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Finding inheritance hierarchies in interval-valued fuzzy concept-networks

Shyi-Ming Chen*, Yih-Jen Horng

Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan, ROC

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Abstract

This paper extends the work of Itzkovich and Hawkes (1994) to present the concepts of interval-valued fuzzy concept-networks and to present an algorithm for finding the collection of inheritance hierarchies in interval-valued fuzzy concept-networks, where the similarity relations and the generalization relations between concepts are represented by interval values in [0, 1]. The proposed method is more flexible than the one presented in Itzkovich and Hawkes (1994) due to the fact that it allows the grades of similarity relations and the generalization relations between concepts to be represented by interval-values rather than crisp real values between zero and one.

Keywords: Inheritance hierarchy; Interval-valued fuzzy concept-network; Similarity relation; Generalization relation

1. Introduction

In [4], Itzkovich and Hawkes presented a fuzzy extension of inheritance hierarchies. They pointed out that inheritance hierarchies provide significant descriptive capability using only the generalization. They also pointed out that inheritance hierarchies have been used in knowledge representation and object-oriented software development. Firstly, they presented the theory of concept-networks, and then extended it to fuzzy concept-networks. Furthermore, they also discussed how fuzzy concept-networks can be used in the application of reusable software retrieval in object oriented software development, where a concept-network is defined by two kind of relations between concepts: synonymy relations and generalization relations, and a fuzzy concept-network is defined by similarity relations and graded generalization relations. In [5], Lucaralla and Morara also presented a kind of concept-networks for fuzzy information retrieval, where the relevant values between concepts are represented by real values between zero and one. In [3, 8], we presented knowledge-based fuzzy information retrieval techniques based on [5]. However, the concept-networks presented in [4, 5] all assume that the relevant values (degrees of graded generalization or degrees of similarity) in a concept-network are represented by crisp real values between zero and one. If we can allow

* Corresponding author. Address: No. 11, Lane 118, Minchuan W. Road, Taipei, Taiwan, ROC.

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the relevant values (degrees of graded generalization or degrees of similarity) between concepts to be represented by an interval in [0, 1] rather than crisp real values between zero and one, then there is room for more flexibility. In [7], Turksen proposed interval valued fuzzy sets for the representation of combined concepts based on normal forms. In [6], Mukaidono introduced interval logic and its extension, where two partial ordered relations on the set of truth values of interval logic were introduced.

In this paper, we extend the work of [4] to present the concepts of interval-valued fuzzy concept-networks based on [6, 7] and to present an algorithm for finding the collection of inheritance hierarchies in interval-valued fuzzy concept-networks, where the similarity relations and the generalization relations between concepts in an interval-valued fuzzy concept-network are represented by a real interval in [0, 1]. The proposed method is more flexible than the one presented in [4] due to the fact that it allows the similarity relations and the graded generalization relations between concepts to be represented by interval-values rather than crisp real values between zero and one.

2. Interval-valued fuzzy concept-networks

In 1994, Itzkovich and Hawkes presented a fuzzy extension of inheritance hierarchies [4]. They pointed out that the purpose of presenting a fuzzy extension of inheritance hierarchies is to provide a more refined construction that facilitates the representation of relations among concepts under uncertain conditions. The extension is done in the following two steps:

Step 1: Incorporate the synonymy relation in the inheritance hierarchy, resulting in a new construction denoted as a concept-network.

Step 2: the relations of the concept-network are fuzzified to yield a new construction denoted as a fuzzy concept-network.

In this section, we present the concepts of interval-valued fuzzy concept-networks based on [4]. In [4], it was pointed out that a fuzzy concept-network is an extension of the concept-network. The definition of fuzzy concept-networks is reviewed from [4] as follows

Definition 1. The similarity relation R_{sim} over a finite set of concepts C, $C = \{c_1, c_2, \ldots, c_n\}$, is a binary fuzzy relation which satisfies all of the following properties:
(1) Reflexive: $\mu_{\text{sim}}(c_i, c_i) = 1$.
(2) Symmetric: $\mu_{\text{sim}}(c_i, c_j) = \mu_{\text{sim}}(c_j, c_i)$.
(3) Transitive: $\mu_{\text{sim}}(c_i, c_k) \geqslant \bigvee_{c_j}(\mu_{\text{sim}}(c_i, c_j) \wedge \mu_{\text{sim}}(c_j, c_k))$.

Definition 2. The graded generalization relation R_g over a finite set of concepts C, $C = \{c_1, c_2, \ldots, c_n\}$, is a binary fuzzy relation which satisfies all of the following properties:
(1) Reflexive: $\mu_g(c_i, c_i) = 1$.
(2) Anti-symmetric: If $\mu_g(c_i, c_j) > 0$ and $\mu_g(c_j, c_i) > 0$, then $c_i = c_j$.
(3) Transitive: $\mu_g(c_i, c_k) \geqslant \bigvee_{c_j}(\mu_g(c_i, c_j) \wedge \mu_g(c_j, c_k))$.

Definition 3. A fuzzy concept-network is denoted by $\text{FCN}(C, R)$, where C is a finite set of concepts and R consists of two relations R_{sim} and R_g over C as defined in Definitions 1 and 2.

For example, Fig. 1 shows a fuzzy concept-network.

In the following, we present the concepts of interval-valued fuzzy concept-networks. In an interval-valued fuzzy concept-network, the degrees of similarity and the degrees of generalization between concepts are represented by a real interval in [0, 1]. Two intervals $[a, b]$ and $[c, b]$ are called equal if and only if $a = c$ and $b = d$. If $[a, b] > [c, d]$, then it implies that $a > c$ and $b > d$ or $a = c$ and $b > d$ or $a > c$ and $b = d$.

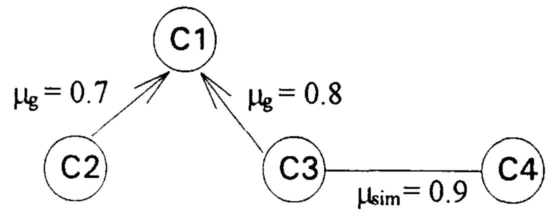

Fig. 1. A fuzzy concept-network.

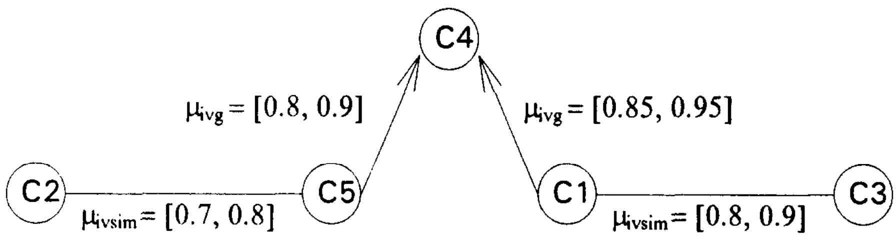

Fig. 2. An interval-valued fuzzy concept-network.

Definition 4. The interval-valued similarity relation R_{ivsim} over a finite set of concepts C, $C = \{c_1, c_2, \ldots, c_n\}$, is a binary fuzzy relation which satisfies all of the following properties:

(1) Reflexive: $\mu_{\text{ivsim}}(c_i, c_i) = [1, 1]$.

(2) Symmetric: $\mu_{\text{ivsim}}(c_i, c_j) = \mu_{\text{ivsim}}(c_j, c_i)$.

(3) Transitive: Let the degree of interval-valued similarity between any concepts c_x and c_y be represented by $\mu_{\text{ivsim}}(c_x, c_y)$, where $\mu_{\text{ivsim}}(c_x, c_y) = [S^l(c_x, c_y), S^h(c_x, c_y)]$ and $0 \leqslant S^l(c_x, c_y) \leqslant S^h(c_x, c_y) \leqslant 1$. Then,

$$S^l(c_i, c_k) \geqslant \bigvee_{c_j} (S^l(c_i, c_j) \wedge S^l(c_j, c_k)),$$

$$S^h(c_i, c_k) \geqslant \bigvee_{c_j} (S^h(c_i, c_j) \wedge S^h(c_j, c_k)).$$

Definition 5. The interval-valued generalization relation R_{ivg} over a finite set of concepts C, $C = \{c_1, c_2, \ldots, c_n\}$, is a binary fuzzy relation which satisfies all of the following properties:

(1) Reflexive: $\mu_{\text{ivg}}(c_i, c_i) = [1, 1]$.

(2) Anti-symmetric: If $\mu_{\text{ivg}}(c_i, c_j) > [0, 0]$ and $\mu_{\text{ivg}}(c_j, c_i) > [0, 0]$, then $c_i = c_j$.

(3) Transitive: Let the degree of interval-valued generalization between any concepts c_x and c_y be represented by $\mu_{\text{ivg}}(c_x, c_y)$, where $\mu_{\text{ivg}}(c_x, c_y) = [g^l(c_x, c_y), g^h(c_x, c_y)]$ and $0 \leqslant g^l(c_x, c_y) \leqslant g^h(c_x, c_y) \leqslant 1$. Then,

$$g^l(c_i, c_k) \geqslant \bigvee_{c_j} (g^l(c_i, c_j) \wedge g^l(c_j, c_k)),$$

$$g^h(c_i, c_k) \geqslant \bigvee_{c_j} (g^h(c_i, c_j) \wedge g^h(c_j, c_k)).$$

Definition 6. An interval-valued fuzzy concept-network is denoted by IVFCN(C, R), where C is a finite set of *concepts* and R consists of two relations R_{ivsim} and R_{ivg} over C as defined in Definitions 4 and 5.

For example, Fig. 2 shows an interval-valued fuzzy concept-network.

3. An algorithm for finding the inheritance hierarchies in interval-valued fuzzy concept-networks

In this section, we present an algorithm for finding the inheritance hierarchies in interval-valued fuzzy concept-networks. Firstly, we present a method to model the interval-valued fuzzy concept-network by using a concept matrix M. If there are n concepts in an interval-valued fuzzy concept-network, then a $n \times n$ concept matrix M will be used to model the interval-valued fuzzy concept-network. The method for modeling interval-valued fuzzy concept-networks by means of a concept matrix M is presented as follows:
If $\mu_{\text{ivsim}}(c_i, c_j) = \mu_{ij}$, where $0 \leqslant \mu_{ij} \leqslant 1$, then let $M(i,j) = M(j,i) = [\mu_{ij}, \mu_{ij}]$;
if $\mu_{\text{ivg}}(c_i, c_j) = \mu_{ij}$, where $0 \leqslant \mu_{ij} \leqslant 1$, then let $M(i,j) = [\mu_{ij}, \mu_{ij}]$ and $M(j,i) = [0, 0]$;
if $\mu_{\text{ivsim}}(c_i, c_j) = [\mu_{ij}^l, \mu_{ij}^h]$, where $0 \leqslant \mu_{ij}^l \leqslant \mu_{ij}^h \leqslant 1$, then let $M(i,j) = M(j,i) = [\mu_{ij}^l, \mu_{ij}^h]$;
if $\mu_{\text{ivg}}(c_i, c_j) = [\mu_{ij}^l, \mu_{ij}^h]$, where $0 \leqslant \mu_{ij}^l \leqslant \mu_{ij}^h \leqslant 1$, then let $M(i,j) = [\mu_{ij}^l, \mu_{ij}^h]$ and $M(j,i) = [0,0]$;
if there are no relationships between the concepts c_i and c_j, then let $M(i,j) = M(j,i) = [0,0]$.

Furthermore, we let $M(i,i) = [1,1]$, where $1 \leqslant i \leqslant n$, due to the fact that each concept c_i is reflexive to itself.

Example 1. Given an interval-valued fuzzy concept-network IVFCN(C, R), where $C = \{c_1, c_2, c_3, c_4\}$, c_1 is an interval-valued generalization of c_2 with $\mu_{\text{ivg}}(c_2, c_1) = [0.5, 0.7]$, c_3 is similar to c_2 with $\mu_{\text{ivsim}}(c_2, c_3) = [0.3, 0.4]$, and c_2 is an interval-valued generalization of c_4 with $\mu_{\text{ivg}}(c_4, c_2) = [0.8, 0.9]$. The interval-valued fuzzy concept-network is shown in Fig. 3.

In this case, we can use a 4×4 concept matrix M to model the interval-valued fuzzy concept-network shown as follows:

$$M = \begin{bmatrix} [1,1] & [0,0] & [0,0] & [0,0] \\ [0.5,0.7] & [1,1] & [0.3,0.4] & [0,0] \\ [0,0] & [0.3,0.4] & [1,1] & [0,0] \\ [0,0] & [0.8,0.9] & [0,0] & [1,1] \end{bmatrix}$$

In the following, we present a method for performing α-cuts operations in an interval-valued fuzzy concept-network.

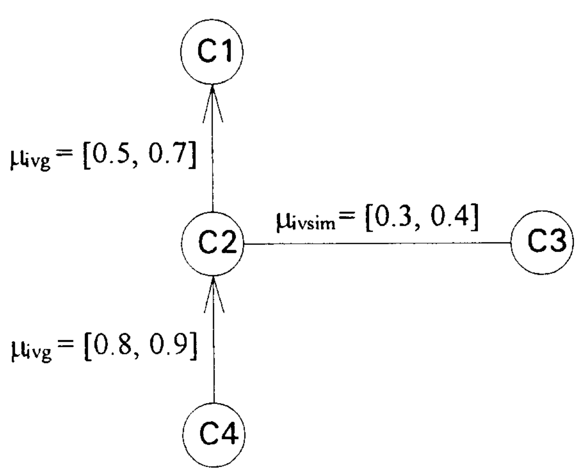

Fig. 3. Interval-valued fuzzy concept-network of Example 1.

Let P be a probability matrix derived from the α_1-cut concept matrix M, where α_1 is a threshold value between zero and one. In a probability matrix P, $P(i,j) = 1$ indicates that the degree of relationship (graded generalization relationship or similarity relationship) between the concepts c_i and c_j is larger than or equal to α_1, where $\alpha_1 \in [0, 1]$; If $M(i,j) = [\mu_{ij}^l, \mu_{ij}^h]$ and $\mu_{ij}^l = \mu_{ij}^h$, then

Case 1: If $\mu_{ij}^h \geqslant \alpha_1$, then we let $P(i,j) = 1$.

Case 2: If $\mu_{ij}^h < \alpha_1$, then we let $P(i,j) = 0$.

Otherwise, if $\mu_{ij}^l \neq \mu_{ij}^h$, then we let

$$P(i,j) = \frac{\mu_{ij}^h - \max(\mu_{ij}^l, \alpha_1)}{\mu_{ij}^h - \mu_{ij}^l}$$

$P(i,j) = 0$ indicates that the degree of probability in which the degree of relationship between the concepts c_i and c_j is less than α_1, where $\alpha_1 \in [0, 1]$; $P(i,j) = \beta$, $\beta \in [0, 1]$, indicates that the degree of probability β in which the degree of relationship between the concepts c_i and c_j is represented by an interval $[a, b]$ is larger than or equal to α_1, where $0 \leqslant a \leqslant \alpha_1 \leqslant b \leqslant 1$, and

$$\beta = \frac{b - \max(a, \alpha_1),}{b - a}.$$

The larger the value of β, the more the degree of the probability that the relationship between the concept c_i and c_j is larger than α_1.

Let Q be a confidence matrix derived from P, and let α_2 be a threshold value between zero and one. If $P(i,j) \geqslant \alpha_2$, where $\alpha_2 \in [0, 1]$, then we let $Q(i,j) = 1$. Otherwise, we let $Q(i,j) = 0$. $Q(i,j) = 1$ indicates that the degree of probability β, in which the degree of relationship between the concepts c_i and c_j is larger than or equal to α_1, is larger than or equal to α_2, where $\alpha_2 \in [0, 1]$.

In the following, we assume that an interval-valued fuzzy concept-network consists of n concepts which has been modeled by an $n \times n$ concept matrix M, where $M(i,j) = [\mu_{ij}^l, \mu_{ij}^h]$, $0 \leqslant \mu_{ij}^l \leqslant \mu_{ij}^h \leqslant 1$, $1 \leqslant i \leqslant n$, and $1 \leqslant j \leqslant n$. The algorithm for performing α-cuts operations in an interval-valued fuzzy concept-network to obtain the probability matrix P and the confidence matrix Q is now presented as follows:

α-Cuts Operations Algorithm

for $i \leftarrow 1$ to n do
  for $j \leftarrow 1$ to n do
  begin
    if $(\mu_{ij}^l = \mu_{ij}^h)$ then
      if $(\mu_{ij}^h \geqslant \alpha_1)$ then $P(i,j) \leftarrow 1$
      else $P(i,j) \leftarrow 0$

      else if $(\mu_{ij}^h \geqslant \alpha_1)$ then $P(i,j) = \dfrac{\mu_{ij}^h - \max(\mu_{ij}^l, \alpha_1)}{\mu_{ij}^h - \mu_{ij}^l}$

        else $P(i,j) \leftarrow 0$;
      if $(P(i,j) \geqslant \alpha_2)$ then $Q(i,j) \leftarrow 1$
      else $Q(i,j) \leftarrow 0$
  end.

Example 2. Given an interval-valued fuzzy concept-network IVFCN(C, R) shown in Fig. 4. Assume that $\alpha_1 = 0.6$ and $\alpha_2 = 0.6$, then we can use a concept matrix M to model the interval-valued fuzzy concept-network of Fig. 4 as follows

$$M = \begin{bmatrix} [1,1] & [0.1,0.2] & [0,0] & [0,0] & [0,0] & [0,0] & [0,0] & [0,0] \\ [0,0] & [1,1] & [0.8,0.8] & [0,0] & [0.4,0.7] & [0,0] & [0.7,0.9] & [0,0] \\ [0,0] & [0,0] & [1,1] & [0,0] & [0,0] & [0.8,0.9] & [0,0] & [0,0] \\ [0,0] & [0,0] & [0,0] & [1,1] & [0,0] & [0,0] & [0,0] & [0,0] \\ [0,0] & [0,0] & [0,0] & [0.5,0.8] & [1,1] & [0,0] & [0,0] & [0.8,0.9] \\ [0,0] & [0,0] & [0.8,0.9] & [0,0] & [0,0] & [1,1] & [0,0] & [0,0] \\ [0,0] & [0.7,0.9] & [0,0] & [0,0] & [0,0] & [0,0] & [1,1] & [0,0] \\ [0,0] & [0,0] & [0,0] & [0,0] & [0.8,0.9] & [0,0] & [0,0] & [1,1] \end{bmatrix}$$

By performing the α-cuts operations, the probability matrix P and confidence matrix Q can be obtained as follows:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2/3 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

In the following, we present the definition of concept classes in an interval-valued fuzzy concept-network based on [4].

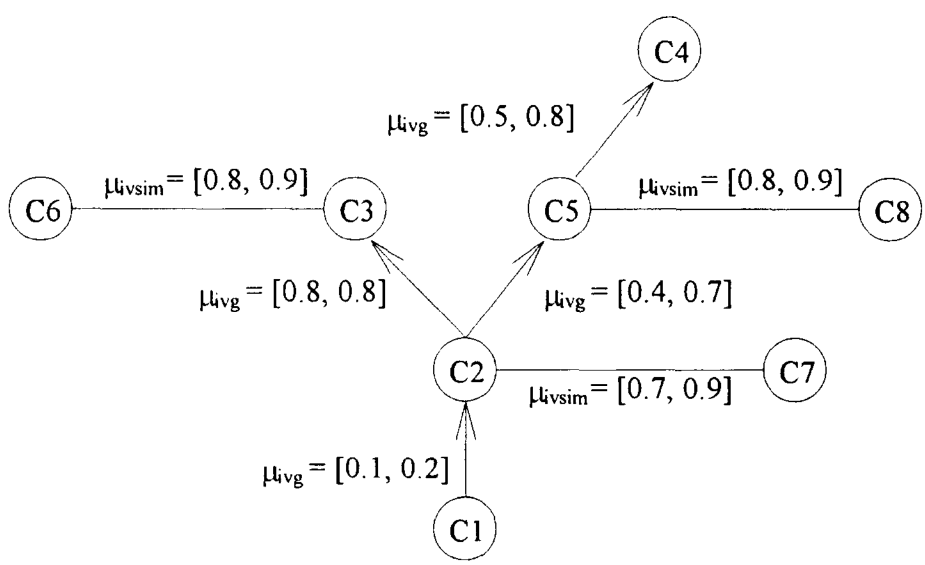

Fig. 4. Interval-valued fuzzy concept-network of Example 2.

A concept class P_i in an interval-valued fuzzy concept-network is a set of concepts, such that the set of concepts C in an interval-valued fuzzy concept-network is the union of each concept class, i.e., $C = \bigcup_i P_i$.

Furthermore, after performing the α-cuts operations of an interval-valued fuzzy concept-network, we can define the set of synonymous concepts in each concept class.

Definition 7. In a concept class P_i, $\forall c_i, c_i \in P_i$, if $\mu_{\text{ivsim}}(c_i, c_j) > 0$, then we say that c_i and c_j are in the same set of synonymous concepts.

The algorithm for finding the inheritance hierarchies in an interval-valued fuzzy concept-network is now presented as follows:

Inheritance Hierarchy Generation Algorithm

```
for i ← 1 to n do
  for j ← 1 to n do
  begin
    if (i = j) then
      if (c_i is not in any concept class) then
      generate a new concept class, and put c_i in the new generated concept class;
      if [(i ≠ j) and (Q(i,j) = 1)] then
        if (Q(j,i) = 1) then
        begin
          if (c_i is not in any concept class) then
            generate a new concept class, and put c_i and c_j in the new generated concept class
          else
            put c_j in the same concept class with c_i;
          if (c_i is not in any set of synonymous concepts) then
            generate a new set of synonymous concepts, and put c_i and c_j in the new generated set of
            synonymous concepts
          else
            put c_j in the same set of synonymous concepts with c_i
        end
  else
    begin
      if ((c_i is not in any concept class) and (c_j is not in any concept class)) then
        generate a new concept class, and put c_i and c_j in the new generated concept class;
      if ((c_i is in a concept class) and (c_j is not in any concept class)) then
        put c_j in the same concept class with c_i;
      if ((c_i is not in any concept class) and (c_j is in a concept class)) then
        put c_i in the same concept class with c_j;
      if ((c_i is in a concept class) and (c_j is in a concept class)) then
      begin
        put all concepts in the concepts class containing c_j in the same concept class with c_i;
        put all interval-valued generalizations in the concept class containing c_j in concept class
        containing c_i
      end;
      let ⟨c_i, c_j⟩ be an interval-valued generalization relation in concept class containing c_i
    end
  end;
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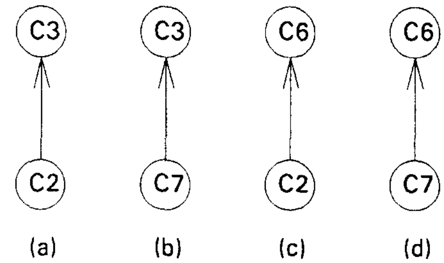

Fig. 5. Inheritance hierarchies.

find the concept class containing concept c_k;
list all interval-valued generalization relations in this concept class which form an inheritance hierarchy;
for all c_i in this inheritance hierarchy do
begin
  find the set of synonymous concepts containing c_i;
  for each c_j in this set of synonymous concepts do
  begin
    substitute c_i in the interval-valued generalization relation by c_j;
    list all interval-valued generalization relations in this concept class which form a new inheritance hierarchy
  end
end.

Example 3. We make same assumptions as in Example 2, where the interval-valued fuzzy concept-network is modeled by the concept matrix M, and the probability matrix P and the confidence matrix Q have been obtained. By applying the inheritance hierarchy generation algorithm, we can obtain three concept classes: $\{c_1\}$, $\{c_2, c_3, c_6, c_7\}$, $\{c_4, c_5, c_8\}$, and three sets of synonymous concepts: $\{c_2, c_7\}$, $\{c_5, c_8\}$, $\{c_3, c_6\}$. Assume that we are interested in the concept class containing c_2, then after performing the algorithm, we can find the inheritance hierarchy $\{\langle c_2, c_3\rangle\}$ containing c_2, graphically as shown in Fig. 5(a). By using replacements among synonymous concepts, we can obtain the other three inheritance hierarchies: $\{\langle c_7, c_3\rangle\}$, $\{\langle c_2, c_6\rangle\}$, $\{\langle c_7, c_6\rangle\}$ as shown in Figs. 5(b)–(d), respectively.

4. Conclusions

In this paper, we have extended the work of [4] to present the concepts of interval-valued fuzzy concept-network and to present an algorithm for finding the collection of inheritance hierarchies in interval-valued fuzzy concept-networks. The proposed method is more flexible than the one presented in [4] due to the fact that it allows the similarity relations and the generalization relations between concepts to be represented by interval values in [0, 1] rather than crisp real values between zero and one.

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