行政院國家科學委員會專題研究計畫　成果報告

擴展式卡門濾波於檢定含水層參數的特性探討與改進

計畫類別：個別型計畫
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中 文 摘 要

本研究計畫目的是利用擴展式卡門濾波，透過遞迴的方式重複使用抽水試驗的洩降數據，以檢定含水層參數，此方法可應用於線上及時系統，並避免因為洩降數據不足導致無法得到收斂的結果。此外，有些含水層參數（例如比出水）對於洩降的影響僅在某個時期，若未能在這段時間內檢定出參數值，則其他時間的洩降資料將無法反應這些參數的特性。因此，本研究進一步分析含水層參數對洩降的影響時間與程度的敏感度，並配合參數即時檢定模式推求水層參數，討論抽水試驗時間對參數檢定結果的影響。目前計完成一篇期刊論文和兩篇研討會論文。在期刊論文中，利用遞迴的方式重複使用洩降數據，並推求滲漏水層的參數，發表於 2005 年 2 月的 Journal of hydrology 期刊。一篇研討會論文分析模式與數據對 EKF 檢定參數的影響，發表在 2004 年 8 月的 Western Pacific Geophysics Meeting，另一篇研討會論文則探討抽水時間對於含水層參數檢定的影響，發表於九十三年度農業工程研討會。
關鍵詞：抽水試驗，擴展式卡門濾波，參數檢定，敏感度分析

Abstract

The purpose of this study is to identify the hydraulic parameters of the aquifers using the extended Kalman filter (EKF). By re-iterating measurement data, this approach can overcome the problem of on-line analyzing the real-time measurement data. Besides, the aquifer parameters may not be identified accurately because some aquifer parameters (e.g., specific yield) only influence the drawdown at certain period after the start of pumping. Therefore, this study provides the sensitivity analysis of aquifer parameters to the drawdown for exploring the detail of the parameter properties. Concurrently, the real-time model is applied to identify the aquifer parameter for clarifying the influence of the pumping time to the results of parameter identification. Some results of this study have been published at Journal of Hydrology, February, 2005, 2004 Western Pacific Geophysics Meeting, Hawaii, and the Agriculture Engineering Conference, 2004, Taiwan.
Keywords: pumping test, extended Kalman filter, parameter identification, sensitivity analysis

報 告 內 容

1. Introduction

The conventional EKF can quickly identify the parameters, using only part of observed drawdown data, and the obtained parameters are shown to have good accuracy. In the field pumping test, a long pumping time may not be necessary if the proposed method is implemented on a computer which is connected to pressure transducers and a data logger. Besides, differing from other conventional methods, the EKF can reflect physical nature of the aquifer system in the identified procedures. This property indicates that the EKF may identify specific parameter when that parameter starts affecting the drawdown. However, this procedure may introduce new errors in the algorithm and lead to the divergence during the identified procedures. Accordingly, this study concentrates on two purposes: (1) to recursively use the field measurement data during the stepwise identification process for eliminating the extra errors; (2) to analyze the properties of the EKF in parameter estimation processes.

2. Methodology

2.1 The Extended Kalman filter (EKF)

A nonlinear dynamical state vector is described as (*Grewal and Andrews*, 1993)

$$x_k = f(x_{k-1}, k-1) + w_k, \quad w_k \sim (0, Q_k) \tag{2.1}$$

where x_k is state vector of system, $f(x_{k-1}, k-1)$ is the function for the state vector, and w_k is the state noise assumed to be normally distributed with zero mean white (uncorrelated) sequence with known covariance structure Q_k.

The nonlinear implementation equations for the state vector can be described as

$$\hat{x}_k(-) = f(\hat{x}_{k-1}(+), k-1) \tag{2.2}$$

where $\hat{x}_k(-)$ denotes the prior (or *a priori*) estimate at *k* step and $\hat{x}_k(+)$ represents the posterior (or *a posteriori*) estimate at *k*-1 step.

A measurement model of system can be described as (*Grewal and Andrews*, 1993)

$$z_k = h(x_k, k) + \upsilon_k, \quad \upsilon_k \sim (0, R_k) \tag{2.3}$$

where z_k is the measurement vector, $h(x_k, k)$ is the function for the measurement system, and υ_k is the measurement noise assumed to be a white sequence with known covariance structure R_k.

The nonlinear implementation equation for the measurement may be expressed as

$$\hat{z}_k = h(\hat{x}_k(-), k) \tag{2.4}$$

The recursive processes of the EKF can be described as

$$P_k(-) = E\left[e_k(-)e_k^T(-)\right] = E\left[(x_k - \hat{x}_k(-))(x_k - \hat{x}_k(-))^T\right] \tag{2.5}$$

$$P_k(-) = \Phi_{k-1}P_{k-1}(+)\Phi_{k-1}^T + Q_{k-1} \tag{2.6}$$

$$\bar{K}_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \tag{2.7}$$

$$P_k(+) = \{I - \overline{K}_k H_k\}P_k(-) \tag{2.8}$$

$$\hat{x}_k(+) = \hat{x}_k(-) + \bar{K}_k(z_k - \hat{z}_k) \tag{2.9}$$

where $P_k(-)$ is *a priori* error covariance matrix, $e_k(-)$ is defined as $x_k - \hat{x}_k(-)$, Φ_{k-1} is state transition matrix, $\overline{K}_k$ is defined as the Kalman gain, $P_k(+)$ is *a posteriori* covariance, H_k is measurement matrix, and $\hat{x}_k(+)$ is the updated estimate at step *k*.

The state transition matrix Φ_{k-1} and measurement matrix H_k can, respectively, expressed as

$$\Phi_{k-1} \approx \left.\frac{\partial f(x,k-1)}{\partial x}\right|_{x=\hat{x}_{k-1}(-)} \tag{2.10}$$

and

$$H_k \approx \left.\frac{\partial h(x,k)}{\partial x}\right|_{x=\hat{x}_k(-)} \tag{2.11}$$

With Eqs. (2.2), (2.4)-(2.7), and (2.13), the recursive process of EKF is then established.

2.2 Sensitivity Analysis

Sensitivity analysis has been widely used because the most engineering, physical, chemical, and biological systems can be viewed as input-output models that relate the output information to the appropriate input parameters. The form of the traditional sensitivity (Kabala, 2001) analysis is shown as

$$\frac{\partial O}{\partial P_i} \tag{2.12}$$

where *O* is the output of the system (i.e., the aquifer drawdown in this study) and P_i is the ith input parameter of the system. However, the traditional sensitivity analysis should not be used to compare the influence of one parameter to the influence of another one when the two have different dimensions. This study utilizes normalized sensitivity (Kabala, 2001) which can be used to compare the influence of one parameter with the influence of another one when the two have different dimensions because, as is apparent from its definition, it measures the influence that the fractional change in the parameter, or its relative error, exerts on the output. The form of normalized sensitivity is

$$P_i \frac{\partial O}{\partial P_i} = \frac{\partial O}{\partial P_i / P_i} \quad (2.13)$$

$$O = O(P_i, t) \quad (2.14)$$

where O is the aquifer drawdown, P_i is the ith input parameter, and t is the pumping time.

3. Results and discussions

3.1 Parameter identification while analyzing the pumping data being recursively used

Three time-drawdown data sets measured from observation wells, as reported in Cooper (1963) and cited by Lohman (1972, p.31, Table 11), are selected for data analyses. The distances between the pumping well and the observation well 1, 2, and 3 are respectively 30.48 *m*, 152.4 *m*, and 304.8 *m*. The pumping rate Q is 5450.98 m^3/day, the thickness of the aquitard is 30.48 *m* and total pumping time is 1000 minutes (16.67 hours). Table 1 illustrates the results of estimated parameters and prediction errors while measurement data is recursively used in the EKF identification process using the three-parameter and four-parameter models, respectively. The results indicate that the EKF method gives slightly more accurate results, even though many more time steps are needed, due to the measurement data that is recursively used.

Table 1
Initial guess values for EKF and the estimated parameters and prediction errors when using EKF to analyze Cooper's data (Cooper, 1963) for leaky aquifer without considering the effect of aquitard storage

three-parameter model							
	Initial guesses for hydraulic parameters			Initial error covariance matrix for hydraulic parameters			
Case No.	*T*	*S*	*L*	*T*	*S*	*L*	
1	1000	2.00E-04	1.00E-02	55000	1.00E-09	1.00E-03	
2	1000	2.00E-04	1.00E-01	55000	1.00E-09	2.00E-02	
3	1000	2.00E-04	1.00E-01	55000	1.00E-09	1.00E-02	
	Estimated parameters			Prediction errors			
Case No.	*T*	*S*	*L*	ME	RMSE	SEE	Step
EKF on interpolated data							
1	1257.9	9.09E-05	4.82E-02	-6.53E-04	1.46E-02	1.69E-02	236
2	1311.4	9.29E-05	2.28E-01	3.72E-03	7.46E-03	8.62E-03	1508
3	1228.0	1.00E-04	5.08E-01	-2.44E-04	3.42E-03	4.09E-03	7603
EKF on recursively used data							
1	1239.4	9.78E-05	4.94E-02	1.56E-04	1.15E-02	1.33E-02	14656

Table 2 lists the estimated parameters and prediction errors in different number of observations when used EKF to identify the drawdown data from the well one. The standard error estimated (SEE) decreases when the number of observations increase and the magnitude of SEE ranges from 4.3×10^{-1} to 1.33×10^{-3}. Note that the prediction errors are calculated based on all of the observed drawdown data to demonstrate whether those two methods can provide reasonable results when using only part of the drawdown data. The relative errors of T are compared with the estimated parameters when analyzing 12 observed drawdown data. The relative error of T when using EKF ranges from 14.36% to 0.19%. The relative error of T is less than 1.5% if the number of the observed drawdown data is more than 9, i.e., 100 min. Both the parameter S and L are slightly overestimated when utilizing part of the observed drawdown data. Thus, the estimated drawdown is smaller than the observed one in the last two observed data. The overestimate of these two parameters may attribute to aquifer heterogeneity. However, these errors are quite small and negligible.

Table 2 The estimated results and related errors when using EKF to analyzing field data

Number of Observed Drawdown	Last Observed Time (min)	Estimated values			Errors		Relative errors of T
		T	$S\times10^{-4}$	$L\times10^{-2}$	*ME*	*SEE*	
4	2	1061.40	1.12	15.7	-2.82E-01	4.32E-01	14.36%
5	5	1182.24	1.05	6.78	-5.01E-02	8.86E-02	4.61%
6	10	1183.07	1.05	6.75	-4.93E-02	8.73E-02	4.54%
7	20	1203.61	1.03	5.86	-2.08E-02	4.12E-02	2.89%
8	50	1216.34	1.01	5.51	-1.22E-02	2.79E-02	1.86%
9	100	1222.51	1.00	5.32	-7.43E-03	2.00E-02	1.36%
10	200	1232.49	0.99	5.09	-2.79E-03	1.45E-02	0.56%
11	500	1237.05	0.98	4.99	-8.24E-04	1.35E-02	0.19%
12	1000	1239.39	0.98	4.94	1.56E-04	1.33E-02	0.00%

3.2 Sensitivity analysis of aquifer parameters

The drawdown due to a pumping in an unconfined aquifer with assumed parameter values is estimated using Neuman's model (1974). The thickness of the aquifer, *b*, is 10 m, pumping rate *Q* is 3000 m^3/day, and the distance between the pumping well and observation well *R* is 10 m. The radial hydraulic conductivity *Kr*, vertical hydraulic conductivity *Kz*, storage coefficient *S*, and specific yield *Sy* are set to 1×10^{-3} m/sec, 1×10^{-4} m/sec, 1×10^{-4}, and 1×10^{-1}, respectively.

The synthetic drawdowns and the results of the sensitivity analysis are plotted in Fig. 1. This figure clearly indicates that all aquifer parameters have their own influence period. The influence period of parameter *S* ranges from 1 to 10 seconds, *Kz* is in the range of 1 to 1000 seconds, and *Sy* appears from 80 seconds to the end of pumping. The parameter *Kr* is most sensitive to the drawdown except the early period of the pumping and continuously increasing through the end of the pumping. Physically, the drawdown in an unconfined aquifer can be divided into three segments (Charbeneau, 2000). In the early stage, water is instantaneously released from storage by the compaction of the aquifer and the expansion of the water. The volume of water removed from aquifer per unit surface area per unit change in hydraulic head is defined as the coefficient of storage *S*. In the second stage, the vertical gradient near the water table causes drainage of the porous matrix. The vertical hydraulic conductivity *Kz* starts to contribute to the pumping and the rate of decline in the hydraulic head slows or stops after a period of time. Finally, when the flow is essentially horizontal and most of the pumping is supplied by the specific yield, *Sy*. The sensitivity analysis displays similar behaviors as those physical phenomena. The sensitivity coefficient of *S* begins with highest value and drops quickly after the start of the pumping. The sensitivity coefficient of *Kz* reaches its highest value in the stage between 10 and 1000 seconds, implying that the slow decline of the water table is attributed to the contribution of the *Kz* at the moderate pumping time. The increasing of the drawdown in the observation well stops when the magnitude of *Kz*'s contribution approaches its maximum. The sensitivity analysis shows that the aquifer parameter *Sy* does not contribute to the pumping at the beginning of the test and starts to react at about 80 seconds. Therefore, the parameter estimation model may not obtain accurate results for *Sy* if the time-drawdown data is too short to cover the period of *Sy* reaction.

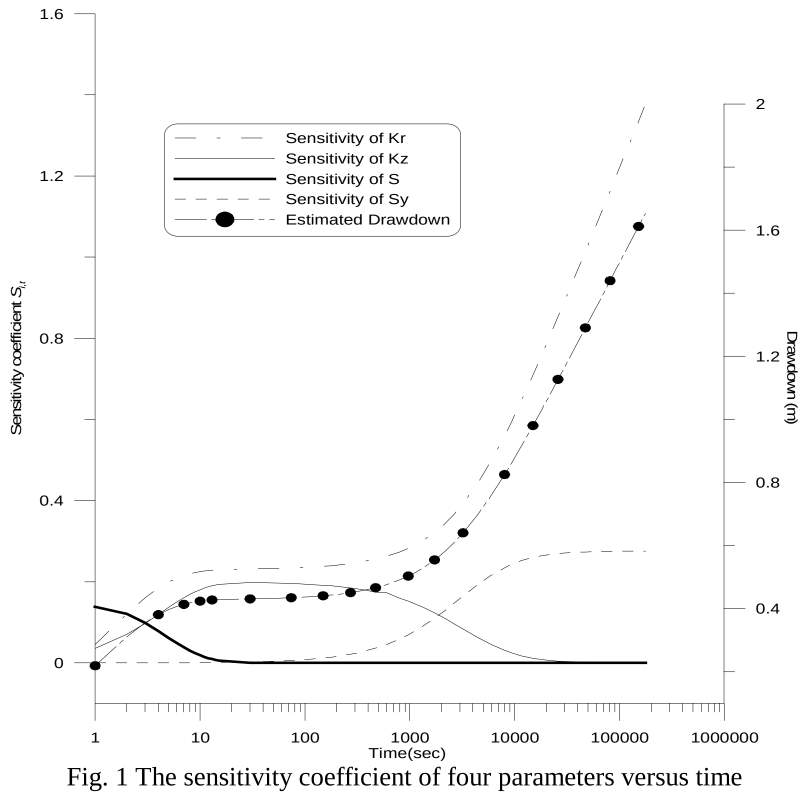

Fig. 1 The sensitivity coefficient of four parameters versus time

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計畫成果自評

The results shown above demonstrate that two purposes of this study are successfully accomplished. By re-iterating measurement data, the EKF method takes significantly less number of observations in the identification process to get convergent results. From the sensitivity analysis, the parameter *Sy* has been shown to have a time lag in response to a pumping. This phenomenon indicates that the parameter estimation model for analyzing unconfined aquifer data on-line may be inaccurate if the pumping time is too short. In addition, the estimated result of *Sy* approach to a constant and keep stably when the *Sy* begins contributing to the pumping. Then this is an indication for an on-line parameter estimation model to terminate the test. Some results of this study have been published at journal and conference. In sum, the results of this study are valuable in both engineering applications and researches.

附錄一

Yeh, H.D. and Y.C. Huang, 2005, Parameter estimation for leaky aquifers using the extended Kalman filter, and considering model and data measurement uncertainties, Journal of Hydrology, 302(1-4), 28-45.

Parameter Estimation for Leaky Aquifers Using the Extended Kalman Filter, and Considering Model and Data Measurement Uncertainties

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Abstract

A method using the extended Kalman filter (EKF) is proposed to identify the hydraulic parameters in leaky aquifer systems both with and without considering the aquitard storage. In the case without considering the aquitard storage, Hantush and Jacob’s model combined with EKF can optimally determine the parameters for the leaky aquifer when analyzing the drawdown data. Coupled with Neuman and Witherspoon’s model, the EKF is also employed to estimate the four parameters of aquifers. The observed drawdown data may be either interpolated using the

Lagrangian polynomial or recursively used while implementing the EKF. The proposed method can identify the parameters, using part of the interpolated drawdown data or recursively used data, and obtains results with good accuracy. In the field pumping test, a long pumping time may not be necessary if the proposed method is implemented on a computer which is connected to pressure transducers and a data logger. In the process of parameter estimation, the leakage coefficient changes marginally for the first few observations. This phenomenon reflects the fact that there is a time lag between the start of pumping and the leakage effect on the drawdown. The analyses of the data uncertainty demonstrate that the EKF approach is applicable for drawdown data even when it contains white noise or temporal correlated noise. Finally, the choice between Hantush and Jacob's model and Neuman and Witherspoon's model depends on the hydrogeological condition of the aquifer system indicated in the analyses of the model uncertainty. Hantush and Jacob's model is shown to be a good choice for representing the leaky aquifer system if the aquitard storage is comparatively small.

Key words: Parameter estimation, Kalman filter, Lagrangian polynomial, groundwater, leaky aquifer, model uncertainty.

附錄二

Huang, Y.C. and H.D. Yeh, 2004, Uncertainty and Sensitivity Analyses in Identifying Leaky Aquifer Parameters using Extended Kalman Filter, Western Pacific Geophysics Meeting, AGU, Hawaii, WP60.

Uncertainty and Sensitivity Analyses in Identifying Leaky Aquifer Parameters using Extended Kalman Filter

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Abstract

Hydrogeologic parameters are very important in site characterization, so groundwater hydrologists often conduct pumping tests to determine hydrogeologic parameters, such as hydraulic conductivity and storage coefficient. These parameters are necessary information for quantitative and/or qualitative groundwater studies. Hantush and Jacob (1955) described non-steady radial flow to a well in a fully penetrated leaky aquifer under a constant pumping rate. In their model, the aquitard is overlain by an unconfined aquifer, and the main aquifer is underlain by an impermeable bed. Their solution is herein called the three-parameter model. Neuman and Witherspoon (1969) gave a solution describing the drawdown of the lower and pumped aquifer in a hydrogeologic system which is composed of two confined aquifers and one aquitard. Their solution, which considers the effect of aquitard storage and neglects the drawdown in the unpumped aquifer, is called the four-parameter model. In this study, the uncertainties of the measurement data are represented by white noise and temporally correlated noise, and sensitivity analyses for the extended Kalman Filter (EKF) method are performed for data with those two types of noise. The MATLAB function randn is first chosen to generate a realization of white noise (The MatlabWorks, 1995). The elements in this realization are normally distributed random numbers with zero mean and unit variance. The original realization of white noise is employed to generate temporally correlated noise. The MATLAB function Hamming(n) with n = 5 is used to produce five coefficients of a Hamming window (The MatlabWorks, 1995). The MATLAB function conv(A, B) is applied to convolve vectors A and B (The MatlabWorks, 1995), where vector A represents the original realization and vector B represents the coefficients of the Hamming window. Algebraically,

convolution is an operation which multiplies two polynomials with coefficients containing the elements of A and B. The estimated parameters have no significant difference of all cases and the standard error estimate (SEE) values are on the same order of magnitude. Therefore, the effect of data with either white noise or temporal noise is negligible in the identification procedure. The third type of uncertainty listed in Eisenberg et al. (1989) is the conceptual model uncertainty regarding to the geometrical configuration, major features, and boundary conditions. Generally speaking, field hydrogeologic information is never known in sufficient detail. Also, the development of a mathematical model usually depends on some assumptions and/or simplifications. Thus, the selection of a model for describing a target aquifer system is always subject to some degree of uncertainty. The model uncertainty in the parameter estimation is assessed for the case of employing both three-parameter and four-parameter models for analyzing three data sets. The first two data sets are taken from Cooper (1963) and Sridharan (1987), and the third data set is taken from Batu (1998, p. 265). The choice between Hantush and Jacob's model and Neuman and Witherspoon's model for representing the leaky aquifer system depends on the hydrogeological condition of the system indicated in the analyses of the model uncertainty. However, Hantush and Jacob's model is suggested for use if the ratio of the aquitard storage to the aquifer storage is less than 10-3.

附錄三

黃彥禎、葉弘德，，93 年 10 月，含水層參數即時檢定的敏感度分析，九十三年度農業工程研討會，中國農業工程學會，桃園，論文摘要集 258 頁，論文集光碟版 1637-1644 頁。

含水層參數檢定的敏感度分析

Sensitivity analysis for aquifer parameter identification

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Abstract

Generally speaking, the conventional pumping test is considered to require more than 24 hours to obtain drawdown data for analyzing aquifer parameters. However, the pumping test would spend a lot of time, money, and groundwater resources. Those drawbacks can be avoided if the aquifer parameters are simultaneously identified when the drawdown data are measured, i.e., identifying parameters on-line by a parameter estimation model. However, the drawdown of an unconfined aquifer in response to the pumping may have time lag. The estimated parameters may be in poor accuracy if the measured drawdown data is too short to reflect the hydrogeologic characteristics of the aquifer. In addition, the shut-down time for the pumping test is difficult to decide when applying a parameter estimation model on-line to analyze the aquifer parameters. The purpose of this study is to explore when the aquifer parameters starts to affect the drawdown using the sensitivity analysis for unconfined aquifer. The result may provide useful information for aquifer parameter estimation.

Keywords: pumping test, sensitivity analysis, parameter estimation model

可供推廣之研發成果資料表

□ 可申請專利　■ 可技術移轉　日期：94 年 9 月 15 日

國科會補助計畫	計畫名稱：擴展式卡門濾波於檢定含水層參數的特性探討與改進 計畫主持人：葉弘德 計畫編號：NSC-93-2218-E-009-056　學門領域：水利工程
技術/創作名稱	應用擴展式卡門濾波檢定水層參數
發明人/創作人	葉弘德
技術說明	中文： 本研究所提出的擴展式卡門濾波法，可應用於水層參數檢定。擴展式卡門濾波可以即時反應水層參數特性，所以可結合現地的 data logger 進行水層參數之檢定，在水層參數特性均反應在洩降數據後，提前終止抽水試驗，節省時間與成本。
	英文： The proposed approach, EKF, can be applied to identify aquifer parameters. From the results of this study, the EKF can reflect the characteristics of the aquifer parameter during the process of the identification. Therefore, EKF can be combined with the field data logger and the pumping test would be terminated earlier for saving the cost when all aquifer parameter have contributed to the drawdown.
可利用之產業及可開發之產品	地下水資源管理、地下水流模式、地下水污染調查前置作業
技術特點	正確推求水層參數、計算時間短、操作簡單
推廣及運用的價值	可發展成視窗介面，以利各相關領域人士之應用