行政院國家科學委員會專題研究計畫成果報告

形態影像金字塔分解法之研究

A Study on Image Decomposition by Morphological Pyramids

計畫編號：NSC 93-2213-E-009-063

執行期限：2004 年 8 月 1 日至 2005 年 7 月 31 日

主持人：薛元澤　國立交通大學資訊科學系

計畫參與人員：莊逢軒、何昌憲　國立交通大學資訊科學系

中文摘要

在多重解析度影像分解法裡，影像金字塔是非常簡單有效的表示法。影像金字塔一般分為線性與非線性兩種。Goutsias 與 Heijmans 於 2000 年提出了影像金字塔之一般性理論，稱之為金字塔轉換。金字塔轉換之主要觀念乃將影像金字塔之建構與重建過程分別視為分解運算與合成運算，其範圍則涵蓋了線性與非線性影像金字塔表示法。分解與合成運算如能表示成形態運算，則金字塔轉換所產生的影像金字塔亦稱之為形態金字塔。

形態金字塔表示法包含了某些形式的形態骨架法並可融合影像量化法。故本計畫之主要目的乃在探討形態金字塔表示法在影像壓縮上之應用。為了表示細微影像與重建，減法與加法是必須的運算。本計畫之另一目的則再提供一個非常方便定義此二運算的數學架構。

關鍵詞：多重解析度影像分解法、影像金字塔、金字塔轉換、形態金字塔

Abstract

In multiresolution image representation, the method of image pyramids is simple and powerful. Depending on the underlying operators, image pyramids are roughly classified as linear and nonlinear pyramids. In 2000, Goutsias and Heijmans proposed a general theory, called pyramid transform, to unify the notions of linear and nonlinear image pyramids. They implement the decomposition procedure in constructing an image pyramid by analysis operator and the approximation procedure by synthesis operator. In a pyramid transform, the analysis and synthesis operators possess some properties quite similar to those possessed by adjunctions in mathematical morphology. Indeed, many morphological operators can be employed to form pyramid transform. In such cases, the resulting image pyramids are called morphological pyramids.

It is interesting that morphological skeletons are examples of morphological pyramids. Besides, image quantization, which is a frequently used technique in image compression, can also be used to form image pyramids. Therefore, it is our first purpose to investigate different morphological pyramids and their applications in image compression..

In order to represent detail images and reconstruct images from their pyramid transforms, subtraction and addition operators are essential. In this report, we suggest to define these two operators under the structure of a *clc*-monoid.

Keywords: Multiresolution Image Decomposition, Image Pyramid, Pyramid Transform, Morphological Pyramid

1. INTRODUCTION

Image pyramids are simple and powerful in multiresolution image representation [1, 2, 4, 8, 9, 10, 11, 12, 13]. Depending on the filters used, they are classified as linear and nonlinear pyramids. In 2000, Goutsias and Heijmans [3, 5] proposed a general theory, called pyramid transform, to unify the notions of linear and nonlinear image pyramids. They implement the decomposition procedure in constructing an image pyramid by analysis operator and the approximation procedure by synthesis operator. If morphological operators are employed as the analysis and synthesis operators to form pyramid transforms, the resulting image pyramids are called morphological pyramids.

In order to represent detail images and reconstruct images from their pyramid transforms, subtraction and addition operators are essential. However, the usual subtraction and addition operators on integers are not operators on gray scales, for instance the gray scale {0, 1, …, 255}. In this report, we suggest to define these two operators under the structure of a *clc*-monoid [6].

2. PYRAMID TRANSFORMS

In the general framework of pyramidal image decomposition, there are given complete lattices

$$L_0, L_1, L_2, \cdots$$

with the following analysis and synthesis operators:

(1) analysis operators:

$$U_j : L_j \to L_{j+1}, \quad j = 0,1,\cdots$$

(2) synthesis operators:

$$S_j : L_{j+1} \to L_j, \quad j = 0,1,\cdots$$

The operators U_j and S_j are said to satisfied the **pyramid condition** if the composition $U_j S_j$ is the identity operator on L_{j+1}.

For an image $x \in L_0$, we define

$$\begin{aligned} y_0 &= x \\ y_{j+1} &= U_j(y\ ), \quad j = 0, 1, 2, \cdots \\ x_j &= S_j(y_{j+1}), \quad j = 0, 1, 2, \cdots \end{aligned}$$

The image y_j is called the **level *j* approximation** of *x* and the image x_j is called the **level *j* prediction** of *x*. The approximation images $y_0, y_1, y_2, \cdots$ are used to form the image pyramid representation of *x*.

Furthermore, suppose that there exit a subtraction operator $-_j$ and an addition operator $+_j$ on L_j. Then the image r_j given by $r_j = y_j -_j x_j$ is called the **level *j* prediction residual** of *x*. We will have a **perfect reconstruction** if $r_j +_j x_j = y_j$. Under such perfect reconstruction conditions, the recursive analysis scheme

$$y_0 = x \to \{y_1, r_0\} \to \{y_2, r_1, r_0\} \to \cdots$$

where $y_{j+1} = U_j(y_j)$, $r_j = y_j -_j S_j(y_{j+1})$, for each $j \geq 0$, is called the **pyramid transform** of *x*. Moreover, the reconstruction procedure

$$y_j = S_j(y_{j+1}) +_j r_j, \quad j \geq 0 \text{ and } x = y_0$$

is called the **inverse pyramid transform** of *x*.

3. MORPHOLOGICAL PYRAMIDS

Let L_j be the set of all digital images defined on Z^2 with gray scale *G*, where *G* is a complete lattice. Then the following analysis and synthesis operators are used to form the **morphological pyramid transform** of an image *x*:

$$U(x)(n) = \bigwedge_{k \in Z^2} \varepsilon_{k-2n}(x(k))$$
$$S(x)(k) = \bigvee_{n \in Z^2} \delta_{k-2n}(x(n))$$

if the following assumptions are made:

(1) For each $k \in Z^2$, ε_k is an erosion and δ_k is a dilation such that

$$\delta_k(x) \le y \quad \text{if and only if} \quad x \le \varepsilon_k(y)$$

(2) Let *A* be the set of all $k \in Z^2$ such that ε_k and δ_k are nontrivial. Denote $Z^2[n]$ as the set $\{k \in Z^2 \mid k - n \in 2Z^2\}$ and *A*[*n*] as the set $A \cap Z^2[n]$. Then there exists a pixel *a* such that *A*[*a*] = {*a*} and δ_a is injective.

An interesting consequence of the above framework is that the morphological skeletonization is a special case of the morphological pyramid transformation.

4. APPLICATIONS

In the construction of a morphological pyramid transform, the following assumptions are made:
(1) the underlying structure of the set of digital images is a complete lattice or a chain;
(2) there exist subtraction and addition operators that satisfied the perfect reconstruction condition $y + (x - y) = x$ if $y \le x$.

In most applications, the gray scale *G* is the set $\{0, 1, \ldots, 255\}$ and the subtraction and addition operations are the usual subtraction and addition operations on integers. However, it should be noted that the usual subtraction and addition operations are actually not operations on *G*. Therefore, we propose the following structure to make the construction of morphological pyramid transform more theoretically justifiable.

In this report, we assume that the underlying structure of the gray scale *G* form a complete *clc*-monoid $(G, \vee, \wedge, *, id)$. In this structure, the element

$$a : b = \vee\{c \mid c * b \le a\}$$

is called the residual of *a* by *b*. Then, the subtraction and addition operators on *G* are defined as

$$a + b = (a^d * b^d)^d$$
$$a - b = (a^d : b^d)^d$$

For instance, if $a * b = a \wedge b$, then

$$a + b = a \vee b$$
$$a - b = \begin{cases} a & if \ a > b \\ 0 & if \ a \le b \end{cases}$$

Observe that $b + (a - b) = a$ if $b \le a$ in this instance. In some other instances, the perfect reconstruction condition might not be satisfied. Then, we can only obtain lossy compression results in those cases.

5. CONCLUSIONS

In this study we investigate the morphological approach to pyramid transform. This approach uses morphological operators to form the analysis and synthesis operators. In order to obtain detail images and to reconstruct the original image from pyramid transform, two additional operators subtraction and addition are defined. We propose the definitions of these two operators under the structure of a *clc*-monoid.

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計畫類別：□個別型計畫　　□整合型計畫

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中　華　民　國　94　年　10　月　31　日