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Research paper

Application of modified nonlinear storage function on runoff estimation

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Abstract

This study proposed a modified nonlinear storage function runoff model to take into account the storage hysteresis effect, in which there exists difference of the storage–discharge relationship between the rising and recession limb. Since the modified storage function runoff has seven parameter, a parameter-calibration method, which combines the genetic algorithm with the least square criterion. For model calibration and validation, twenty rainfall–runoff events (1968–2005) recorded at Wudu gage in Keelung River in northern Taiwan were used in the study. The results of model validation reveal that the modified storage function runoff model not only produces the realistic storage–discharge relationship, but also provides a good estimation of the runoff.
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Keywords: Storage function runoff model; Storage hysteresis effect; Genetic algorithm

1. Introduction

The runoff estimation and prediction play an important role in the design and performance evaluation of hydraulic structures and water resource management. According to the transformation relationship between rainfall and runoff, the rainfall-runoff models commonly used are classified into three types: (1) the linear model, (2) the semi-linear model, and (3) the nonlinear model. For example, the unit hydrograph is a typical linear model, and the tank model is a semi-linear model. Of the above three types of models, the storage function model (SFRM) not only describes more realistic rainfall and runoff relationships, but also embeds a simple underlying equation $s = kq^p$ for storage (s) and runoff (q) (e.g., Kimura, 1961; Aoki et al., 1976; Zbigniew and Jaroskaw, 1986; Sugiyama et al., 1997, 1999; Sujono et al., 2003). In addition to parameters k and p, the SFRM has two other parameters, namely, the lag time T_l and runoff coefficient f_r. Park et al. (1999) analyzed runoff characteristics of three small watersheds in the Su-Young River Basin in Korea using three rainfall-runoff models, i.e. the storage function method, liner reservoir cascade model, and the discrete linear input–output model. Their study concluded that the storage function model is the most accurate among the three models. In literature, a number of forms of storage functions can be found (e.g. Prasad, 1967; Kimura, 1975).

In the SFRM, the storage–discharge relationship for the SFRM is a single-valued monotonic function. Thus, of five parameters of the SFRM, the parameters k and p are commonly calibrated using the regression analysis (e.g. Prasad, 1967; Kimura, 1975; Sugiyama et al., 1997). The runoff coefficient f_r is equal to a ratio of runoff volume to that of rainfall. For the lag time T_l and cumulative saturated rainfall R_{sa}, a trial-and-error procedure is applied with an objective function for minimizing the mean absolute error of estimated runoff. Note that the parameters k, p, and f_r should be calibrated under the specific T_l and R_{sa}. Hence, the process of calibrating SFRM parameters is complicated and inflexible. In addition, due to storage hysteresis effect which means that the storage–discharge relation is a loop-shaped function, different storages exist on the rising and recession limbs of a hydrograph with the same discharge (Kimura, 1975). Therefore, the objectives of this study are to modify the nonlinear storage

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function by considering the storage hysteresis effect, and to develop a corresponding parameter-calibration method. To demonstrate the proposed model, twenty-four rainfall-runoff events recorded at the Wudu station in the Keelung River of northern Taiwan are used for the parameter calibration and model validation.

2. Storage function runoff model

2.1. Brief concept

The storage function runoff model (SFRM) with a loss mechanism was developed by Kimura (1961). In the SFRM, relationship between the storage (s) and runoff (q) is defined as:

$$s = kq^p \quad (1)$$

where k and p are coefficients representing watershed characteristics. As $p = 1$, Eq. (1) reduces to the linear reservoir cascade model. In the runoff estimation, the SFRM incorporates the modified Plus method, which is derived from the continuity equation, as (Kimura, 1961):

$$\begin{aligned} r_e(t - T_l) - q(t) &= \frac{ds}{dt} \\ r_e(t - T_l) &= \begin{cases} f_r \times r(t - T_l), & \text{if } \sum r(t - T_l) \le R_{sa} \\ r(t - T_l), & \text{if } \sum r(t - T_l) > R_{sa} \end{cases} \end{aligned} \quad (2)$$

where $r(t)$ and $q(t)$, respectively, denote the inflow and outflow at time t; f_r is the runoff ratio; T_l is the lag time; and R_{sa} stands for the cumulative saturated rainfall. Note that the outflow rate $q(t)$ (mm/h) in Eq. (2) is calculated by:

$$q(t) = \frac{Q(t)}{A} \times 3.6 \quad (3)$$

where A is the catchment area (km^2); and $Q(t)$ is the discharge (m^3/s).

2.2. Traditional storage–discharge relationship

At the beginning of a rainstorm event, the rainfall mostly infiltrates into the ground, and the storage is proportional to the discharge. When the soil is saturated, the rainfall mostly becomes surface runoff so that the storage is inversely related to the runoff. Thus, the storage–discharge relationship has a loop-like shape, which is called the storage hysteresis effect (see Fig. 1). In Fig. 1, it can be seen the storage has different magnitude for the same discharge on the rising and recession limbs. However, Eq. (1) assumes that the storage–discharge relationship is a one-to-one function, so that it hardly describes the looped-shaped storage–discharge relationship attributed to the storage hysteresis effect. Hence, Prasad (1967) proposed a relationship between storage and discharge as:

$$s = k_1 q^p + k_2 \frac{dq}{dt} \quad (4)$$

in which k_1 and k_2 are watershed characteristics. The added term $k_2(dq/dt)$ in Eq. (4) differentiates the storage–discharge

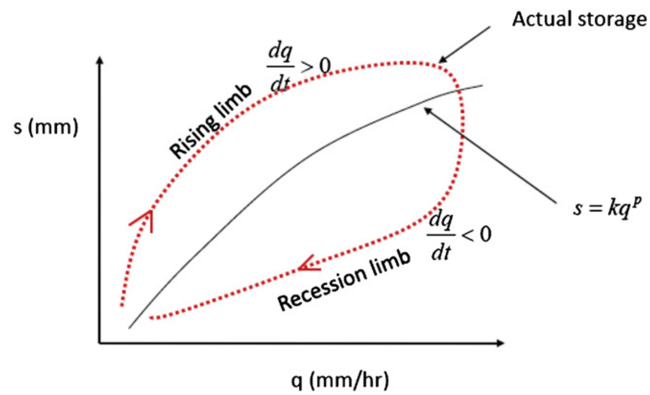

Fig. 1. Graphical illustration of storage–discharge relationship.

relationship on the rising and recession limbs, which cannot be captured by Eq. (1). Therefore, this study adopts the Prasad storage–discharge relation in the SFRM resulting in a six-parameter rainfall–runoff model, called the PSFRM herein with the parameters f_r, p, k_1, k_2, T_l, and R_{sa}.

2.3. Development of the modified storage function runoff model

2.3.1. Basic concept

Although the Prasad model can capture the storage hysteresis effect in the watershed runoff modeling, it has a limitation of using a constant ratio of difference in discharge on the rising and recession limbs. In reality, the slope of storage–discharge relationship on the rising limb could be different from that on the recession limb, meaning the different coefficients could be associated with the term dq/dt on the rising and recession limbs. Therefore, a modified storage–discharge relationship is proposed based on the Prasad storage–discharge function as:

$$\begin{aligned} s &= k_1 q^p + k_2 \times I_2 \times \frac{dq}{dt} + k_3 \\ &\times I_3 \frac{dq}{dt} \begin{cases} \text{if } \dfrac{dq}{dt} \ge 0, \ I_2 = 1, \ I_3 = 0 \ \text{(rising limb)} \\ \text{if } \dfrac{dq}{dt} < 0, \ I_2 = 0, \ I_3 = 1 \ \text{(recession limb)} \end{cases} \end{aligned} \quad (5)$$

where k_1, k_2, k_3, and p are parameters previously defined; and I_2, and I_3 are indicator variables. The physical meanings of k_1 and p resemble those of the SFRM and PSFRM. In this study, the Kimura storage function relation, Eq. (1), is replaced by Eq. (5), resulting in a seven-parameter rainfall–runoff model, i.e. parameters f_r, T_l, k_1, k_2, k_3, p, and R_{sa}, denoted herein as the MPSFRM. When $k_2 = k_3 = 0$, Eq. (5) reduces to a linear storage function. Furthermore, the SFRM and PSFRM are the special cases of the MPSFRM.

2.3.2. Sensitivity analysis for model parameters

Although the storage–discharge relationship focuses on the behavior between the storage and runoff, it is expected to have influences on runoff characteristics, i.e. the peak discharge Q_p, time-to-the peak T_p, total runoff volume V, runoff volume on

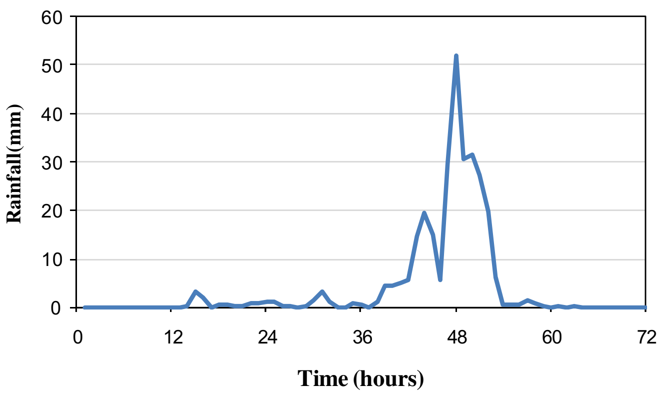

Fig. 2. Rainfall hyetograph used in sensitivity analysis of MPSFRM.

the rising and recession limb V_u and V_d, respectively. Hence, the sensitivity analysis for model parameters is performed and the results could be used as the basis to develop the proposed MPSFRM. In detail, the sensitivity analysis is carried out using the various ratios of model parameters for estimating runoff; thus, the associated ratio of runoff characteristics r_ϕ can be calculated by Eq. (6), for a particular rainstorm hyetograph (see Fig. 2).

$$\begin{aligned} r_\phi &= \frac{\phi_i - \phi_0}{\phi_0} \\ \phi_0 &= f(\theta_0) \\ \phi_i &= f[\theta_0(1+\rho)] \end{aligned} \quad (6)$$

where ρ is the varying ratio of model parameters, which are 0.1, 0.2, 0.3, 0.4 and 0.5; φ_0 denotes the runoff characteristics estimated from an assigned nominal value of model parameters θ_0; and φ_1 is the estimated runoff characteristics with model parameters $\theta_0(1+\rho)$. Since the lag time T_l is well known to mainly produce the delay effect on the hydrograph (Sonu, 1989), it can be excluded from the sensitivity analysis. From the analysis, the sensitivity of runoff characteristics due to various model parameter values can be assessed.

The nominal values of model parameters in Eq. (6) used in the sensitivity analysis are hypothesized as $f_r = 0.5$, $T_l = 0$, $p = 0.5$, $k_1 = k_2 = k_3 = 10$, and $R_{sa} = 300$. Fig. 3 shows the comparison of varying ratios of runoff characteristics with respect to ratios of model parameters. In view of Fig. 3, the peak discharge Q_p is related positively with the parameter f_r, whereas negatively with the remaining parameters. For the total runoff volume V, increasing parameters p, k_1, and R_{sa} leads to decrease in the runoff volume. On the contrary, the runoff volume increases with ratios of parameters f_r and k_3. Of the runoff volumes on the rising and recession limbs, V_u and V_d, increasing k_2 and k_3, respectively, can result in more V_u and V_d.

In summary, the model parameters k_2 and k_3 could have different effect on runoff characteristics, especially for the runoff volume on the rising and recession limbs. As a result, it is reasonable to hypothesize that there exist a different coefficient associated with dq/dt in the Prasad storage–discharge relation for the rising and recession limbs.

2.4. Parameter-calibration method

Since three storage function runoff models (SFRM, PSFRM, and MPSFRM) have five and more parameters, the genetic algorithm (GA) method, which is widely used in the parameter-calibration of conceptual rainfall-runoff models with the multi-parameter (Wang, 1991; Franchini, 1996), would be employed in the parameter-calibration of the three models. The GA

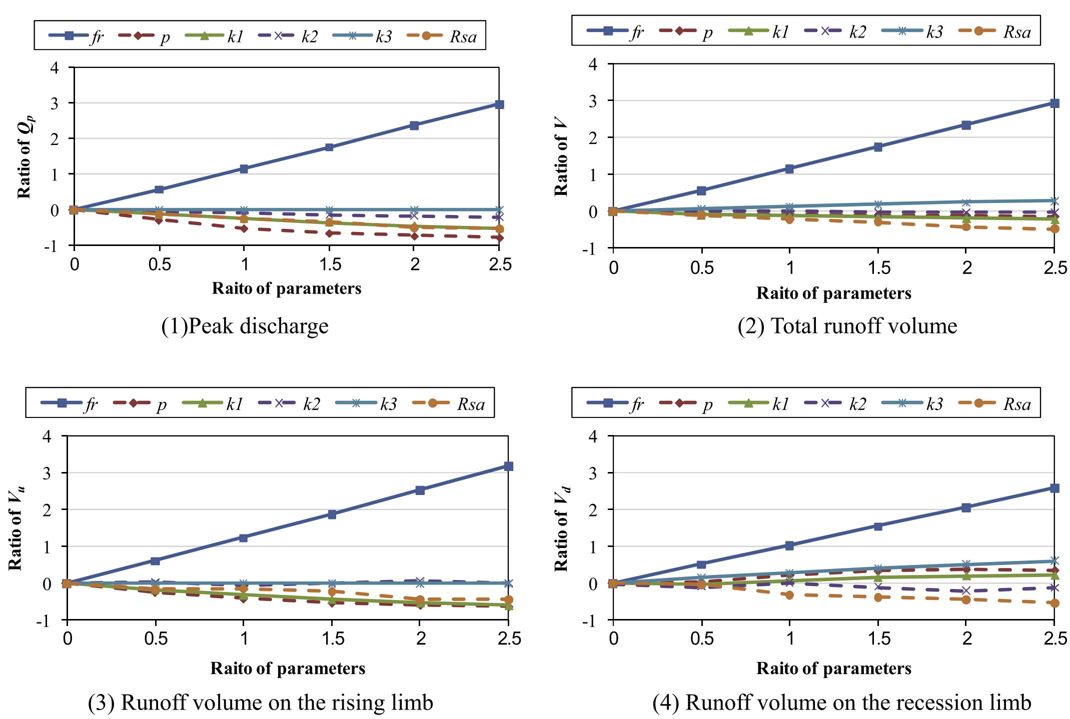

Fig. 3. Comparison of runoff characteristics with various ratios of model parameters.

method should be associated with a fitness function, also named as the objective function, to find the optimal model parameters. In general, the objective functions for the parameter calibration of hydrologic models can base on four criteria: (1) a good agreement between the average simulated and observed runoff volumes; (2) a good overall agreement of the shape of the hydrographs; (3) a good agreement of the peak flow with respect to time, and volumes; and (4) a good agreement for low flows (Madsen, 2000). Based on the above four criteria, the weighting mean square error is used as the objective function F_{obj} in the parameter-calibration of the three models (Madsen et al., 2002):

$$F_{\text{obj}} = \min\left\{\frac{1}{n_{\text{f}}}\left[\sum_{t=1}^{n_{\text{f}}} w(t) \times (q_{\text{obs}}(t) - q_{\text{est}}(t))^2\right]^{0.5}\right\} \qquad w(t) = \frac{\overline{q}_{\text{obs}} + q_{\text{obs}}(t)}{2\overline{q}_{\text{obs}}} \tag{7}$$

where n_{f} is the runoff period; $q_{\text{obs}}(t)$ and $q_{\text{est}}(t)$ are the observed and estimated discharges at time t; and $\overline{q}_{\text{obs}}$ is the mean of observed discharge. The objective function F_{obj} is used as the fitness function in the GA. Since Eq. (5) is a nonlinear function, taking the log transform on to both sides of Eq. (5) yields a linear relation as shown in Eq. (8). Then, the optimal p and k_1 can be obtained by the least square (LS) method conditioned on the parameters f_{r}, T_{l}, R_{sa} and k_2 produced by GA.

$$\log\left[s - \left(I_2 k_2 \frac{\mathrm{d}q}{\mathrm{d}t} + I_3 k_3 \frac{\mathrm{d}q}{\mathrm{d}t}\right)\right] = \log(k_1) + p \times \log(q) \tag{8}$$

The proposed parameter-calibration method based on GA and LS method is outlined below:

Step[1] : Define the range of model parameters f_{r}, k_2, k_3, T_{l} and R_{sa}.
Step[2] : Produce model parameters f_{r}, k_2, k_3, T_{l} and R_{sa} by using the GA.
Step[3] : Calculate the storage using Eq. (2) with generated parameters f_{r}, k_2, k_3, T_{l} and R_{sa} at Step[2].
Step[4] : Calculate parameters p and k_1 using LS method.
Step[5] : Estimate runoff and calculate the objective function with trial values of model parameters obtained at Step [2] and Step[4].
Step[6] : If the objective function is less than the convergence criteria, and the optimal value of model parameters are determined; otherwise, return to Step[2].

3. Model validation

The model validation is made to compare estimated runoff by the three models, SFRM, PSFRM, and MPSFRM. In general, the validation of rainfall–runoff model can be carried out visually and by calculating performance indices. The performance indices commonly used herein for evaluating rainfall–runoff models are listed below:

1. Efficiency coefficient (CE) (Nash and Sutcliffe, 1970)

$$\text{CE} = 1 - \frac{\sum(q_{\text{obs}}(t) - q_{\text{est}}(t))^2}{\sum(q_{\text{obs}}(t) - \overline{q}_{\text{obs}})^2} \tag{9}$$

If CE approaches to 1, this means that the estimated runoff hydrograph can fit the observed one very well.

2. Error of peak discharge (EQP)

$$\text{EQP} = \frac{(q_{\text{p,est}} - q_{\text{p,obs}})}{q_{\text{p,obs}}} \times 100\% \tag{10}$$

where $q_{\text{p,est}}$ and $q_{\text{p,obs}}$ are estimated and observed peak discharges, respectively.

3. Error of time to peak (ETP)

$$\text{ETP} = T_{\text{p,est}} - T_{\text{p,obs}} \tag{11}$$

where $T_{\text{p,obs}}$ is the time to observed peak discharge; and $T_{\text{p,est}}$ is the time to estimated peak discharge.

4. Root mean square error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{n_{\text{f}}}\left[\sum_{t=1}^{n_{\text{f}}}(q_{\text{obs}}(t) - q_{\text{est}}(t))^2\right]} \tag{12}$$

Note that indices EQP and ETP could be negative value, which indicate that estimated value is less than observed data.

In the study, the estimated runoff hydrograph and storage–discharge relationship are not only visually compared, but also used to calculate performance indices for evaluating SFRM, PSFRM, and MPSFRM. In addition, to avoid sampling error in the model performance assessment, cross-validation method is applied in that numerous rainfall–runoff events are selected for the parameter-calibration (called calibration events), and the remaining event are used for the model validation (called validation events).

Table 1
Geographical information of flow-gauge and rainfall-gauges in study area Wudu sub-basin.

Flow-gauge	Location		Area (km^2)	Rain-gauge	Location		Elevation (m)
	TM_X	TM_Y			TM_X	TM_Y	
Wudu	319141	2774866	204.41	Huo-Shao-Liao	324813	2764092	380
				Rang-Fu	330186	2778442	101
				Wudu	319447	2774911	16

4. Results and discussion

For illustration, the study area is chosen to be Wudu sub-basin in the Keelung River watershed of northern Taiwan in which observations at three rainfall-gauges and one flow-gauge are available, geographical information of the Wudu basin is shown in Table 1. Twenty rainstorm events were selected and their relevant rainfall and runoff information are summarized in Table 2.

4.1. Parameter calibration

According to the parameter-calibration method described previously, parameters f_r, k_2, k_3, T_l and R_{sa} are calibrated by GA combined with LS method. The range of SFRM, PSFRM and MPSFRM parameters f_r, k_2, k_3, T_l and R_{sa} are specified in advance. Since the runoff coefficient f_r is generally less than or equal to 1, the associated range is supposed to be between 0.1 and 1.0. For the lag time T_l, the range is assumed from 0 to 5 h. As for the coefficients associated with the term dq/dt, k_2 and k_3, are regarded between 0.1 and 50. Since the cumulative saturated rainfall R_{sa} depends on the antecedent moisture conditions of ground and rainfall depth, a wider range of R_{sa}, from 50 mm to 1000 mm, is adopted.

Performing the proposed parameter-calibration method along with the assumed ranges of model parameters, the optimal parameters of the three models for each of twenty rainstorm events are listed in Table 3. By comparing the objective function, for the most of rainstorm events, the values of F_{obj} for the MPSFRM are the lowest, followed by the SFRM and PSFRM. This means that the MPSFRM can perform the runoff estimation better than the SFRM and PSFRM. It is also observed the parameters R_{sa} for the three models are substantially different, and this could be caused by the antecedent moisture conditions of ground. Also, in Table 3, the k_2 and k_3 for the MPSFRM have different statistical properties. Particularly, the 95% confidence interval of k_2 is greater than that of k_3. It can be said that the coefficient associated with dq/dt for the rising limb has higher uncertainty than that for the recession limb. Therefore, it is advisable to separate coefficients for dq/dt for the storage–discharge relationship.

The parameters of SFRM, PSFRM, and MPSFRM are related to the watershed features. However, as the model parameters are calibrated using the observations, the sample variation in the observations probably contributes to the variation in parameters (e.g. Chaubey et al., 1999; Yu et al., 2001; Jin et al., 2009). Hence, the mean values of optimal model parameters can probably represent the average features of watershed. Thereby, the average of parameters f_r, p, k_1 for the three models, which are approximately 0.7, 0.8, and 35 (see Table 3), could be regarded as the regional optimal values.

Table 2
Rainstorm events used for model validation.

No of event	Date	Peak discharge (cm)	Max rainfall intensity (mm/h)	Rainfall depth (mm)
1	19680908–19681002	1150.0	53.3	695.2
2	19700905–19700908	564.0	26.2	300.7
3	19710921–19710924	1030.0	52.0	297.7
4	19720816–19720818	708.0	18.5	211.2
5	19781012–19781010	1370.0	40.2	603.6
6	19790814–19790816	1030.0	21.0	303.4
7	19810719–19810720	1250.0	22.1	227.5
8	19820811–19820812	682.0	37.8	276.4
9	19860822–19860815	690.0	60.0	446.0
10	19871022–19871028	1980.0	77.0	1755.2
11	19880928–19881003	734.0	40.5	698.5
12	19890728–19890731	946.0	36.1	461.7
13	19900903–19900904	857.0	52.5	239.0
14	19911028–19911031	583.0	18.6	262.9
15	19970817–19970820	1040.0	49.3	351.6
16	19981015–19981017	1050.0	56.0	571.6
17	20020709–20020711	608.0	43.9	162.0
18	20010916–20010919	2040.0	111.1	977.7
19	20041024–20041027	1574.5	60.9	356.3
20	20050831–20050902	580.8	26.0	283.3

4.2. Comparison of estimated runoff with single-event optimal parameters

Fig. 4 lists the performance indices for estimating runoff by SFRM, PSFRM, and MPSFRM with the event-based optimal parameters. It can be seen that the performance indices of estimated runoff by the three models vary with the rainstorm events. In average, the efficiency coefficient (CE) for the MPSFRM (0.934) is greater than those for SFRM (0.913) and PSFRM (0.927). Correspondingly, the MPSFRM has a less average of RMSE (63.1) than the SFRM (75.4) and PSFRM (68.3). Although the three models have the similar error of peak discharge, the corresponding 95% confidence interval for the MPSFRM is narrower than those for the SFRM and PSFRM. Since the 95% confidence interval represents the degree of uncertainty for the model output, a narrower 95% confidence interval is associated with less uncertainty. This implies that the uncertainty of runoff estimated by the MPSFRM is lower than those by the SFRM and PSFRM.

In view of Fig. 4, the SFRM, PSFRM and MPSFRM have significantly different values of performance indices, especially for events, EV5, EV7, EV12, and EV18. Therefore, a visual comparison of the runoff hydrograph for the above four events estimated by the three models is shown in Fig. 5. It can be observed that the runoff hydrographs, especially on the recession limb, estimated by the PSFRM and MPSFRM match closer to observed line, and the SFRM overestimates the peak discharge for four rainstorm events. Although the PSFRM and MPSFRM also overestimate the peak discharge, their RMSE are less than one by the SFRM.

4.3. Comparison of estimated storage with event-based optimal parameters

In this section, the comparison of storage estimated by SFRM, PSFRM and MPSFRM are made using the event-based optimal parameters. The storage can be calculated using the continuity equation as:

Table 3
Summary of event-based optimal parameters.

(1) SFRM

Event	f_r	p	k_1	T_l	R_{sa}	F_{obj}
1	0.299	1.106	6.439	1	318.591	94.467
2	0.938	0.399	62.970	2	531.490	27.335
3	0.239	1.287	2.805	0	84.009	42.715
4	0.951	1.045	2.953	4	113.807	59.585
5	0.325	1.475	1.049	1	900.344	152.625
6	1.000	0.290	43.386	4	63.373	144.031
7	0.266	0.888	4.244	2	133.551	73.621
8	0.850	1.006	5.527	2	811.824	58.234
9	1.000	0.543	11.169	5	195.355	125.314
10	0.880	0.330	68.003	2	359.952	31.237
11	0.964	0.540	109.131	0	414.410	189.993
12	0.732	0.174	74.120	0	953.981	88.435
13	0.886	0.494	36.310	3	257.191	48.552
14	0.866	0.496	35.905	3	255.071	49.083
15	1.000	1.060	4.072	3	156.085	71.189
16	0.114	0.836	11.429	1	163.580	62.379
17	0.736	1.021	5.540	2	345.753	72.081
18	0.786	0.329	81.911	1	773.433	247.503
19	0.252	0.950	4.190	2	149.146	59.895
20	0.992	0.586	38.721	0	340.049	53.833
Statistics						
Mean	0.704	0.743	30.494	1.900	366.050	87.605
Standard deviation	0.310	0.360	32.033	1.411	274.150	55.527
95% Lower limit	0.155	0.212	1.625	0.000	70.133	28.613
95% Upper limit	1.000	1.445	104.748	4.839	945.344	238.243

(2) PSFRM

Event	f_r	p	k_1	k_2	T_l	R_{sa}	F_{obj}
1	0.193	1.110	6.283	4.485	1	257.032	95.171
2	0.931	0.391	64.323	1.518	2	317.401	25.155
3	0.867	0.667	17.018	5.095	0	425.627	59.459
4	0.841	1.043	3.074	2.917	3	101.274	58.884
5	0.299	2.161	0.077	2.295	1	848.181	157.546
6	0.178	2.139	0.086	32.210	1	87.701	102.500
7	0.543	0.259	67.673	4.819	1	108.293	55.282
8	0.826	0.988	5.724	10.390	1	337.441	50.399
9	1.000	0.593	14.494	5.290	3	97.000	113.705
10	0.937	0.310	77.418	1.025	2	322.717	30.946
11	1.000	0.526	115.179	7.867	0	176.862	187.779
12	0.672	1.398	1.948	0.531	1	600.649	66.521
13	0.931	0.431	51.056	4.403	2	222.180	44.139
14	0.316	0.507	38.215	4.742	2	57.674	44.913
15	1.000	0.922	6.688	6.680	2	140.440	63.673
16	0.259	0.550	27.395	6.694	0	186.696	38.647
17	0.693	1.127	3.840	0.843	2	341.804	71.750
18	0.683	1.229	1.879	2.062	1	429.038	256.483
19	0.710	0.328	61.262	2.593	1	204.683	23.149
20	1.000	0.574	40.175	0.273	0	146.970	54.151
Statistics							
Mean	0.694	0.863	30.190	5.337	1.300	270.483	80.013
Standard deviation	0.287	0.539	32.078	6.691	0.900	190.392	57.906
95% Lower limit	0.183	0.276	0.080	0.357	0.000	67.510	23.807
95% Upper limit	1.000	2.158	109.099	28.696	3.000	808.324	245.420

Table 3 (*continued*)

(3) MPSFRM

Event	f_r	p	k_1	k_2	k_3	T_l	R_{sa}	F_{obj}
1	0.262	1.097	6.620	0.041	0.354	1	281.510	94.581
2	0.894	0.413	58.946	1.914	3.406	2	768.023	24.949
3	0.312	1.079	5.000	0.135	0.522	0	85.297	47.810
4	1.000	1.419	1.950	42.072	2.221	1	684.647	44.594
5	0.330	1.846	0.371	0.457	4.050	2	715.101	168.740
6	1.000	0.260	48.222	0.568	17.850	5	58.571	112.620
7	0.617	0.287	61.312	0.112	4.230	2	132.924	61.096
8	0.834	0.922	6.739	9.269	8.014	1	597.652	47.214
9	1.000	0.562	12.194	12.619	0.239	3	212.000	102.399
10	0.838	0.359	58.670	2.573	2.828	2	342.189	29.904
11	1.000	0.526	114.721	4.201	0.406	0	105.700	187.250
12	0.645	1.699	0.558	14.410	1.666	1	584.684	71.440
13	1.000	0.419	58.228	0.334	9.312	2	428.835	44.207
14	1.000	0.417	58.242	0.815	8.439	2	697.478	43.954
15	0.893	2.134	0.195	0.111	28.357	1	451.859	81.611
16	0.593	0.344	44.155	3.937	14.564	1	1140.577	39.785
17	0.669	1.073	4.438	0.267	1.510	2	329.869	71.603
18	0.769	0.903	7.519	1.225	1.358	1	728.565	153.892
19	0.818	0.284	84.570	1.335	4.304	1	195.446	30.041
20	1.000	0.573	40.182	1.718	0.657	0	536.939	55.055
Statistics								
Mean	0.774	0.831	33.641	4.906	5.714	1.500	453.893	75.637
Standard deviation	0.240	0.555	32.688	9.467	7.033	1.118	280.272	46.346
95% lower limit	0.278	0.268	0.252	0.064	0.277	0.000	67.326	26.572
95% upper limit	1.000	2.088	109.866	37.618	26.665	4.678	1080.589	184.270

$$S_{t+1} = S_t + \frac{\Delta t}{2}[(I_{t+1} + I_t) - (O_{t+1} + O_t)] \tag{13}$$

where I_t and O_t are the inflow and outflow at time t, respectively; Δt represents the time step; and S_t denotes the storage at time t. Given the inflow and outflow, the storage can be computed by Eq. (13).

Similar to the comparison of estimated runoff hydrograph, a comparison of storage–discharge relationship estimated by the three models with observed data is made. Fig. 6 shows that the PSFRM and MPSFRM can fit the loop-shaped storage–discharge relationship for EV5, EV7, EV12, and EV18 better than the SFRM. Although the observed storage–discharge curves for EV5 and EV7 are close to be a line, the MPSFRM produces a better fit of observed lines than the SFRM. It is also observed that if a rainstorm event has multi-peaks, such as EV12 and EV18, its storage–discharge relationship has a complicated shape. However, the MPSFRM can have a better fit of this kind of storage–discharge relationship than the SFRM and PSFRM. As the result, the MPSFRM not only enhances the performance of estimating runoff, but also is superior to the SFRM and PSFRM in describing the storage–discharge relationship, especially for the loop-shaped relation.

Table 4 lists that the mean and standard deviation of the efficiency coefficient (CE) and root mean square error (RMSE) of estimated storage by the three models. It shows that the mean of CE from MPSFRM (0.672) is closer to one than those from

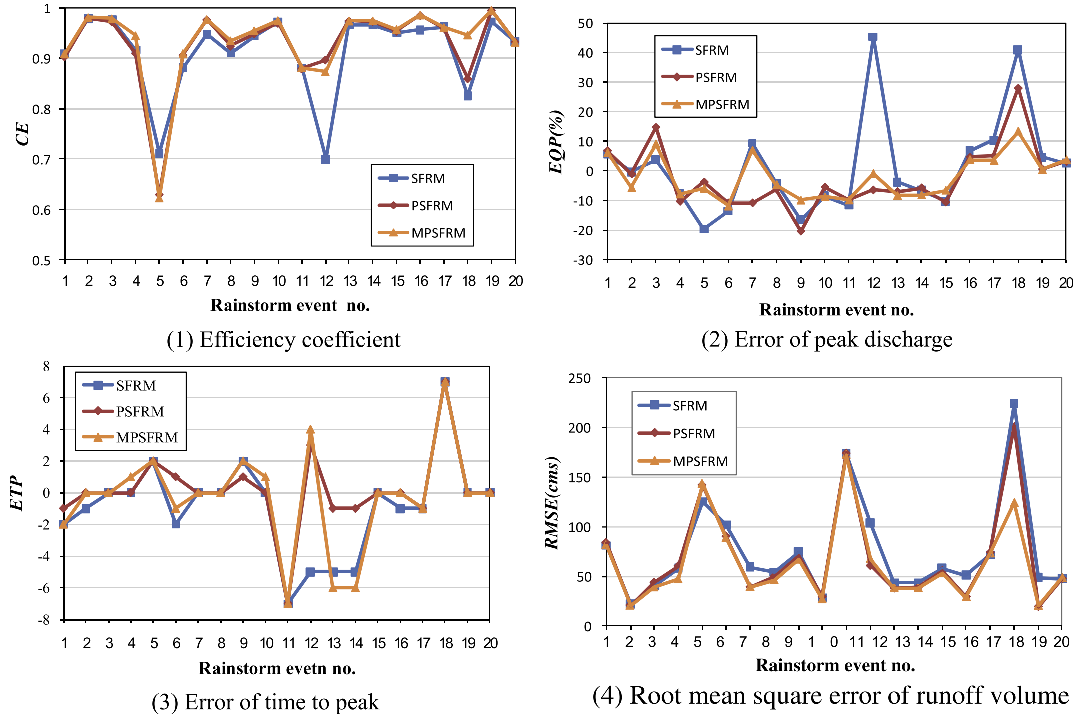

Fig. 4. Comparison of performance indices of estimated runoff with event-based optimal parameters.

SFRM (0.322) and PSFRM (0.59). The average of RMSE from MPSFRM (22.2) is mostly less than those from SFRM (38.3) and PSFRM (28.7). In addition, since the standard deviation of CE and RMSE for MPSFRM are less than those from SFRM and PSFRM, it indicates that the estimated storage from the MPSFRM is more precise than those from the SFRM and MPSFRM. In other words, the MPSFRM can provide the more reliable storage estimation than the SFRM and PSFRM.

4.4. Comparison of predicted runoff with multi-event optimal parameters

To evaluate the overall performance of SFRM, PSFRM and MPSFRM in the runoff prediction, the cross-validation method is adopted. By cross-validation, the total rainstorm events selected are separated into two groups: i.e. the calibration events and validation events defined previously.

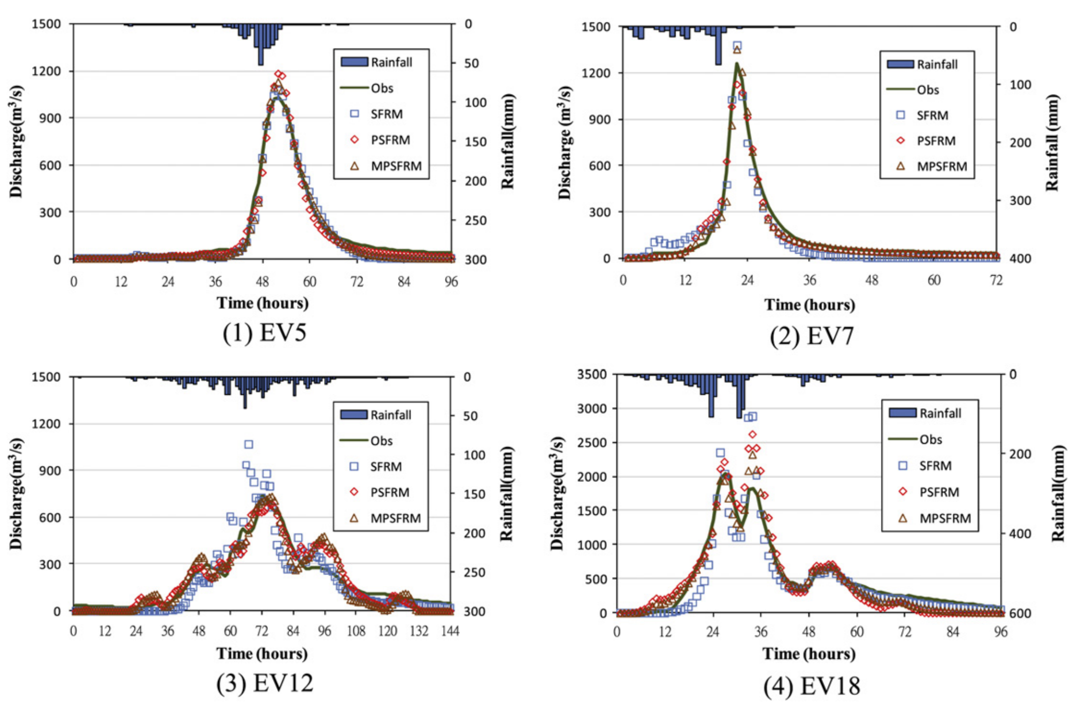

Fig. 5. Comparison of estimated hydrograph with event-based optimal parameters.

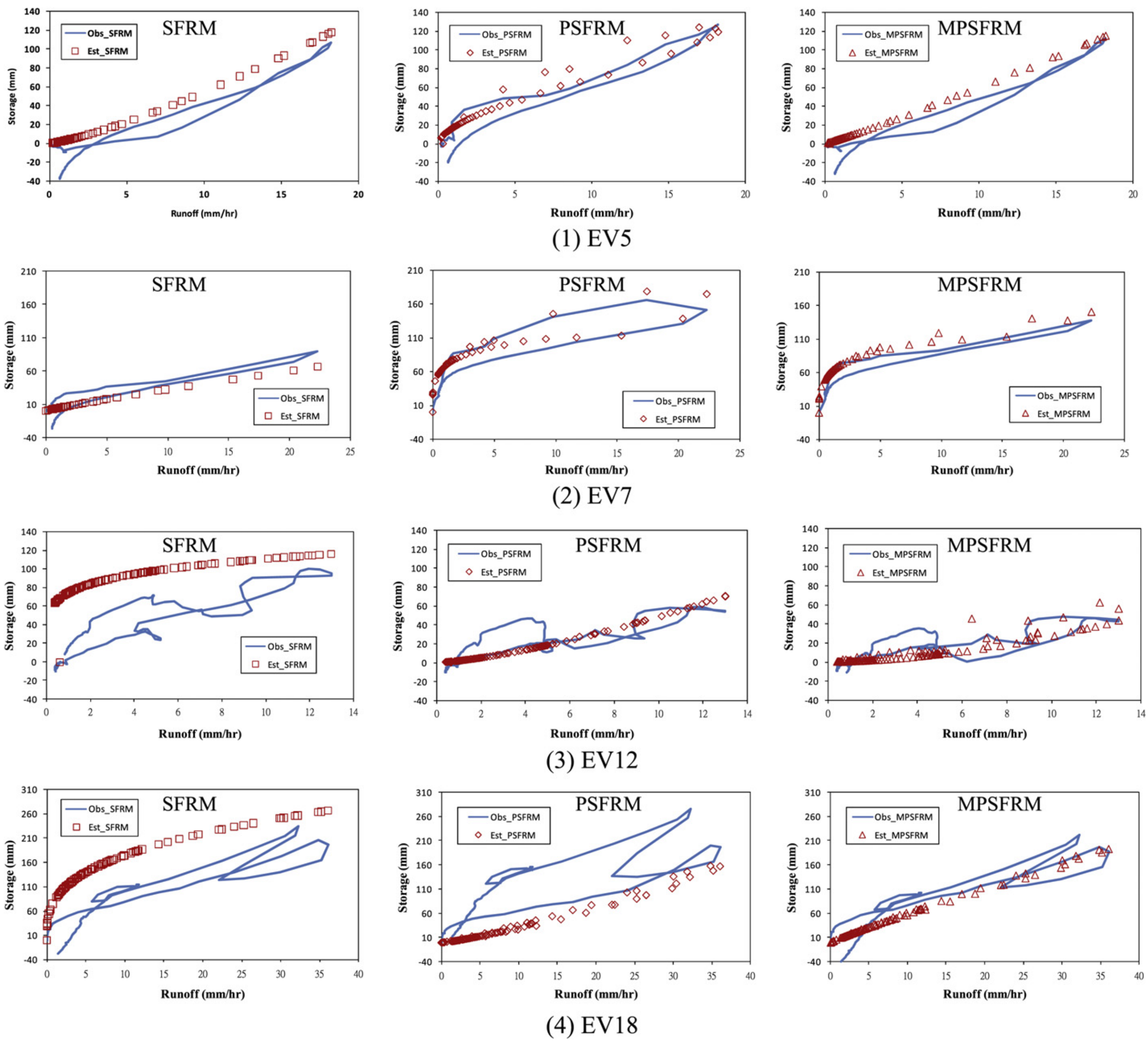

Fig. 6. Comparison of estimated storage with event-based optimal parameters.

Although the mean of model parameters can probably represent the average features of watershed, the objective function F_{obj} for event-based optimal parameters (see Table 3) represents that the fitness of the estimated runoff to observed data of individual event. Therefore, this study calculates the weighted average of model parameters based on objective function and event-based parameters, defined as multi-event optimal parameters, by:

$$\overline{\theta} = \sum_{i=1}^{N_{\text{event}}} w_i \times \theta_i \qquad w_i = \frac{1/F_{\text{obj},i}}{\sum_{i=1}^{N_{\text{event}}} 1/F_{\text{obj},i}} \tag{14}$$

where N_{event} is the number of rainstorm events; $\overline{\theta}$ is the multi-event model parameters; and θ_i and $F_{\text{obj},i}$ denote the event-based optimal parameters and the corresponding objective function using the ith rainstorm event. Table 5 shows the resulting multi-event multi-parameters of three models using the first 4, 8, 12, and 16 calibrated events. In Table 5, although the PSRM and MPSFRM have similar optimal values of parameter f_r, T_l, p, and k_1, the parameters k_2 and k_3 are significantly different. This implies that there is a quite different storage–discharge property on the rising and recession limbs.

Table 4
Statistics of CE and RMSE of estimated storage by SFRM, PSFRM and MPSFRM.

Statistics	SFRM		PSFRM		MPSFRM	
	CE	RMSE	CE	RMSE	CE	RMSE
Mean	0.322	38.295	0.585	28.668	0.672	22.194
Standard deviation	0.841	40.703	0.527	30.381	0.480	18.836

Table 5
Summary of multi-event optimal parameters.

Model	Parameter	Number of calibration events			
		4	8	12	16
SFRM	f_r	0.608	0.603	0.773	0.774
	p	0.873	0.916	0.561	0.408
	k_1	26.671	21.159	49.848	46.046
	T_1	2	2	2	2
	R_{sa}	317.941	400.739	399.984	205.399
PSFRM	f_r	0.714	0.64	0.812	0.741
	p	0.713	0.926	0.628	0.626
	k_1	32.71	27.608	52.891	47.824
	k_2	3.119	6.054	3.286	3.834
	T_1	2	1	2	2
	R_{sa}	293.191	315.789	324.401	291.843
MPSFRM	f_r	0.638	0.637	0.764	0.769
	p	0.872	0.909	0.651	0.646
	k_1	26.328	24.553	42.794	42.875
	k_2	7.359	6.429	5.092	4.483
	k_3	1.891	3.092	2.688	5.082
	T_1	1	1	2	2
	R_{sa}	497.256	497.494	401.085	488.032

Fig. 7 shows the statistics of performance indices for predicted runoff by the three models with multi-events optimal parameters using various number of calibration events. It can be seen that the performance indices of predicted runoff by the three models vary with the number of calibrated events. In detail, the 95% confidence intervals of performance indices from MPSFRM are narrower than ones from SFRM and PSFRM. The mean values of the efficiency coefficient CE from MPSFRM are closer to one than those from SFRM and PSFRM. Moreover, the MPSFRM has smaller mean values of EQP and RMSE than the SFRM and PSFRM. The above results indicate that the MPSFRM can produce more accurate and reliable runoff prediction than the SFRM and PSFRM.

Fig. 8 shows the visual comparison of runoff hydrographs for EV17, EV18, EV19, and EV20 predicted by the three models with multi-event parameters based on the first 16 events for calibration. It is observed that the performance of predicted runoff by the three models depends on validation events. The MPSFRM predicts runoff hydrographs for EV17 and EV18 which match the observed data well, but the peak discharge for EV19 and EV20 are underestimated. Although the SFRM predicts runoff hydrographs for EV19 and EV20 closer to observed ones, the MSPSRM can produce the suboptimal runoff hydrograph.

In summary, the MPSFRM can describe a nonlinear storage function well by means of taking into account the different behaviors of storage−discharge relationship on the rising and recession limbs. As the result, the proposed MPSFRM can enhance the accuracy of runoff estimation.

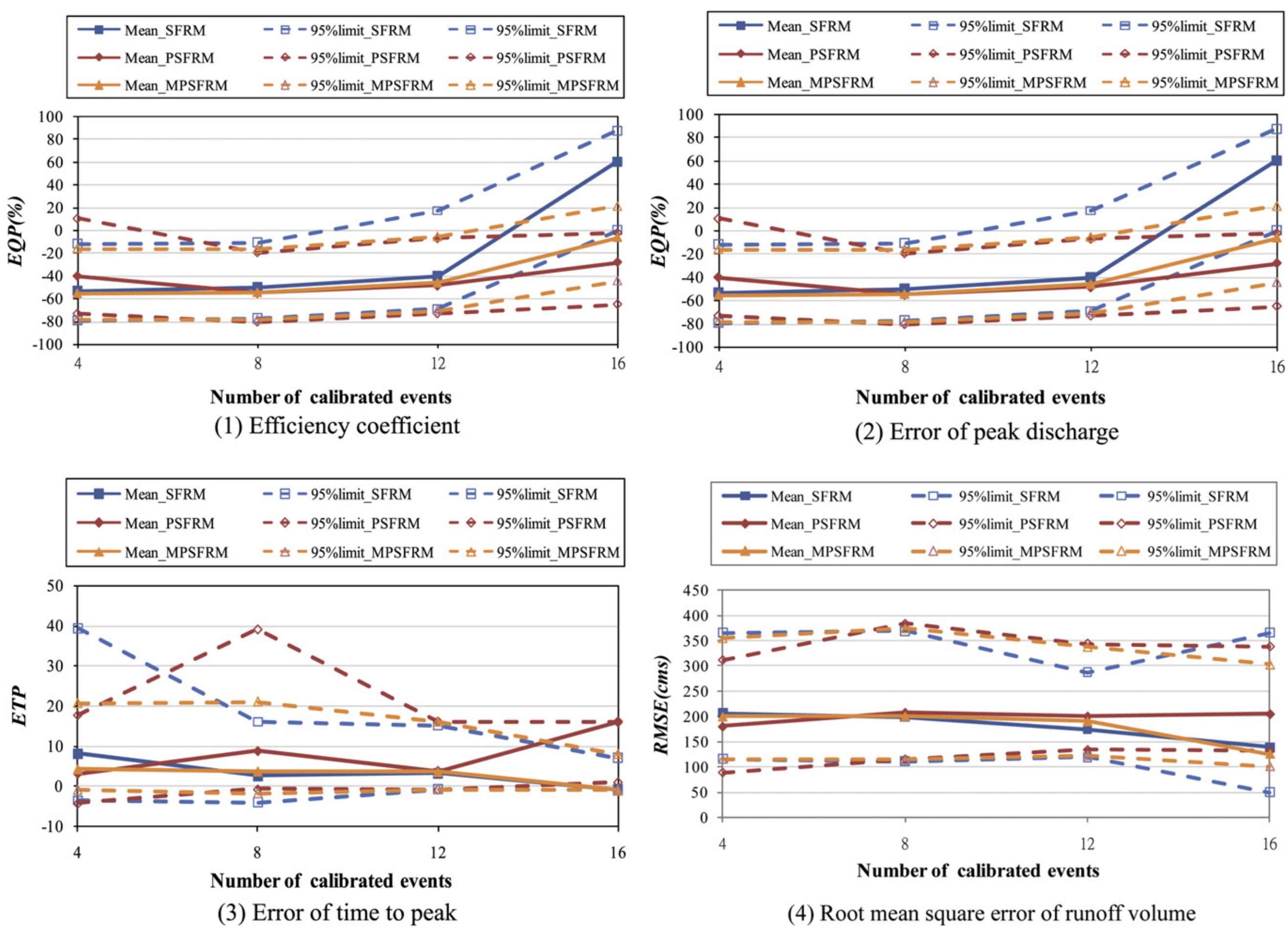

Fig. 7. Comparison of statistics of performance indices of estimated runoff with multi-event optimal parameters.

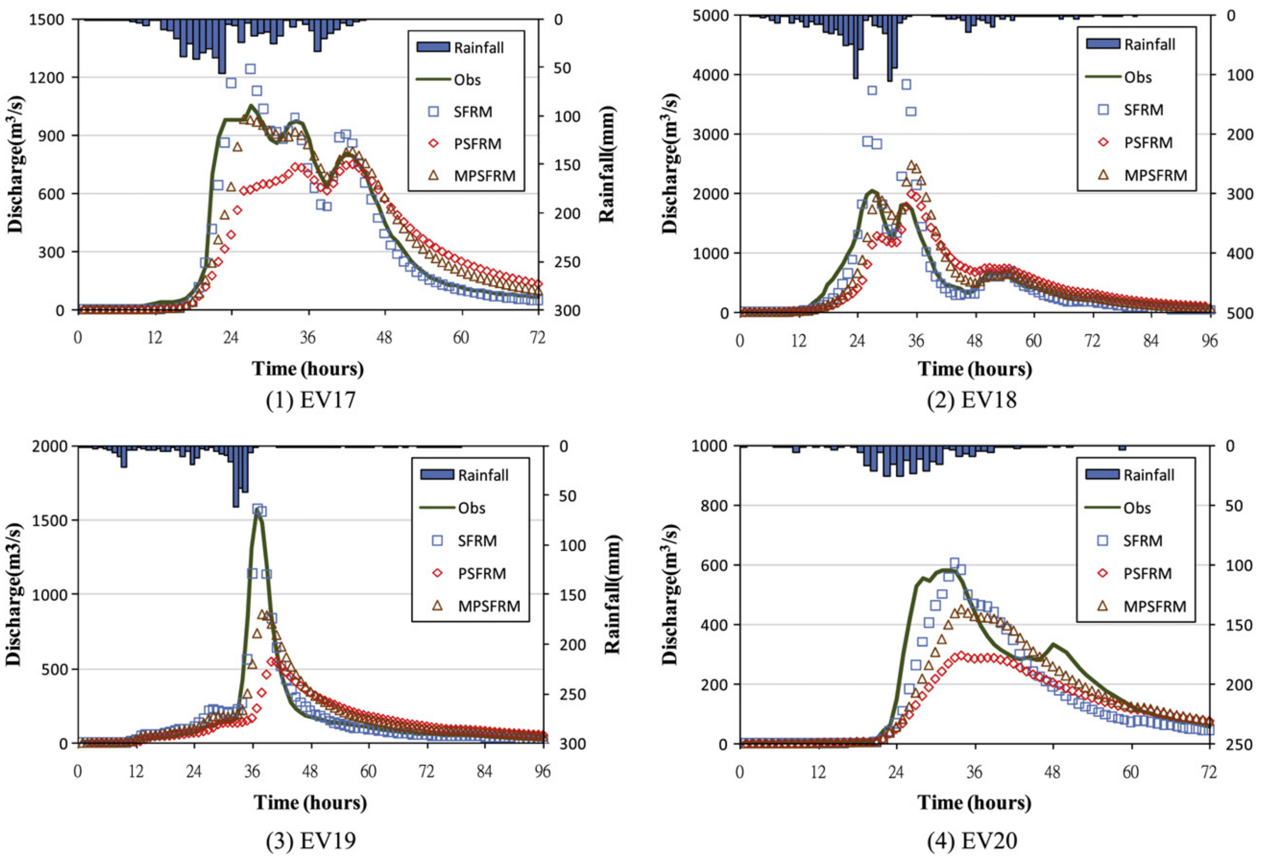

Fig. 8. Comparison of predicted hydrograph with multi-event optimal parameters.

5. Conclusions

This study modifies the storage function runoff model (SFRM), which is denoted as MPSFRM, based on the Prasad storage function runoff model (PSFRM), by considering different coefficients for dq/dt on the rising and recession limbs. To calibrate parameters of SFRM, PSFRM, and MPSFRM, a method which combines the genetic algorithm with the least square method was adopted in this study. The model validation was made using twenty rainstorm events recorded at the Wudu gauge in the Keelung River of northern Taiwan. The results of the model validation indicate that the MPSFRM has a best capability to capture the actual storage–discharge relationship, and hence, can more accurately estimate runoff than the SFRM and PSFRM. Furthermore, the MPSFRM provides a more flexibility in the runoff estimation than the SFRM and PSFRM as the latter two are special cases of the former.

As for future work, the regional formula for the MPSFRM parameters should be developed based on physical meaning of model parameters and geometric characteristics of the watershed, such as the area, river length, and catchment slope. Such regional formula will be useful in estimating runoff at an ungauged watershed. Moreover, other global optimization methods with various calibration strategies could be utilized to obtain model parameters which might produce more accurate estimated runoff.

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