近共振時的電流
發明人/創作人	趙學永
技術說明	中文：電場積分方程式在近共振時的電流含有不正確的共振分量。利用迭代性正則化修正及適當的參數選取可得到正確的電磁遠場甚至表面電流。
	英文：The current obtained from EFIE near resonant frequencies consists of undesirable resonant modes. Through iterative regularization and proper selection of parameters, one can obtain correct far fields and surface currents.
可利用之產業 及 可開發之產品	國防工業、電腦輔助軟體設計、半導體及電子業。 電磁及高頻電子相關電腦輔助設計軟體。
技術特點	易於加於現有基於電場積分方程式的電磁模擬程式中。
推廣及運用的價值	計畫中發展的數值技術可節省電磁及高頻電子模擬軟體開發及測試時間。

※　1. 每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送貴單位研發成果推廣單位（如技術移轉中心）。

※　2. 本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。

※　3. 本表若不敷使用，請自行影印使用。

附件二

出席國際會議心得報告

The training course on Monday covered several domain decomposition methods for solving large-scale eletromagnetics (EM) scattering problems by integral-equation and finite-difference methods. There had been always questions about how the boundary conditions were maintained at the interfaces. Prof. Mittra from University of Pennsylvania and his colleagues proposed several clever methods to tackle the problems, but it seemed all their methods were restricted to specific applications and could not be incorporated in general EM solvers. Only the SVD-based preconditioner and multi-resolution basis functions presented in the course were promising for enhancing general integral-equation-based EM solvers.

The Tuesday session on frequency-domain fast algorithms included several presentations from University of Illinois at Urbana-Champaign (UIUC). UIUC still maintained the leading position that fast algorithms originated from the school had been extended for multi-region and layered media. Especially the EM solvers developed by UIUC research groups had been incorporated with circuit solvers that both EM and circuit problems could be solved in one shot. I had also discussed a joint project with my former advisor, Prof. Weng Cho Chew from UIUC, about co-developing a low-frequency integral equation formulation for layered media.

The Thursday session on time-domain fast algorithms covered most recent development in UIUC and Ansoft Corporation. Development of time-domain integral-equation (TDIE) solvers had been an interest in recent years mainly for EMC/EMI applications. Two major problems of TDIE solvers, the late time instability and EM-circuit co-simulation, had been solved cleverly by temporal basis functions and model order reduction techniques.

I had presented two talks on Friday about iterative regularization and adaptive mesh generation for integral-equation-based EM solvers. The talks were well accepted and the audience also asked me many questions about the implementation issues. In addition, I also had meetings with former colleagues and classmates at UIUC about most recent trends in computational electromagnetics. Those experiences were the most rewarding for me to attend the conference.

Iterative Regularization Methods for Analyzing Radiation from Nearly Resonant Structures

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The method of moments (MOM) based on the electric field integral equation has been widely applied to the analysis of radiation and scattering from metallic objects. However, the impedance matrices formed in the MOM are ill-conditioned when the objects consist of nearly resonant structures, such as waveguides or cavities with small apertures. Even if the condition of the system is reduced due to the finite discretization of the simulation domain, the system is still ill-conditioned enough that one cannot obtain reliable solutions via Gaussian elimination. For occasions that the ill-conditioned system can only be solved by iterative solvers, such as when the MOM is accelerated by the fast multipole method (FMM) or the adaptive integral method (AIM), the iteration process may diverge or the exceedingly large number of iterations renders the fast algorithms inefficient in terms of simulation time.

In this work, iterative regularization methods based on the singular value decomposition (SVD) are applied to solve the above discrete ill-posed problems. The iteration scheme initially picks up the slowly varying components corresponding to large singular values, where the iteration number is considered as a regularization parameter. Then, the process terminates when the solution has more rapidly varying components coming from the undesired least squares solution. In other words, the scheme intends to filter out the components of less physical significance but have the potential to be the amplified high-frequency oscillations associated with discretization errors. Compared with other approaches for stabilizing solutions of nearly resonant problems, such as the combined field integral equation and the complexification of wave numbers, the iterative regularization methods can be applied to both open and closed structures and do not need re-calculation of impedance matrices. The presentation will include simulations showing the advantages of the regularized iterative solvers for analyzing radiation from wire antennas inside nearly closed cavities and waveguides.

Adaptive Mesh Refinement for Integral-Equation-Based Electromagnetics Solvers

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When the method of moments (MOM) is applied to solve the electric field integral equation (EFIE) for electromagnetic radiation and scattering problems, the accuracy of solutions greatly depends on a proper discretization of the simulation domain. The quality of the discretization is even more critical when radiation sources are close to conducting objects or when fields are strongly coupled between nearby objects. For both scenarios, a nicely graded mesh is required to capture rapid current variations without incurring excessive computation, where the regions with stronger current distribution are discretized by smaller elements and vice versa. Although it is possible to manually tune meshes generated by commonly available mechanics CAD tools, such case-by-case and labor-intensive process does not always guarantee high-quality meshes for electromagnetics simulation. Instead, the mesh should be automatically generated and refined based on the solution of the current distribution.

We will present a mesh refinement algorithm that adapts meshes to EFIE solutions by splitting triangular elements (h-refinement) and relocating nodes (r-refinement). Using a divide-and-conquer Delaunay triangulation, an initial mesh is generated with equally spaced seeds on the surfaces of conducting objects. Then, the mesh is iteratively refined according the slopes and the convergence rates of surface currents. The refinement process automatically terminates when preset criteria, such as the smallest edge length and the maximum pass of refinement, are met. In order to expedite the iterative refinement process, only self and nearby terms of the MOM impedance matrix are calculated. Moreover, the MOM matrix equation is solved by an iterative solver fed with interpolated solutions while switching from a coarse grid to a fine one. The presentation will also demonstrate the effectiveness of the adaptive mesh refinement algorithm for capturing singular current distribution on metal wedges and at the vicinity of radiation sources.