# 行政院國家科學委員會專題研究計畫　期中進度報告

## 利用廣義 p 值，廣義信賴區間及特徵函數的統計推論(1/3)

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# 行政院國家科學委員會專題研究計畫期中報告

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本報告含三篇完成之研究成果。

## 一、**On Modeling Data from Degradation Sample Paths over Time**

此乃與博士生林宗儀教授（目前任教於東海大學統計系）合作的文章。本文已發表於 Australian & New Zealand Journal of Statistics (an SCI journal)。其中、英文之摘要如下。

### （一）、中文摘要

本文主要對衰退資料的建模應用成長曲線模型加上 Box-Cox 轉換、隨機效應、及 ARMA (p,q)相關來分析一組衰退的資料。導出最佳概似估計及利用最佳線性不偏預測法求出未來值的預測。本文從預測的觀點比較所提之方法與一個非線性衰退模型。我們也比較不同樣本長度下之斷點(failure times)的預測。

### （二）、英文摘要

This paper is mainly concerned with modelling data from degradation sample paths over time. It uses a general growth curve model with Box–Cox transformation, random effects and ARMA *(p, q)* dependence to analyse a set of such data. A maximum likelihood estimation procedure for the proposed model is derived and future values are predicted, based on the best linear unbiased prediction. The paper compares the proposed models with a nonlinear degradation model from a prediction point of view. Forecasts of failure times with various data lengths in the sample are also compared.

### （三）、報告內容

In this paper, we are concerned with modelling degradation data such as measurements of the growth of fatigue cracks. In general, engineers need to produce units of material with acceptable reliability and at an acceptable life-cycle cost. Most material accumulates irreversible damage during its life, which leads to failure. The cumulative damage reduces the reliability of the material as time increases. To maintain an acceptable reliability in the unit, inspections and repairs must be made, which increase life-cycle costs. Thus, cumulative damage plays a very important role in the design of the unit.

For degradation data such as the fatigue crack growth data described above, it is very important to develop a model capable of predicting the fatigue crack growth and, most important of all, predicting the time to failure. Then engineers can order repair or replacement before the failure actually occurs. Once the unit or material has failed, it is too late to repair, and failure could cause heavy physical and/or economic losses.

This type of data is quite typical in studies such as accelerated life testing, because the product usually takes a long time to wear out. One important characteristic of the observations obtained in degradation studies is that they are measurements of several units, and each unit is measured over time. The measurements on a single unit are not independent because they are time-series in nature. If there are only a few measurements on the unit, say fewer than 20, then the dependence may be too hard to estimate. Fortunately, such data are usually obtained for several similar and independent units. Also, the linearity of growth function can be enhanced by the well known Box–Cox transformation (Box & Cox, 1964), as seen in Figure 1(b). These phenomena occur in many studies including technology substitutions as reported by Keramidas & Lee (1990). This paper predicts that a general growth curve model having ARMA*(p, q)* dependence coupled with the Box–Cox transformation can be applied to degradation data. A model is proposed and compared, in terms of its prediction accuracy and failure time prediction, with the degradation model of Lu & Meeker (1993) using the fatigue crack data of Bogdanoff & Kozin (1985). The failure time is the time to grow a crack from 0.90 inches to the critical crack length of 1.60 inches. A credible prediction of failure time is important, particularly to engineers.

From the results presented in Section 4 we know that if there are only a few measurements on each unit, it may be too hard to estimate the autocorrelation. Also, the data in each unit are time-series in nature and hence are not independent. Therefore, we can use the general growth curve model with ARMA*(p, q)* covariance structures to analyse this kind of data, using measurements from similar units to get better prediction results. The advantage of our modelling for this type of data is evident in the comparisons of forecast accuracy in future values and in failure times.

As remarked in Rochon (1992), ARMA*(p, q)* covariance structures are worth considering and may have better performance than AR(1) dependence in many applications. For the modelling of degradation data, with appropriate ARMA*(p, q)* covariance structure and coupled with random effects and the Box–Cox transformation, our modelling approach makes the prediction results quite appealing.

（四）、參考文獻


Barndorff-Nielsen, O.E. & Schou, G. (1973). On the reparameterization of autoregressive models by partial autocorrelations. *J. Multivariate Anal.* **3**, 408–419.

Bogdanoff, J.L. & Kozin, F. (1985). *Probabilistic Models of Cumulative Damage.* NewYork:Wiley.

Box, G.E.P. & Cox, D.R. (1964). An analysis of transformations (with discussion). *J. Roy. Statist. Soc. Ser.B* **26**, 211–252.

Box, G.E.P., Jenkins, G.M. & Reinsel, G.C. (1994). *Time Series Analysis Forecasting and Control.* 3rd edn. San Francisco: Holden–Day.

Chi, E.M. & Reinsel, G.C. (1989). Models for longitudinal data with random effects and AR(1) errors. *J. Amer. Statist. Assoc.* **84**, 452–459.

Dempster, A.P., Laird, N.M. & Rubin, D.B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *J. Roy. Statist. Soc. Ser. B* **39**, 1–38.

Diggle, P.J. (1988). An approach to the analysis of repeated measurement. *Biometrics* **44**, 959–971.

Geisser, S. (1975). The predictive sample reuse method with applications. *J. Amer. Statist. Assoc.* **70**, 320–328.

Jennrich, R.I. & Schluchter, M.D. (1986). Unbalanced repeated–measures models with structured covariance matrices. *Biometrika* **42**, 805–820.

Keramidas, E.M. & Lee, J.C. (1990). Forecasting technological substitutions with concurrent short time series. *J. Amer. Statist. Assoc.* **85**, 625–632.

Laird, N.M. & Ware, J.H. (1982). Random effects models for longitudinal data. *Biometrics* **38**, 963–974.

Lee, J.C. (1988). Prediction and estimation of growth curve with special covariance structures. *J. Amer. Statist. Assoc.* **83**, 432–440.

Lindstrom, M.J. & Bates, D.M. (1988). Newton–Raphson and EM algorithms for linear mixed-effects models for repeated measures data. *J. Amer. Statist. Assoc.* **83**, 432–440.

Liu, C. & Rubin, D.B. (1994). The ECME algorithm: a simple extension of EM and ECM with faster monotone convergence. *Biometrika* **81**, 633–648.

Lu, C.J. & Meeker, W.Q. (1993). Using degradation measures to estimate a time-to-failure distribution. *Technometrics* **35**, 161–174.

Meng, X.L. & Rubin, D.B. (1993). Maximum likelihood estimation via the ECM algorithm: a general framework. *Biometrika* **80**, 267–278.

Monahan, J.F. (1984).A note on enforcing stationarity in autoregressive moving average models. *Biometrika* **71**, 403–404.

Morrell, C.H., Pearson, J.D., Carter, H.B. & Brant, L.J. (1995). Estimating unknown transition times using a piecewise non-linear mixed-effects model in men with prostate cancer. *J. Amer. Statist. Assoc.* **90**, 45–53.

Paris, P.C. & Erdogan, F. (1963). A critical analysis of crack propagation laws. *J. Basic Engineering* **85**, 528–534.

Potthoff, R.F. & Roy, S.N. (1964). A generalized multivariate analysis of variance model useful especially for growth curve problems. *Biometrika* **51**, 313–326.

Rao, C.R. (1987). Prediction of future observations in growth curve model. *Statis. Sci.* **2**, 434–471.

Rochon, J. (1992). ARMA covariance structures with time heteroscedasticity for repeated measures experiments. *J. Amer. Statist. Assoc.* **87**, 777–784.

Verbeke, G., Lesaffre, E. & Brant, L.J. (1998). The detection of residual serial correlation in linear mixed models. *Statist. Med.* **17**, 1391–1402.

（五）、計畫成果自評

本研究成果發表於SCI的期刊Australian & New Zealand Journal of Statistics，這是個頗被肯定的國際期刊。

## 二、**Bayesian Estimation for Time Series Regressions with Applications**

此乃與博士生林淑惠教授(目前任教於台中技術學院)合作的文章。本文已發表於 Journal of Statistical Planning and Inference (an SCI journal)。其中、英文摘要如下。

### (一)、中文摘要

根據廣義 p 值及廣義信賴區間的概念，我們研發有關兩個常態分配母體平均數比例(ratio)的假設檢定與信賴區間。我們利用兩個不同的方法來取得關鍵值(pivotal qualities)。當中之一是直接利用平均數的比例來找關鍵值。另一方法則是將此問題當作類似 Behrens-Fisher 之問題來處理，然後利用假設檢定之法來建構信賴區間。我們的模擬結果發現本文所提之信賴區間優於其他兩種方法。這個優點更顯示於兩個母體之變異數相當不同之情況。

### (二)、英文摘要

Based on the generalized *p*-values and generalized confidence interval developed by Tsui and Weerahandi (1989), Weerahandi (1993), respectively, hypothesis testing and confidence intervals for the ratio of means of two normal populations are developed to solve Fieller's problems. We use two different procedures to find two potential generalized pivotal quantities. One procedure is to find the generalized pivotal quantity based directly on the ratio of means. The other is to treat the problem as a pseudo Behrens-Fisher problem through testing the two-sided hypothesis on $\theta$, and then to construct the $1-\alpha$ generalized confidence interval as a counterpart of generalized *p*-values. Illustrative examples show that the two proposed methods are numerically equivalent for large sample sizes. Furthermore, our simulation study shows that confidence intervals based on generalized *p*-values without the assumption of identical variance are more efficient than two other methods, especially in the situation in which the heteroscedasticity of the two populations is serious.

### (三)、報告內容

Much attention has been paid to Fieller's problems, because they occurred frequently in many important research areas such as bioassay and bioequivalence. In bioassay problem, the relative potency of a test preparation as compared with a standard is estimated by (i) the ratio of two means for direct assays, (ii) the ratio of two independent normal random variables for parallel-line assays and (iii) the ratio of two slopes for slope-ratio assays. In biological assay problems (Fieller (1954), Finney (1978)) and bioequivalence problems (Chow and Liu (1992), Berger and Hsu (1996)), one is interested in the relative potency of two drugs or treatments. Traditionally, Fieller (1944, 1954) provides a widely used general procedure for the construction of confidence intervals (often called Fieller's theorem) for the ratio of means (also discussed by Rao (1973), Finney (1978), Koschat (1987) and Hwang (1995)). Under homoscedasticity case, Koschat (1987) has also shown that within a large class of sensible procedures the Fieller solution is the only one that gives exact coverage probability for all parameters. However, the conventional procedures are often restricted to the assumption of a common variance or pairwise observations for mathematical tractability. Thus, the exact approaches to Fieller's problems under the unequal variances assumption have also been intensively investigated. Consider the following problem: Let $\boldsymbol{X}=(X_1, X_2, \cdots, X_{n_1})$ and $\boldsymbol{Y}=(Y_1, Y_2, \cdots, Y_{n_2})$ be two independent sets of observations for the potency of a standard drug and a new drug, respectively. Assume that $X_i$ are independently and identically distributed as $N(\mu_1, \sigma_1^2)$, $Y_i$ are independently and identically distributed as $N(\mu_2, \sigma_2^2)$, where $\mu_1$ and $\mu_2$ are the true potencies. The problem is to determine, with any desired probability, the range of values for the ratio of means $\theta = \frac{\mu_2}{\mu_1}$, which is the relative potency of the new drug to the standard.

Under the assumption of identical variance, Fieller (1954) constructed a confidence interval based on the statistic

$$T = \frac{(\overline{Y} - \theta\overline{X})}{\sqrt{(\frac{1}{n_2} + \frac{\theta^2}{n_1})S^2}},$$

where $\overline{X} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i$, $\overline{Y} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_j$ and $S^2 = \frac{\sum_{i=1}^{n_1}(X_i-\overline{X})^2 + \sum_{j=1}^{n_2}(Y_j-\overline{Y})^2}{n_1+n_2}$. It is obvious that $\sqrt{\frac{(n_1+n_2-2)}{n_1+n_2}}\, T$ has the Student's $t$ distribution with $(n_1+n_2-2)$ degrees of freedom. Solving the inequality

$$\begin{aligned} &\{\theta : \sqrt{\tfrac{(n_1+n_2-2)}{n_1+n_2}}\,|T| < t_{1-\frac{\alpha}{2}}\} \\ &= \{\theta : |\overline{y} - \theta\overline{x}| \le t_{1-\frac{\alpha}{2}}\sqrt{\frac{(n_1+n_2)(1/n_2+\theta^2/n_1)s^2}{n_1+n_2-2}}\}, \end{aligned}$$

where $t_{1-\frac{\alpha}{2}}$ is the $(1-\frac{\alpha}{2})th$ quantile of the $t$ distribution, the exact $1-\alpha$ confidence interval for $\theta$ will be obtained.

On the other hand, if variances are related to the means, such as $\sigma_i^2 = (c+\mu_i)^k\sigma^2$ with $c + \mu_i > 0$, $i = 1, 2$ and $k$ is known, Cox (1985) provided a interval estimate based on the statistic

$$T^* = \frac{(N-2)l^2}{aS^*},$$

with $l = (c+\mu_2)(c+\overline{x}) - (c+\mu_1)(c+\overline{y})$, $a = (c+\mu_2)^2\frac{(c+\mu_1)^k}{n_1} + (c+\mu_1)^2\frac{(c+\mu_2)^k}{n_2}$ and $\frac{S^*}{\sigma^2} = \frac{\sum_{i=1}^{n_1}(X_i-\overline{X})^2}{\sigma_1^2} + \frac{\sum_{i=1}^{n_2}(Y_i-\overline{Y})^2}{\sigma_2^2}$. It is noted that $T^*$ has the Fisher-Snedecor's $F$ distribution with 1 and $n_1+n_2-2$ degrees of freedom. Define $\theta = \frac{c+\mu_2}{c+\mu_1}$, the $100(1-\alpha)\%$ confidence interval for $\theta$ is obtained by solving the inequality

$$\{\theta : \frac{(N-2)l^2}{aS^*} \leq F_{1-\alpha}(1, n_1+n_2-2)\}.$$

For $c = 0$ and $k = 2$, the $100(1-\alpha)\%$ confidence interval for $\theta = \frac{\mu_2}{\mu_1}$ is based on solving the quadratic inequality

$$\{\theta : (\overline{y}-\theta\overline{x})^2 \leq \frac{F_{1-\alpha}(1, n_1+n_2-2)}{n_1+n_2-2}[\frac{s_2^2}{n_1/(n_1+n_2)} + \theta^2\frac{s_1^2}{n_2/(n_1+n_2)}]\},$$

with $s_1^2 = \frac{1}{n_1}\sum_{i=1}^{n_1}(x_i-\overline{x})^2$ and $s_2^2 = \frac{1}{n_2}\sum_{i=1}^{n_2}(y_i-\overline{y})^2$, respectively.

In this article, we propose two different exact approaches based on generalized $p$-values and generalized confidence intervals, as defined by Tsui and Weerahandi (1989), Weerahandi (1993), respectively, to construct confidence intervals for the ratio of means of two normal populations under heteroscedasticity. The lack of exact confidence intervals in many applications can be attributed to the statistical problems involving nuisance parameters. The possibility of exact confidence interval can be achieved by extending the definition of confidence interval. To generalize the definition of confidence intervals, first examine the properties of interval estimates obtained by the conventional definition. To fix ideas, consider a random sample $\boldsymbol{X}=(X_1, X_2, \cdots, X_n)$ from a distribution with an unknown parameter $\theta$. Let $\mathrm{A}(\boldsymbol{X})$ and $\mathrm{B}(\boldsymbol{X})$ be two statistics satisfying the equation

$$P[\mathrm{A}(\boldsymbol{X}) \leq \theta \leq \mathrm{B}(\boldsymbol{X})] = \gamma,$$

where $\gamma$ is a prespecified constant between 0 and 1. Let $a$=A($\boldsymbol{x}$) and $b$=B($\boldsymbol{x}$) be the observed values of the two statistics, then, in the commonly used terminology, $[a, b]$ is a confidence interval for $\theta$ with the confidence coefficient $\gamma$. For example, if $\gamma = 0.95$, then the interval $[a, b]$ obtained in this manner is a 95% confidence interval. This approach to constructing interval estimates is conceptually simple and easy to implement, but in most applications involving nuisance parameters it is not easy or impossible to fnd A($\boldsymbol{x}$) and B($\boldsymbol{x}$) so as to satisfy the above equation for all possible values of the nuisance parameters. The idea in generalized confidence intervals is to make this possible by making probability statements relative to the observed sample, as done in Bayesian and nonparametric methods. In other words, we allow the functions A(·) and B(·) to depend not only on the observable random vector $\boldsymbol{X}$ but also on the observed data $\boldsymbol{x}_{obs}$.

In this article, we propose two different exact generalized approaches based on generalized $p$-values and generalized confidence intervals to solve the well-known Fieller-Creasy problem, which is widely used in many important research areas such as bioassay and bioequivalence. Under homogeneous case, Fieller's solution gives exact coverage probability for all parameters. Unfortunately, in the presence of serious heteroscedasticity, the methods under the restriction of identical variance can not yield decent confidence intervals. Through the proposed methods in this article, an exact $1-\alpha$ generalized confidence intervals for the ratio of two means can be obtained under unequal variances and unequal sample sizes. According to our findings, the existing procedures ignoring the mild heteroscedasticity will perform well. However, they will perform very poorly in the situation in which serious heteroscedasticity is present. Thus our proposed methods are very valuable in practice, especially when the two variances are quite different.

（四）、參考文獻

Berger R. L. and Hsu J. C. (1996), "Bioequivalence Trials, Intersection Union Tests and Equivalence Confidence Sets," *Statistical Science,* 11(4), 283-319.

Chow, S. C. and Liu, J. P. (1992), "Design and Analysis of Bioavailability and Bioequivalence Studies," *New York: Marcel Dekker, Inc.*

Cox, C. P. (1985), "Interval Estimates for the Ratio of the Means of Two Normal Populatins with Variances Related to Means," *Biometrics,* 41, 261-265.

Fieller, E. C. (1944), "A Fundamental Formula in the Statistics of Biological Assay, and Some Applications," *Quarterly Journal of Pharmacy and Pharmacology,* 17, 117-123.

Fieller, E. C. (1954), "Some Problems in Interval Estimation," *J. Roy. Statist. Soc,* Series B 16, 175-185.

Finney, D. J. (1978), "Statistical Method in Biological Assay," *London: Griffin,* 3rd ed.

Hwang, J. T. (1995), "Fieller's Problems and Resampling Techniques," *Statistica Sinica,* 5(1), 161-171.

Jarvis, M. J., Tunstall-Pedoe, H., Feyerabend, C., Vesey, C., and Saloojee, Y. (1987), "Comparison of Tests Used to Distinguish Smokers from Nonsmokers," *American Journal of Public Health,* 77, 1435-1438.

Koschat, M. A. (1987), "A Characterization of the Fieller Solution," *The Annals of Statistics,* Vol. 15, No. 1, 462-468.

Pagano, M. and Gauvreau, K. (1993), "Principles of Biostatistics," *California: Cuxbury.*

Rao, C. R. (1973), "Linear Statistical Inference and Its Applications," Second Edition, *New York: Wiley.*

Tsui, K. W., and Weerahandi, S. (1989), "Generalized *p*-Values in Significance Testing of Hypotheses in the Presence of Nuisance Parameters," *Journal of American Statistical Association,* 84, 602-607.

Weerahandi, S. (1993), "Generalized Confidence Intervals," *Journal of American Statistical Association,* 88, 899-905.

（五）、計畫成果自評

本研究成果乃計畫所提的一部分，刊登的期刊是SCI統計期刊當中不錯的雜誌，相當值得。

## 三、On the Inverse of the Autocorrelation Matrix for an AR (p) Process

此乃與博士生王仁聖及林宗儀教授（目前任教於東海大學統計系）合作的文章。本文已發表於Journal of Chinese Statistical Association。這是個CIS的期刊。其中英文之摘要如下。

（一）、中文摘要

假設 $C_N = (\rho_{|t-s|}), \quad t, s = 1, ..., N,$ 是穩定p次自我迴歸向量 $X_N = (x_1, ..., x_N)'$ 的自我相關矩陣，其中 $N \geq p$。本文在沒有分配的假設下導出直接且容易寫程式的 反矩陣，稱之為 $C_N^{-1}$。這個 $C_N^{-1}$ 的形成有助於研究時間序列、多變量線性模型 MANOVA 及成長曲線模型當中誤差項具自我迴歸，這可簡化參數統計的計算程序。AR(2)、AR(3)、及AR(4)的示範也列舉於本文中。

（二）、英文摘要

Let $\boldsymbol{C}_N = (\rho_{|t-s|})$, $t, s = 1, \ldots, N$, be the autocorrelation matrix of a vector $\boldsymbol{X}_N = (x_1, \ldots, x_N)'$ from a stationary autoregressive process of order $p$, where $N \geq p$. In this paper, we derive a general formula without any distributional assumption, which is easy to program for directly solving the inverse of $\boldsymbol{C}_N$, denoted by $\boldsymbol{C}_N^{-1}$. The formulation of $\boldsymbol{C}_N^{-1}$ is useful in time series analysis, general linear model, multivariate linear model, MANOVA and growth curves model with high order autoregressive errors, and can simplify the computational procedure of parameter estimation. Some demonstrations of $\boldsymbol{C}_N^{-1}$ including AR(2), AR(3), and AR(4) are given.

（三）、報告內容

In time series model, let $\{x_t\}$ be the observations on a variate at time $t$, for $t = 1, 2, \ldots$. It is assumed that the underlying model is a stationary autoregressive process of order $p$ (AR($p$)),

i.e.,

$$x_t - \phi_1 x_{t-1} - \cdots - \phi_p x_{t-p} = z_t,$$

with the following assumptions:

$$\rho_{|t-s|} = \frac{\mathrm{E}(x_t x_s)}{\sigma_x^2}.$$

Similar to Siddiqui (1958), let

$$\begin{aligned}
\boldsymbol{X}_N &= (x_1, \ldots, x_N)', \\
\boldsymbol{X}_N^* &= (x_N, \ldots, x_1)', \\
\boldsymbol{Y}_N &= (x_1, \ldots, x_p, z_{p+1}, \ldots, z_{N-1}, z_N)', \\
\boldsymbol{Y}_N^* &= (z_N, z_{N-1}, \ldots, z_{p+1}, x_p, \ldots, x_1)',
\end{aligned}$$

then $\mathrm{E}(\boldsymbol{X}_N) = \mathbf{0}$, $\mathrm{E}(\boldsymbol{Y}_\mathbf{N}) = \mathbf{0}$, and $\rho_0 = 1$, $\rho_{-t} = \rho_t$.

Let $\boldsymbol{\Sigma}_N$ stand for the covariance matrix of $\boldsymbol{X}_N$, i.e., $\mathrm{Cov}(\boldsymbol{X}_N) = \boldsymbol{\Sigma}_N$, and $\boldsymbol{\Sigma}_N^*$ for $\mathrm{Cov}(\boldsymbol{X}_N^*)$, then by (1.2), $\boldsymbol{\Sigma}_N \equiv \sigma_x^2 \boldsymbol{C}_N = \boldsymbol{\Sigma}_N^*$, where $\boldsymbol{C}_N$ is the autocorrelation matrix, that is, $\boldsymbol{C}_N = (\rho_{|t-s|})$, $t, s = 1, \ldots, N$, and $\rho_i = \sum_{j=1}^{p} \phi_j \rho_{|i-j|}$, for $i \geq 1$. Siddiqui (1958) noted that $\boldsymbol{\Sigma}_N$ is persymmetric (or doubly symmetric), and so is $\boldsymbol{\Sigma}_N^{-1}$.

By ( A.3 ), $\mathrm{Cov}(\boldsymbol{Y}_N) = \sigma_z^2 \left( \begin{array}{c|c} \boldsymbol{\Omega}_p & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{I}_{N-p} \end{array} \right)$,
where

$$\sigma_z^2 \boldsymbol{\Omega}_p = \mathrm{Cov}(\boldsymbol{X}_p) = \sigma_x^2 \boldsymbol{C}_p.$$

Wise (1955) proposed a method to obtain $\sigma_z^2 \boldsymbol{\Sigma}_N^{-1}$, where $N > 2p$, based on the spectral density function and under the assumption of $\sigma_z^2$ being equal to $\sigma_x^2$. Siddiqui (1958) derived $\boldsymbol{\Sigma}_N^{-1}$, where $N > 2p$, from the properties of persymmetric matrix under the normality assumption of $\boldsymbol{X}_N$ with $\sigma_z^2$ being one. Galbraith and Galbraith (1974) provided an explicit formula for $\sigma_z^2 \boldsymbol{\Sigma}_N^{-1}$, where $N \geq p$, given the distributions of $z_t's$ are i.i.d. Gaussion with mean zero and variance $\sigma_z^2$ for all $t$. Box, Jenkins and Reinsel (1994, p. 296) also gave a somewhat improved version of Siddiqui's method to find $\boldsymbol{\Sigma}_N^{-1}$ iteratively, which is not efficient when $N$ is much bigger than $p$.

In this paper, we give a general derivation, alternative to Galbraith and Galbraith (1974), of obtaining $\boldsymbol{C}_N^{-1}$, and its determinant when $N \geq p$, by using simple matrix algebra and without any distributional assumption. Once $\boldsymbol{C}_N^{-1}$ is obtained, $\boldsymbol{\Sigma}_N^{-1} \equiv \frac{1}{\sigma_x^2} \boldsymbol{C}_N^{-1}$ which is quite useful in time series analysis, general linear model, multivariate linear model, multivariate analysis of variance (MANOVA), and growth curves model as well as other areas where the dependence of the observed variables has an AR($p$) structure. This will facilitate the parameter estimation and prediction of future observations. [ cf. Rao (1967), Lee (1988) and Chib (1993) ].

（四）、參考文獻


Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1994). Time Series Analysis Forecasting and Control, 3nd edition. San Francisco: Holden-Day.

Chib, S. (1993). "Bayes Regression with Autoregressive Errors: A Gibbs Sampling Approach," *Journal of Econometrics.*, 58, 275-294.

Galbraith R. F. and Galbraith J. I. (1974). "On the inverse of some patterned matrices arising in the theory of stationary time series," *J. Appl. Prob.*, 11, 63-71.

Lee, J.C. (1988). "Prediction and estimation of growth curves with special covariance structures," *J. Am. Statist. Assoc.*, 83, 432-440.

Rao, C.R. (1967). "Least squares theory using an estimated dispersion matrix and its application to measurement of signals," *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability*, 1, 355-372.

Siddiqui, M.M. (1958). "On the inversion of the sample covariance matrix in a stationary autoregressive process," *Ann. Math. Statist.*, 29, 585-588.

Wise, J. (1955). "The autocorrelation function and the spectral density function," *Biometrika*, 42, 151-159.


（五）、計畫成果自評

本研究發表於Journal of Chinese Statistical Association。這是個CIS的期刊，所得的方法非常有助於time series、multivariate linear model、及growth curve model等參數估計。