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利用廣義 p 值，廣義信賴區間及特徵函數的統計推論(2/3)

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計畫主持人：李昭勝博士　國立交通大學財務金融研究所

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本成果報告包括以下應繳交之附件：

出席國際學術會議心得報告一份(見附件一)

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本報告含三篇完成之研究成果。

一、A Bayesian Analysis of Mixture Modeling Using the Multivariate t Distribution

此乃與博士生林宗儀教授(目前任教於東海大學統計系)及碩士生倪惠芬(目前就讀台大博士班) 合作的文章。本文已發表於 Statistics and Computing (an SCI journal)。其中英文之摘要如下。

(一)、中文摘要

t 分佈的有限個體的混合模型被用來當常體分佈的混合模型的延伸。本文提出 t 混合模型的貝氏參數推論. 先驗分佈採用弱式的有資訊方式以避免後驗分佈的不可積分, 我們提出兩種有效的 EM-式的運算法用以計算後驗分佈的最高值。馬可夫鏈蒙地卡羅法用來取得後驗分佈的參數估計值, 用實際資料來展示貝氏法優於最大概似法。

(二)、英文摘要

A finite mixture model using the multivariate *t* distribution has been shown as a robust extension of normal mixtures. In this paper, we present a Bayesian approach for inference about parameters of *t*-mixture models. The specifications of prior distributions are weakly informative to avoid causing nonintegrable posterior distributions. We present two efficient EM-type algorithms for computing the joint posterior mode with the observed data and an incomplete future vector as the sample. Markov chain Monte Carlo sampling schemes are also developed to obtain the target posterior distribution of parameters. The advantages of Bayesian approach over the maximum likelihood method are demonstrated via a set of real data.

(三)、報告內容

Finite mixture models introduced by Pearson (1894) have been a useful tool for modeling the data that are thought to come from several different groups with varying proportions. In the past two decades, tremendous improvements and applications have been made in across many research fields. The fundamental idea and usefulness of the mixture models are explained in McLachlan and Basford (1988) and Titterington (1985). A comprehensive introduction to the theory and recent advances can be found in McLachlan and Peel (2000).

Historically, much effort has been devoted to the maximum likelihood (ML) approach for fitting the mixture models. It was first considered by Rao (1948), who used Fisher's scoring method for a mixture of two normal distributions with equal variance. The computation of ML estimates cannot be easily manipulated until the EM algorithm was introduced by Dempster *et al.* (1977). More recently, Peel and McLachlan (2000) considered how to model a mixture of multivariate *t* distributions. They provided the ECM algorithm for parameter estimation and showed the robustness of the model in clustering.

Redner and Walker (1984) pointed out that the ML approach for finite mixture model could encounter unbounded likelihood in some special cases. Hathaway (1985) suggested that using simple constraints in an optimization problem can lead to a strongly consistent and global solution. Hosmer (1973) gave an example of including a portion of labeled observations for each component. However, both solutions are restricted to the univariate case.

In recent developments of computational methods, Bayesian methods are considered an alternative way to deal with mixture models. Diebolt and Robert (1994) used data augmentation and Gibbs sampling as approximation methods for evaluating the posterior distribution and Bayes estimators. They also showed that the duality principle leads to stronger and more general results about the convergence of the simulated Markov chains and of the related moments. Richardson and Green (1997) considered a hierarchical prior that avoid the mathematical pitfalls of using improper priors in mixture model. More recently, Fruhwirth-Schnatter (2001) explored the MCMC output of the random sampler to find suitable identifiability constraints in dealing with label switching problems.

In this article, we extend the ML approach of Peel and McLachlan (2000) to deal with a mixture of *t* distributions from Bayesian viewpoints. Since some observations could be missing in many practical situations, our approach is more general as it allows for some of the observed vectors to be partly known. For the sake of clarity, we only demonstrate one partly known individual and treat it as an incomplete future vector in the model. We compare the prediction and classification results on a real data set between ML and MCMC techniques via cross-validations.

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（五）、計畫成果自評

本研究成果發表於SCI的期刊Statistics and Computing，這是個頗被肯定的國際期刊。

二、Generalized Inferences on the Common Mean of Several Normal Populations

此乃與博士生林淑惠教授(目前任教於台中技術學院)合作的文章。本文將登於Journal of Statistical Planning and Inference (an SCI journal)。其中英文摘要如下。

(一)、中文摘要

本文考慮數個常態分佈其未知變異數為未知且不等的共同平均數的推論問題。利用廣義推論我們提出共同平均數的正確信賴區間的新方法，此法與文獻上一些方法比較涵蓋率及期望的信賴區間長度。當我們把當中的隨機變量以其期望值取代後我們的區間與 Jordan Krishnamoorthy (1996)以等值權數下相同。且兩者皆以著名 Graybill-Deal 的共同平均數估計量為中心點。根據比較的結果，我們提出一些在應用上有關不同模型的選擇手續。

(二)、英文摘要

The hypothesis testing and interval estimation are considered for the common mean of several normal populations when the variances are unknown and possibly unequal. A new generalized pivotal is proposed based on the best linear unbiased estimator of the common mean and the generalized inference. An exact confidence interval for the common mean is also derived. The generalized confidence interval is illustrated with two numerical examples. The merits of the proposed method are numerically compared with those of the existing methods with respect to their expected lengths, coverage probabilities and powers under different scenarios.

(三)、報告內容

Estimating the common mean of several normal populations with unknown and possibly unequal variances is one of the oldest and most interesting problems in statistical inference. This problem arises, for examples, when two or more independent agencies are involved in measuring the effect of a new drug, while utilizing several measuring instruments to measure the products produced by the same production process to estimate the average quality, or when different laboratories are employed to measure the amount of toxic waste in a river. If it is assumed that the samples collected by independent studies are from normal populations with a common mean but possibly with different variances, then the problem of interest may be to estimate or construct a confidence interval for the common mean μ of these populations. If the variances of these populations are assumed to be equal, then there are optimal methods available to make inferences on μ. However, when the variances are unknown and unequal, it is clear that the distribution of any combined estimators of μ will involve nuisance parameters. Consequentially, the standard method has serious limitations for the purpose of finding an exact confidence interval. Thus, intensive studies have been made over the last four decades from both classical and decision theoretic points of view.

In the literature, Meier (1953), Maric and Graybill (1979), Pagurova and Gurskii (1979), Sinha (1985), and Eberhardt et al. (1989) provided approximate confidence intervals for μ, centered at the wellknown Graybill and Deal (1959) estimator $\hat{\mu}_{GD}$ of μ, $\hat{\mu}_{GD} = \dfrac{\sum_{i=1}^{I} n_i\bar{x}_i / s_i^2}{\sum_{i=1}^{I} n_i / s_i^2}$, where $\bar{x}_i$, s_i^2 are sample means and unbiased sample variances for the ith population, i = 1, ,I; Fairweather (1972) and Jordan and Krishnamoorthy (1996) provided exact confidence intervals for μ based on inverting weighted linear combinations of the Student's t statistics and the FisherSnedecor's F statistics, respectively. In general, there is no clear-cut winner between these two intervals. Fairweather's intervals are shorter than Jordan and Krishnamoorthy's when the variance ratios are small; otherwise Jordan and Krishnamoorthy's interval is narrower than Fairweather's. Therefore, some knowledge regarding the relationship between the population variances is needed to choose between these two intervals estimates. However, it should be noted that the method considered by Jordan and Krishnamoorthy (1996) does not always produce nonempty intervals. Yu et al. (1999) considered several confidence intervals that are obtained based on pivots and combinations of appropriately defined p-values. Based on simulation studies, they recommended the methods by Fisher (1932), Fairweather (1972) and Jordan and Krishnamoorthy (1996) for different scenarios. The methods considered by Yu et al. (1999), however, do not always produce nonempty confidence intervals except Fairweather's method (1972). A recent work by Krishnamoorthy and Lu (2003) provided a procedure based on inverting weighted linear combinations of the generalized pivotal quantities, which is similar in spirit to ours, whereas the pivotal quantity derived in this paper is based on the best unbiased estimator of μ. Both works are based on the concepts of generalized p-values and generalized confidence interval, but with different pivotal quantities.

In this paper, we intend to provide a method that is readily applicable for both hypothesis testing and interval estimation of the common mean μ. Our approach is based on the concepts of generalized *p*-values and generalized confidence intervals. The notions of generalized *p*-values and generalized confidence intervals were proposed by Tsui and Weerahandi (1989) and Weerahandi (1993) and since then these ideas have been applied to solve many statistical problems, for examples, Lin and Lee (2003) have provided exact tests in simple growth curve models and one-way ANOVA model, Lee and Lin (2003) have constructed generalized confidence intervals for the ratio of means of two normal populations, etc. The methods are exact in the sense that the tests and the confidence intervals developed are based on exact probability statements rather than on asymptotic approximations. This means that the inferences based on the generalized *p*-values can be made with any desired accuracy, provided that the assumed parametric model and/or other assumptions are correct. Based on the comparison studies, the expected lengths of the new confidence intervals, coverage probabilities and power performances are compared with classical method and the methods proposed by Fairweather (1972), Jordan and Krishnamoorthy (1996) and Krishnamoorthy and Lu (2003). The numerical results in sections 4 and 5 also show that our method performs better than the existing methods.

This article is organized as follows. The theory of generalized *p*-values and generalized confidence interval will be briefly introduced in Section 2. Our procedures for hypothesis testing and constructing the generalized confidence intervals about the common mean μ are presented in Section 3. Three existing procedures including those proposed by Fairweather (1972), Jordan and Krishnamoorthy (1996) and Krishnamoorty and Lu (2003) will be briefly addressed in Section 3. We apply these results to two sets of data, and compare our procedure with the classical method and the other methods with respect to their expected lengths in Section 4. Three simulation studies are presented in Section 5 to compare the expected lengths, the coverage probabilities and power performances of these methods in different combinations of sample sizes and variances.

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（五）、計畫成果自評

本研究成果乃計畫所提的一部分，將登於JSPI，此期刊是SCI統計期刊當中不錯的雜誌，相當值得肯定。

三、**Bayesian Analysis of Box-Cox transformed Linear Mixed Models with ARMA(p,q) Dependence**

此乃與博士生王仁聖及林宗儀教授（目前任教於東海大學統計系）許英麟教授（目前任教於中興大學應數系）及碩士生李國榮合作的文章。本文將登於Journal of Statistical Planning and Inference (an SCI journal)。其中英文摘要如下。

（一）、中文摘要

本文提出Box-Cox中轉換且具ARMA(*p*,*q*)相關之線性混合模式的貝氏推論方法，除了近似法外，也提供馬可夫鏈蒙地卡羅法的結果。兩種先驗分佈用來比較參數估計與未來值的預測。利用實際資料與模擬資料展現貝氏法優於最大概似法。

（二）、英文摘要

In this paper, we present a Bayesian inference methodology for Box-Cox transformed linear mixed model with ARMA(*p*, *q*) errors using approximate Bayesian and Markov chain Monte Carlo methods. Two priors are proposed and put into comparisons in parameter estimation and prediction of future values. The advantages of Bayesian approach over maximum likelihood, method are demonstrated by both real and simulated data.

（三）、報告內容

The main purpose of this paper is to address the problem of analyzing growth curve data from a Bayesian point of view, using an unbalanced linear mixed model with ARMA(*p*, *q*) dependence, while applying the Box-Cox transformation (Box and Cox, 1964) on the observations.

The normal linear mixed models proposed by Laird and Ware (1982) have been widely applied in dealing with longitudinal data. They assumed the within-subjects errors are independent and provided EM algorithms for obtaining the maximum likelihood (ML) estimates and the restricted maximum likelihood (RML) estimates of model parameters. Jennrich and Schluchter (1986) discussed various types of covariance structures, including random effects models and the AR(1) dependence separately. Chi and Reinsel (1989) presented an explicit ML estimation procedure using the scoring method for the model with both random effects and AR(1) errors and remarked that it may be worthwhile to merge higher-order ARMA(*p*, *q*) structures in the model. Bayesian analysis for ARMA(*p*, *q*) regression error models using the Markov chain Monte Carlo (MCMC) methods has been considered by Chib and Greenberg (1994). Rochon (1992) presented a fixed-effects model with ARMA structures of time heteroscedasticity for analyzing repeated measures experiments. More recently, Chib and Carlin (1999) constructed several partially and fully blocked MCMC algorithms for hierarchical mixed models with white noise errors.

Some transformations on the observations could enhance the justification of assumptions such as normality of the distribution or linearity of the growth function. Lee and Lu (1987) and Keramidas and Lee (1990) showed tremendous improvement in predictive accuracy using the Box-Cox transformation for technology substitutions. This is primarily due to the fact that the linearity assumption for the growth function can be enhanced significantly with the Box-Cox transformation, along with incorporating into the model the proper dependence structure among the observations. Enhancement of normality and constancy of variance could have relatively minor roles in the improvement of predictive accuracy.

The model considered here is:

$$Y_i^{(\lambda)} = X_i\beta + Z_i b_i + \varepsilon_i, \quad \text{for } i = 1, 2, \ldots, N,$$

where $Y_i = (Y_{i1}, \ldots, Y_{it_i})'$ *is a* $t_i \times 1$ vector of measurements and is independent of Y_j for all $i \neq j$, β is an unknown $m_1 \times 1$ vector of regression coefficients, X_i and Z_i are known design matrices, b_i is a $m_2 \times 1$ random effects to be sampled from multivariate normal distribution with mean 0 and covariance matrix $\sigma^2\Gamma$, and ε_i is an independent $t_i \times 1$ vector of within subject errors whose components are assumed to follow the ARMA(p, q) model, i.e.,

$$\varepsilon_{ik} = \sum_{j=1}^{p} \phi_j \varepsilon_{i,k-j} - \sum_{j=1}^{q} \theta_j a_{i,k-j} + a_{ik}, \quad \text{for k=1,}\ldots, t_i,$$

where $\{a_{ik}\}$ is a series of shocks or white noise, which are identically and independently distributed as $N(0, \sigma_a^2)$. In our study, we assume the observations for each subject are made at equally spaced intervals. Following Box *et al.* (1994), we write $\phi(B)\varepsilon_{it} = \theta(B)a_{it}$, where $\phi(B) = 1 - \phi_1 B - \Lambda - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \Lambda - \theta_q B^q$ are polynomials of B, which is the backshift operator such that $B\varepsilon_{ik} = \varepsilon_{i,k-1}$. For the process to be stationary and invertible so that there will be a unique model corresponding to the likelihood function, the roots of $\phi(B)$ and $\theta(B)$ must lie outside the unit circle, which constrains the parameter vectors

$\phi = (\phi_1, ..., \phi_p)$ and $\theta = (\theta_1, ..., \theta_q)$to lie in regions C_P and C_q, respectively.

For simplifying the estimating procedure, we shall denote $\sigma^2 C_i$ as the covariance matrix of ε_i and C_i= [$\rho_{|r-s|}$], where $r; s$ = 1, 2,…, t_i. We found that

$$\sigma^2 = (1 - \theta_1 \varphi_1 - \Lambda - \theta_q \varphi_q)\sigma_a^2 / (1 - \phi_1 \rho_1 - \Lambda - \phi_p \rho_p)$$

where $\varphi_j = \sum_{k=1}^{p} \phi_k \varphi_{j-k} - \theta_j$ with θ_j = 0 for $j > q$ and φ_j = 0 for j < 0. It is noted that ρ_j's are implicit functions of ϕ and θ .

The Box-Cox transformation is defined as:

$$Y_{ij}^{(\lambda)} = \begin{cases} \dfrac{(Y_{ij} + \upsilon)^{\lambda} - 1}{\lambda}, & if \lambda \neq 0 \\ \log(Y_{ij} + \upsilon), & if \lambda = 0 \end{cases}$$

where Y_{ij} is the jth component of Y_i, υ is a known constant such that Y_{ij}+ υ > 0, and λ is an unknown parameter. Without loss of generality, we will assume υ = 0 for the rest of the paper. The covariance matrix of $Y_i^{(\lambda)}$ can be written as

$$\sum_i = \sigma^2 (Z_i \Gamma Z_i^{'} + C_i) = \sigma^2 \Lambda_i (\Gamma, \phi, \theta)$$

For the choice of priors, there are two possibilities considered for our Bayesian analysis of model (1). In addition to parameter estimation, we also derive two specific types of prediction problems which is useful in practice. Furthermore, in recent years statisticians have been increasingly drawn to MCMC methods, especially the M-H algorithm (Hastings, 1970; Chib and Greenberg, 1995) and the Gibbs sampler (Geman and Geman, 1984). Therefore, we also consider the problem for the prediction of future observations from a Bayesian point of view.

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（五）、計畫成果自評

本文將發表於Journal of Statistical Planning and Inference。這是個SCI的期刊。相當值得肯定。