

Simulations of an Internal Model-Based Active Noise Control System for Suppressing Periodic Disturbances

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This study proposes an active noise control algorithm using the internal model principle. The structure of the noise is built in the transfer function of the controller in the unity feedback configuration. This method is particularly effective in suppressing periodic disturbance in spite of its simplicity. Perfect disturbance rejection at selected frequencies is achieved by the infinite loop gain of the system. A simulation was performed to investigate the internal model-based algorithm. The guidelines of choosing the control parameters with reference to a robust stabilization theory are summarized.

Introduction

Many kinds of noises fall into the category of periodic disturbances because a great majority of machinery is either rotary or reciprocating. Examples are the low-frequency noises from internal combustion engines, compressors, blowers, air conditioners, electrical motors, and so forth. Traditional means of noise control such as sound-absorbing materials and mufflers are effective in only moderate and high frequency ranges (say above 500 Hz). The inability of traditional methods in attenuating low-frequency noises is what makes the active noise control (ANC) techniques attractive (Nelson and Elliot, 1992). In viewing the control algorithms, the LMS method and its variants have been prevailing in ANC community (Elliot and Nelson, 1993). Some state-of-art feedback ANC methods have been developed for dealing with periodic disturbances (Forsythe et al., 1991; Eriksson, 1991; Elliott and Nelson, 1993). This study proposes an alternative active noise control algorithm by using the internal model principle. The structure of the noise is built in the transfer function of the controller in the unity feedback configuration. This method is particularly effective in suppressing periodic disturbance in spite of its simplicity.

The internal model principle has been successfully applied to many areas. To name a few, Francis and Wonham (1975) developed a control algorithm on the basis of the internal model principle for multivariable regulators. This algorithm was implemented on a repetitive controller to suppress the vibration of the read/write head of a disk drive (Chew and Tomizuka, 1990a). Yamamoto et al. (1988) applied the internal model principle to servo control design and obtained excellent asymptotic tracking performance. Shaw and Srinivasan (1993) found the possibility of a simplified controller tuning procedure by using the internal model principle. Chew and Tomizuka (1990b) solved the infinite gain problem in the repetitive control design.

In this study, the internal model principle is applied to control the one-dimensional plane wave field in a duct. The optimal controller coefficients are determined by the sequential quadratic programming method. In addition, the effects of control parameters are analyzed by using the theory of robust stabilization. Simulations are conducted to validate the developed method.

The Internal Model-based ANC Algorithm

Consider a duct driven at one end with an acoustic source, as shown in Fig. 1. A loudspeaker is used as an actuator to attenuate the noise at the microphone position. The residual field detected by the microphone is then fed back to the controller to drive the loudspeaker. The entire ANC system forms a unity feedback regulator system, as shown in Fig. 2. The plant that consists of the loudspeaker, the secondary acoustic path, and the microphone is described by the transfer function $G(s) = N(s)/D(s)$ between the input signal to the loudspeaker and the output signal from the microphone, where the polynomials $N(s)$ and $D(s)$ are coprime. That is, $G(s)$ completely characterizes the plant and is irreducible (Chen, 1984). On the other hand, the controller is described by the transfer function $C(s) = N_c(s)/D_c(s)$, where the polynomials $N_c(s)$ and $D_c(s)$ are coprime. The signals $r(t)$, $e(t)$, $u(t)$, $y(t)$, and $w(t)$ represent the reference input, the error, the actuator input, the measured output, and the primary noise, respectively. Our goal is to cancel the primary noise such that the residual field $y(t)$ at the microphone position is zero below the cutoff frequency of the duct (Pierce, 1981). This is essentially a one-dimensional ANC problem since only the plane wave modes are of concern.

First, a brief review following the same line of Chen (1984) on the internal model principle is given. Assume that the Laplace transform of the primary disturbance $w(t)$ is

$$W(s) = L\{w(t)\} = \frac{N_w(s)}{D_w(s)} \quad (1)$$

where the polynomial $D_w(s)$ is known and the polynomial $N_w(s)$ is arbitrary so long as $W(s)$ is proper. That is, the structure of the disturbance is assumed to be known a priori. Let $\varphi(s)$ be a polynomial containing the unstable poles of $W(s)$. As the name of the internal model principle suggests, we incorporate $\varphi(s)$ into the controller transfer function such that $D_c(s) = D_0(s)\varphi(s)$. With some manipulations, it can be shown that

$$Y(s) = \frac{D_0(s)D(s)N_w(s)}{D_0(s)\varphi(s)D(s) + N_c(s)N(s)} \frac{\varphi(s)}{D_w(s)} \quad (2)$$

If no root of $\varphi(s)$ is a zero of the plant $G(s)$, the system can be proved to be controllable and observable, i.e., the polynomials $N(s)$ and $D(s)\varphi(s)$ are coprime. Consequently, there exists a compensator $C(s) = N_c(s)/D_c(s)$ such that the unity feedback system is asymptotically stable or, equivalently, all roots of

Contributed by the Technical Committee on Vibration and Sound for publication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received April 1995. Associate Technical Editor: G. Koopmann.

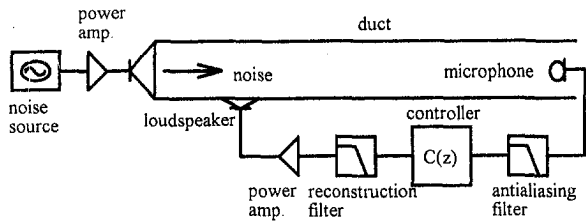


Fig. 1 Schematic diagram of the internal model-based ANC system for a duct

$$D_f(s) \triangleq D_0(s)\varphi(s)D(s) + N_c(s)N(s) \quad (3)$$

have negative real parts. In Eq. (2), all the poles of $Y(s)$ have negative real parts since all unstable poles of $D_w(s)$ have been canceled by $\varphi(s)$. Hence, the employment of the internal model is to create the required numerator to cancel the undesired modes, provided the structure of the disturbance is known. Then, we have $y(t) = -e(t) \rightarrow 0$ as $t \rightarrow \infty$. We see that even though $w(t)$ does not go to zero as $t \rightarrow \infty$, its effect on $y(t)$ diminishes as $t \rightarrow \infty$. The internal model-based controller has thus achieved asymptotic stability and disturbance rejection. Alternatively, we can arrive at the same conclusion by noticing that the loop gain is infinite at the frequency of the disturbance and perfect noise rejection results.

Some cautions regarding the use of the internal model principle are in order (Chen, 1984). First, if we have no knowledge whatsoever about the nature of the noise, then it is impossible to achieve disturbance rejection. Second, the internal model design is robust with respect to parameter perturbation, while the perturbation of $\varphi(s)$ is not permitted. Finally, the condition that no root of $\varphi(s)$ is a zero of the plant $G(s)$ is crucial. Otherwise, if any unstable root of $\varphi(s)$ is a zero of $G(s)$, then the root becomes a hidden mode and will not be affected by any compensation. Hence, the unity feedback system can never be asymptotically stable because of unstable pole-zero cancellation.

The internal model principle is readily applied to the problem of rejecting periodic noises which are the major concern of this study. Since every periodic noise can be treated as the superposition of sinusoids, we begin with the analysis of only a single-frequency component. The Laplace transform of a sinusoid of frequency ω_n is

$$W(s) = \frac{\omega_n^2}{s^2 + \omega_n^2} \quad (4)$$

By the internal model principle, the controller can be chosen to be

$$C(s) = \frac{N_c(s)}{D_c(s)} = \frac{N_c(s)}{s^2 + \omega_n^2} \quad (5)$$

such that the unstable poles $s = \pm j\omega_n$ on the imaginary axis can be canceled to reject the sinusoidal disturbance. In the controller formulation, the polynomial $D_0(s)$ is chosen to be unity for convenience. In addition, the poles of the closed-loop systems, or the roots of $D_f(s)$ in Eq. (3), must be in the left-half s -plane to ensure stability. This imposes a constraint on the polynomial $N_c(s)$.

For an arbitrary periodic noise containing the Fourier components at the frequencies $\omega_1, \omega_2, \dots, \omega_m$, the above-mentioned internal model-based controller for a single frequency can be generalized into

$$C(s) = \frac{N_c(s)}{D_c(s)} = KN_c(s) \sum_{n=1}^m \frac{1}{s^2 + \omega_n^2}, \quad (6)$$

where K is the controller gain.

On the other hand, the discrete-time version of the controller in Eq. (6) would be useful if we are interested in digital implementation.

$$C(z) = \frac{N_c(z)}{z^2 - 2z \cos(\omega_n T) + 1}, \quad (7)$$

where T is the sampling period. For an arbitrary periodic noise containing the Fourier components at the frequencies $\omega_1, \omega_2, \dots, \omega_m$, the digital controller takes the form

$$C(z) = KN_c(z) \sum_{n=1}^m \frac{1}{z^2 - 2z \cos(\omega_n T) + 1}. \quad (8)$$

It should be noted that the gain parameter K and the polynomial $N_c(z)$ must be chosen so that the resulting closed-loop system is stable. That is, the discrete version of Eq. (3)

$$D_f(z) = D(z) \prod_{n=1}^m \frac{1}{z^2 - 2z \cos(\omega_n T) + 1} + KN_c(z)N(z) \quad (9)$$

must have all of the roots inside the unit circle.

$$|\lambda_i| < 1, \quad i = 1, 2, \dots, p, \quad (10)$$

where p is the degree of $D_f(z)$.

In addition to stability, performance is another important ingredient of the controller design. Although the internal model-based controller provides perfect steady-state disturbance rejection, one might be concerned with the settling time of the controller, especially in the cases of frequency-tracking. This leads to a min-max problem in which the farthest closed-loop pole to the origin (in z -plane) is to be minimized:

$$\begin{aligned} \min_{K, N_c(z)} \max_i |\lambda_i| \\ \text{st. } \max_i |\lambda_i| < 1 \end{aligned} \quad (11)$$

To solve the above constrained optimization problem, the sequential quadratic programming (SQP) method was employed in this study. In this method, a quadratic programming subproblem is solved at each iteration by incorporating not only the gradient but also the second order information (the Hessian). An estimate of the Hessian of the Lagrangian is then updated at each iteration, followed by a line search using a merit function. The details of the algorithm can be found in the text by Aurora (1989). Since the optimization procedure is based on the dominant pole criterion of a second order system, it is sometimes possible to fine-tune K and $N_c(z)$ by the root locus method (Franklin, 1994).

Numerical Simulation

A numerical simulation was carried out to validate the internal model-based ANC algorithm. The synthetic plant is chosen to be

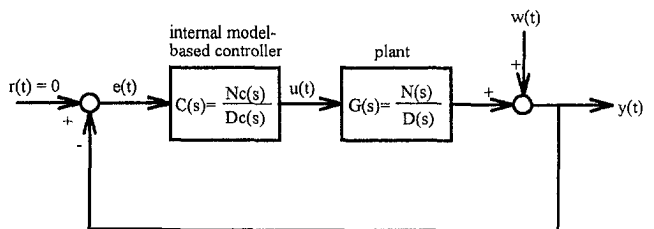


Fig. 2 Block diagram of the internal model-based control system with unity feedback

Table 1 The closed-loop poles and zeros of a digital internal model-based controller (without damping)

pole	zero
$-0.3010 \pm j0.9272$	0
$0.2405 \pm j0.9253$	0
$0.8198 \pm j0.5308$	$-0.3090 \pm j0.9511$
$0.6067 \pm j0.3651$	$0.3090 \pm j0.9511$
0.8202	$0.8090 \pm j0.5878$
-0.1924	$0.8619 \pm j0.1896$

$$G(s) = \frac{\Omega^2}{s^2 + 2\zeta\Omega s + \Omega^2} \exp(-T_d s), \quad (12)$$

where the second order term is used to model the loudspeaker and the pure time delay term is used to model the plane wave propagation in an infinite-length duct. Let $\Omega = 250$ rad/sec, $\zeta = 0.5$, and $T_d = 0.002$ sec, then Eq. (12) becomes

$$G(s) = \frac{62500}{s^2 + 250s + 62500} \exp(-0.002s), \quad (13)$$

which is then discretized by zero-order-hold operation to obtain a digital equivalent.

Suppose that the periodic noise contains three sinusoids of unity amplitude at the frequencies 100, 200, and 300 Hz. The first step consists of choosing a controller in the form of Eq. (9) with the internal model of the disturbance built in. Next, the polynomial $N_c(z)$ can be arbitrarily set to be, for example,

$$N_c(z) = a_5 z^5 + a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0. \quad (14)$$

The sampling period is chosen to be 1 msec for the digital controller. Solving the constrained optimization problem in Eq. (11) yields the following digital controller:

$$C(z) = 0.371 \times \frac{z^5 + 4.0367z^4 - 8.7648z^3 + 2.7239z^2 + 7.6618z - 6.5555}{z^6 - 1.618z^5 + 2.618z^4 - 2.618z^3 + 2.618z^2 - 1.618z + 1}. \quad (15)$$

The closed-loop poles and zeros corresponding to the controller in Eq. (15) are shown in Table 1. All the poles are stable and the farthest one to the origin is $0.8198 \pm j0.5308$.

For the controller in Eq. (15), it is informative to calculate the sensitivity function which equals the frequency response from the disturbance to the output of the unity feedback system (Franklin, 1994).

$$S(e^{j\omega T}) \triangleq \frac{1}{1 + G(e^{j\omega T})C(e^{j\omega T})} = \frac{Y(e^{j\omega T})}{W(e^{j\omega T})}. \quad (16)$$

The result of the sensitivity function of the closed-loop ANC system is shown in Fig. 3. Three sharp dips can be seen at the selected frequencies 100, 200, and 300 Hz. Perfect disturbance rejection is achieved owing to nearly zero sensitivity values at these three frequencies.

It can be verified by, for example, the root locus method, that the system is stable for $0 \leq K \leq 0.577$, where all the closed-loop poles stay inside the unit circle. Beyond $K = 0.577$, the system will become unstable.

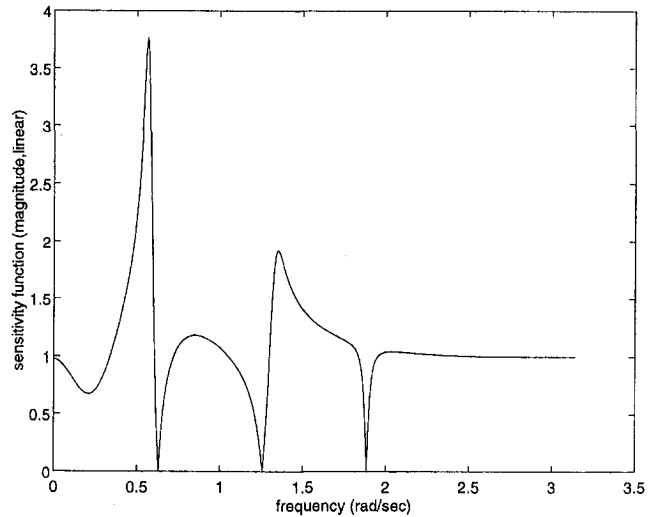


Fig. 3 Magnitude of the sensitivity function of the closed-loop system when the damping factor has not been introduced into the internal model-based controller

There is always discrepancy between the physical plant and the mathematical model because of some unmodeled dynamics in the system. The question regarding the robustness of the internal model-based controller will naturally arise. Namely, what is the ability of the controller in coping with plant uncertainty? This question can be answered by the following robust stability theorem (Doyle et al., 1992).

Theorem

Let $G_0(s)$ be the transfer function of a stable nominal plant and $G_p(s)$ be the transfer function of the perturbed plant. Assume that the plant uncertainty can be described by $G_p(s) = [1 + \Delta W_2(s)]G_0(s)$, $|\Delta| \leq 1$. The controller in the unity feedback system will provide robust stability, i.e., it will stabilize every perturbed plant $G_p(s)$, if and only if $\|W_2 T\| < 1$, where $T(s) = 1 - S(s)$ being the complementary sensitivity function.

Note that the above condition is not only a sufficient but also a necessary one. In the simulation, the perturbation of the pure time delay is used as the major type of plant uncertainty. Other types of uncertainty can be similarly discussed. For the controller in Eq. (15), the allowable range of pure time delay in the plant predicted by the above robust stability theorem is $0.00140 < T_d < 0.00251$.

Next, a technical adjustment concerning the implementability of the controller should be addressed. The internal model-based controller is essentially an unstable controller with the poles on the imaginary axis. An unstable controller is typically not acceptable because of the difficulty in testing either the controller by its-self or the system in open-loop during a bench check-out. To alleviate the problem, a damping factor is introduced into the controller. That is,

$$C_d(z) = K_d N_{cd}(z) \prod_{n=1}^m \frac{1}{z^2 - 2e^{-\alpha T} z \cos(\omega_n T) + e^{-2\alpha T}}. \quad (17)$$

With this modification and employment of the previous optimization procedure, the digital controller becomes

$$C_d(z) = 0.3124 \frac{z^5 + 4.0367z^4 - 8.7648z^3 + 2.7239z^2 + 7.6618z - 6.5555}{z^6 - 1.6108z^5 + 2.5946z^4 - 2.5829z^3 + 2.5713z^2 - 1.5820z + 0.9734}. \quad (18)$$

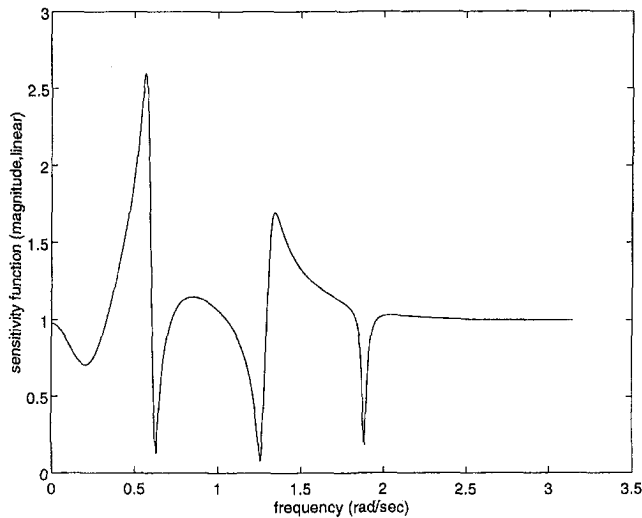


Fig. 4 Magnitude of the sensitivity function of the closed-loop system when the damping factor has been introduced into the internal model-based controller

The sensitivity function of the closed-loop system using the controller in Eq. (18) is shown in Fig. 4. The major difference after the introduction of the damping factor, as one may compare the sensitivity functions in Figs. (3) and (5), is that the sensitivity value is no longer ideally zero at the frequencies of the exciting sinusoids. This implies that the ability of the ANC system in rejecting periodic disturbance will not be as good as before when the damping factor is not incorporated into the controller. Nevertheless, the tolerance against plant uncertainty is improved. In light of the robust stability theorem, the allowable range of pure time delay in the plant for the controller in Eq. (18) can be found to be $0.00116 < T_d < 0.00261$ which is wider than the one without the damping factor. In addition, the range of the gain parameter can be shown to be $0 < K < 0.5891$ for the ANC system with the damping factor to remain stable. This is an improvement over the controller without the damping factor. This analysis of robust stability reveals a classical tradeoff between the performance and the stability of control systems.

To justify the internal model-based controllers in Eqs. (15) and (18), simulation cases of different operating conditions are employed in a numerical experiment. Five simulation cases are shown in Table 2. Case 1 is the reference case, where no perturbation is present in the plant and the internal model. The result obtained by using the controller in Eq. (15) is shown in Fig. 5. This case represents the ideal situation of noise rejection, where the attenuation is approximately 200–300 dB for the tones, by using the internal model-based controller. In practical applications, reasonable deviation from the ideal case such as the perturbation of the plant, the inaccuracy of the internal

Table 2 Simulation case design

case	frequencies of sinusoids (Hz)	plant time delay (second)	sampling rate (kHz)	digital controller
1	100+200+300	0.002	1	Eq. (15)
2	101+202+303	0.002	1	Eq. (15)
3	100+200+300	0.00253	1	Eq. (15)
4	100+200+300	0.00253	1	Eq. (18)
5	110+220+330	0.002	1	Eq. (15)

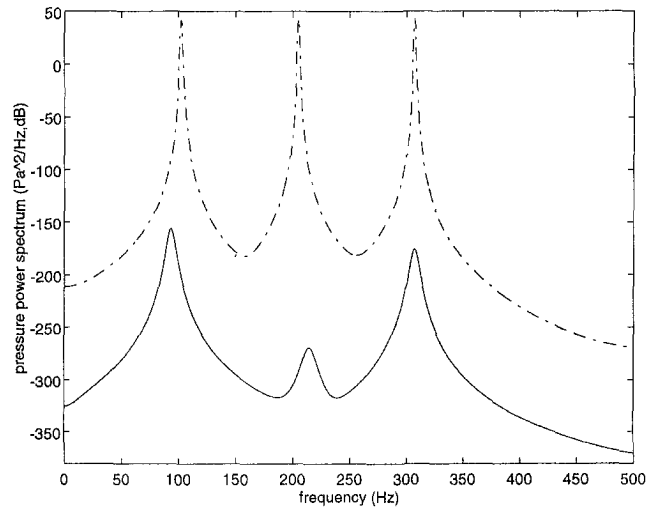


Fig. 5 The pressure power spectrum of the noise field at the microphone position before and after active control has been activated for Case 1. Before control: -.-; after control: —.

model, and the implementability of the controller should be accommodated.

In Case 2, the effect of the inaccuracy of the internal model is investigated. Suppose that 1 percent perturbation of the excitation frequencies is present in the internal model. As can be seen in the result of Fig. 6, the performance of noise rejection is degraded, where the attenuation is within 30 dB for the tones. This confirms the remark made previously that the internal model must be perfect for robust design. However, this does not mean the internal model principle is only of theoretical interest. In the practical design, if the internal model is not employed, the result will be even worse.

Case 3 shows the effect of plant uncertainty. Assume that the pure time delay of the plant is 0.00253 seconds in stead of 0.002 seconds as in the reference Case 1. In this case, the ANC system is driven into the unstable state. The perturbed pure time delay in the plant is not in the range of stability as predicted by the robust stability theorem: $0.00140 \text{ seconds} < T_d < 0.00251 \text{ seconds}$.

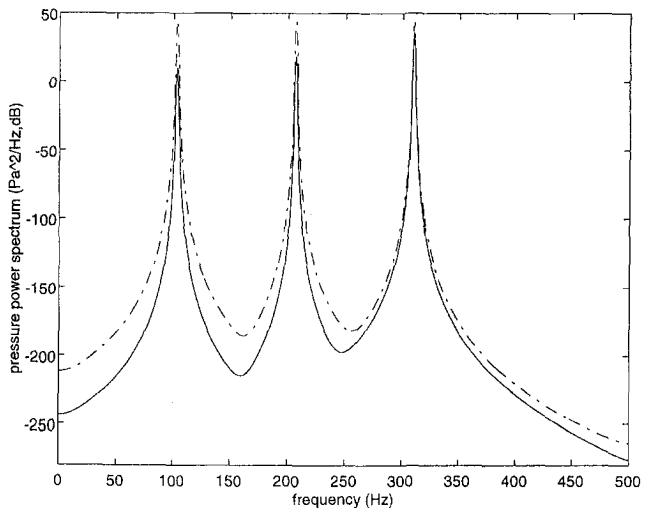


Fig. 6 The pressure power spectrum of the noise field at the microphone position before and after active control has been activated for Case 2. Before control: -.-; after control: —.

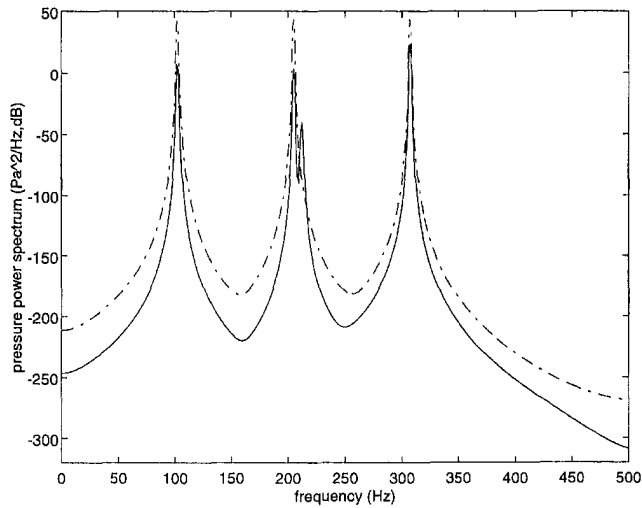


Fig. 7 The pressure power spectrum of the noise field at the microphone position before and after active control has been activated for Case 4. Before control: -.-; after control: —.

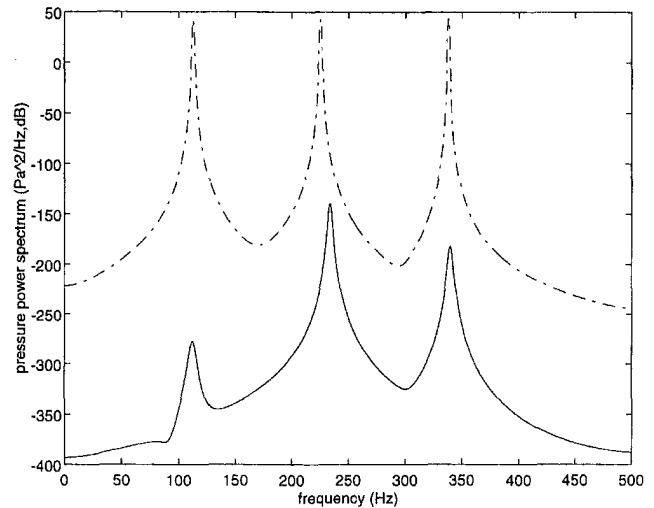


Fig. 8 The pressure power spectrum of the noise field at the microphone position before and after active control has been activated for Case 5. Before control: -.-; after control: —.

The internal model-based controller is essentially unstable because of its purely imaginary poles. In order to avoid the difficulty during the implementation stage, a damping factor can be introduced into the controller. Case 4 explores the effect of plant uncertainty when the damping factor is incorporated into the controller. Similar to Case 3, assume that the pure time delay of the plant is 0.00253 seconds in stead of 0.002 seconds as in the reference Case 1. The response after the ANC system is activated is shown in Fig. 7. Stability has been improved by introducing the damping factor. This result confirms what we have found via the robust stability theorem that the introduction of the damping factor into the controller would produce wider stability range of the pure time delay: 0.00116 seconds $<$ T_d $<$ 0.00261 seconds. Nevertheless, some residual noise can still be found because the perturbed internal model is not able to accurately cancel the undesirable modes in the periodic excitation. In conclusion, good stability margin can only be achieved at the expense of degradation of performance (Doyle, 1992). On the other hand, care must be assumed when dealing with tones corrupted by random noises. For random noises, where we have no knowledge whatsoever about its nature but its statistics, the internal model-based ANC controller will not achieve disturbance rejection.

In some practical applications, frequency tracking may be of interest. The fact that the frequency of disturbance has to be known a priori may appear as a limitation in using the internal-model-based controller. To cope with the problem, reference signals from frequency sensors such as tachometers can be utilized to synchronously adjust the poles of the filter, which is technically not difficult to accomplish in DSP-based controllers. Furthermore, the optimality of the compensation filter $N_c(z)$ seems not critical with respect to the frequency change of disturbances. The robustness of $N_c(z)$ is evidenced from the result of Case 5, where the frequencies of the tones have been shifted from those in Case 1 by 10 percent (Fig. 8). Approximately 200–300 dB attenuation can still be obtained by the controller with new poles, but $N_c(z)$ remains the same as Case 1.

Conclusion

In this study, an ANC algorithm using the internal model principle has been developed. The method is effective in suppressing periodic disturbances. The simplicity of the method lies in the fact that the resulting controller is generally a low-order digital filter, although the off-line search process

for the optimal $N_c(z)$ in Eq. (8) may look somewhat involved. Since this method is based on feedback configuration, one does not have to be concerned with the acoustic feedback problem which often complicates the controller design in many conventional feedforward ANC systems. In addition, the internal model-based controller exhibits robustness with parameter variations, e.g., pure time delay, after a small amount of damping is introduced into the controller. The proposed ANC system requires an accurate internal model to match the structure of disturbance. Optimization methods are then employed to determine the coefficients of the controller. It may happen that numerous local minima exist in the problem of interest. Thus, care must be taken to select appropriate initial guesses. In addition, frequency tracking that represents a large class of practical application might become an issue for the internal model control. Although the frequency of the disturbance has to be known a priori, reference signals from frequency sensors such as tachometers can be utilized to synchronously adjust the poles of the filter. The optimality of the compensation filter is generally not critical with respect to the frequency change of disturbances. Experimental investigations based on DSP controllers are currently under way to further explore the effectiveness and practicality of the internal model-based ANC technique.

Acknowledgment

Special thanks are due to Professor J. S. Hu for the helpful discussions on the internal model principle. The work was supported by the National Science Council in Taiwan, Republic of China, under the project number NSC 83-0401-E-009-024.

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