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計畫主持人：藍武王

計畫參與人員：林豐裕、郭怡雯

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車流複雜系自我組織現象之探索與應用（II）
The Phenomena of Self-organization of Traffic Flow Complex System: Investigation and Application (II)

國立交通大學交通運輸研究所：藍武王、林豐裕、郭怡雯

摘要

每日之車流變化型態具有相似但又不完全重複之特性，針對此種複雜系統，如能找出其規則性，將會有相當之應用價值。惟此種車流變化酷似隨機，很難有一套準確又有效之方法檢測其混沌性。本研究嘗試發展一套有效檢測混沌時間序列之簡潔步驟，第一步引用最大李亞譜諾夫指數以排除趨於定點數列或週期數列，第二步利用頻譜分析進一步排除準週期數列，第三步再比較原始數列與替代（打亂次序）數列之反覆函數系統圖，以篩除隨機數列，依此簡潔步驟即可檢測一數列是否爲混沌。經利用已知數列驗證，發現本簡潔步驟確實有效，最後應用本簡潔步驟檢測高速公路尖峰時段每分鐘交通量，結果發現具有混沌現象。

關鍵字：混沌、車流、時間數列、簡潔檢測步驟、替代數列

Abstract

For a complicated system such as traffic flow dynamics that is characterized with similar patterns but not exactly reproducible from day to day, we might be curious about whether there exist a simple underlying determinism. With only a single sequence of measurements for the traffic flow time series data, however, it is difficult to make distinction between deterministic chaos and nondeterministic randomness because both look very alike. It is argued that testing for chaos is an art rather than a science; nonetheless, this study attempts to develop a parsimony procedure to test for chaos in time series. The first step is to employ the largest Lyapunov exponent to rule out the fixed points or periodic motions. The second step is to use power spectra to further rule out the possibility of quasi-periodic data. The final step is to compare the iterated function system (IFS) clumpiness maps between the original and surrogate data, once we already know that they are not fixed points or periodic or quasi-periodic motions. Our proposed parsimony procedure is validated by some well known time series data generators. It is then applied for testing the existence of chaotic structures in the nature of freeway one-minute flow time series.

Keywords: Chaos; Traffic flow; Time series data; Parsimony testing procedure; Surrogate data

1. Introduction

A time series is a dynamical system that evolves in time. It can be regular predictable or regular unpredictable. It can be chaos of various classes ranging from transient, intermittent, narrow-band, broad-band low-dimensional, to broad-band high-dimensional chaos. It can also be correlated (colored) noise or random noise. Most real-world systems often exhibit a combination of the above dynamical behaviors or in between the two extremes, the regular predictable and the random noise. When we observe complicated systems in nature such as traffic flow variations, we might be curious about whether there exist a simple underlying cause or intrinsic rule that governs the determinism of flow dynamics. With only a single sequence of measurements at successive times (a time series), for instance, we might ask whether the dynamics are deterministic and chaotic or nondeterministic and random. If the time series exhibits a hidden determinism, we can take the advantages of determinism to make prediction, to extract deterministic signals from a noisy background (noise reduction), or to control the system using small perturbations to alter its outcome [29, 30].

Chaotic systems have several important features: (1) they are aperiodic, namely trajectories or orbits never repeat; (2) they exhibit sensitive dependence on initial conditions (SDIC), hence they are unpredictable in the long run; (3) they are governed by one or more control parameters, a small change in which can cause the chaos to appear or disappear; and (4) their governing equations are nonlinear [1]. Chaos is part of the broader field of dynamical systems, which can be stochastic or deterministic. A stochastic system will change with time according to some random processes. A deterministic system, by contrast, will evolve under some deterministic rules (or mathematical equations) such that the present state can be determined by the past states. Such deterministic chaos can only occur when the governing rules or equations are nonlinear.

There are several definitions of chaos in use. A definition similar to the following is commonly found in the literature (for instance, [10]; [11]; [12]; [13]). The series a_t has a chaotic explanation if there exists a system (h, F, x_0) where $a_t = h(x_t)$, $x_{t+1} = F(x_t)$, x_0 is the initial condition at $t = 0$, and where *h* maps the *n*-dimensional phase space, R^n to R^1 and F maps R^n to R^n. It is also required that all trajectories x_t lie on an attractor *A* and nearby trajectories diverge so that the system never reaches equilibrium (i.e., not eventually locating at fixed points) nor exactly repeats its path (i.e., it is aperiodic). For the chaotic time series, if one knows (h, F) and could measure x_t without error, one could forecast x_{t+i} and thus a_{t+i} perfectly. With the divergence property and attractor *A*, in order that *F* generates stochastic-looking behavior, nearby trajectories must diverge (repel) exponentially. Moreover, in order that *F* generates deterministic behavior, locally diverging trajectories must eventually fold back (attract) on themselves. The attractors may be thought of as a subset of the phase space towards which sufficiently close trajectories are asymptotically attracted.

Chaos theory has proven that random input is not the only source of irregular outputs for a nonlinear system. A chaotic time series, which appears stochastic, is in effect governed by deterministic rules. However, it is difficult to make distinction between stochasticity and chaoticity because both have very similar irregularity. To elucidate this feature, we deliberately generate two chaotic time series data and two stochastic time series data as follows. The Henon time series (2000 iterates) generated by eq. (1) and the Lorenz time series (2000 data points at an interval of $\Delta t = 0.1$) generated by eq. (2) are well-known chaotic time series. The Gaussian white noise time series generated by eq. (3) and the random time series generated by eq. (4) are well-known stochastic time series. The Henon and Lorenz time series data with parameters given in eqs. (1) and (2) have been proven as deterministic chaotic systems [14, 15]; while the Gaussian white noise and random time series given in eqs. (3) and (4) have been proven as stochastic systems [16].

$$X_{n+1}=1-1.4X_n^2+Y_n;\ Y_n=0.3X_n \quad (1)$$

$$dX/dt=10(Y-X);\ dY/dt=28X-Y-XZ;\ dZ/dt=XY-8Z/3 \quad (2)$$

$$P(X)\frac{1}{\sqrt{2\pi}}e^{-X^2/2} \quad (3)$$

$$X_{n+1}=AX_n+B(modC) \quad (4)$$

For the one-dimensional (trace) plots, *x(t)* versus *t*, we notice that Henon and Lorenz chaotic time series (Figures 1(a) and 1(b)) are almost indistinguishable from a Gaussian white noise (Figure 1(c)) and a random data (Figure 1(d)). Namely, it is almost impossible to distinguish, by visualization method, between a chaotic system and a stochastic system because both have very similar irregularity. However, we already know that the simplest determinism of chaotic time series has each value dependent solely on its immediate predecessor; thus, through the reconstruction of the state space, some of its geometric plots might reveal very unique patterns, which can be served for the distinction purposes.

If we reconstruct these time series in higher dimensional state spaces, we would look for the difference. For instance, Figure 2 presents the corresponding two-dimensional plots, *x(t)* versus *x(t-n)*, where *n* is the appropriate delay time; and Figure 3 shows the three-dimensional plots, *x(t)* versus *x(t-n)* versus *x(t-2n)*. Notice that both chaotic systems have shown discernible structures (Figures 2(a) and 2(b); Figures 3(a) and 3(b)), in which the trajectories are governed by intrinsically deterministic rules. In contrast, the random system does not reveal any structure at all, which plots just fill up the entire plane in a two-dimensional state space (Figure 2(c) and 2(d)) and look like a “fuzzy ball” or scatter uniformly in a three-dimensional state space (Figure 3(c) and 3(d)).

As pointed out by Sprott [1], a stochastic system with a non-uniform power spectrum can masquerade for chaos. Therefore, it may be necessary to further compare the properties between an original time series and its surrogates, which are designed to mimic the statistical properties of the original data, but with the determinism removed. The surrogate data can be easily generated by randomly shuffling the original data. The shuffling can be done quickly by stepping through the time series, swapping each values with one chosen randomly from anywhere in the series. On average, each point being moved twice will essentially guarantee randomness and this method does require keeping the whole time series in memory. While shuffling the sequences will preserve the same probability distribution as the original data, the surrogates do not preserve the same power spectrum and correlation function [27]. In other words, the surrogates for any original time series data are random.

To generate the surrogate time series Y_n with the same power spectrum as the original time series X_n, we use the following equations to find S_m first and then to construct Y_n with the same Fourier amplitudes but with random phases.

$$S_m = G(0) + G(k)\cos(\frac{2\pi mk}{N}) + 2\sum_{k=1}^{K} G(k)\cos(\frac{2\pi mk}{N}) \tag{5}$$

$$Y_n = \frac{a_0}{2} + \sum_{m=1}^{N/2} \sqrt{S_m}\,\sin 2\pi(mn/N + r_m) \tag{6}$$

where *G(k)* is autocorrelation function, *K* is the maximum of *k*, which usually appears to be about *N/4*. r_m are *N/2* uniform random numbers chosen from $0 \le r_m < 1$. *N* is the number of data [19].

The iterated-function system (IFS) clumpiness maps between the original and surrogate data for the four known time series generators are used for illustration. The IFS clumpiness is a useful plot to present the determinism of the data [24]. For instance, white noise is a space-filled uncorrelated process that uniformly fills its space of representation. Brown noise accumulates over the diagonals and some of the sides of the square leaving most of the representation space empty. Pink noise produces self-similar repeating triangular structure of different sizes and accumulates, albeit in a dispersed way, near the diagonals [1]. Figures 4 and 5 compare the original and surrogate IFS clumpiness maps of Henon, Lorenz, white noise, and random time series data, respectively. We notice that the original chaotic data, particularly for the Lorenz, and its surrogate data have apparently dissimilar IFS clumps; in contrast, the original random data have very similar IFS clumps to its surrogates.

The above illustrations with four known examples provides good evidences that a very simple deterministic time series, which is essentially a chaotic system, can reveal very irregular trace similar to a random system. The illustrated plots have suggested that it is easy to incorrectly think a random system as chaos or a chaotic system as random by only judging their trace plots because they are very much alike. Through the state space reconstruction, however, one can easily distinguish a chaotic system from a random system. Moreover, the comparison of IFS clumpiness maps between the original and surrogate data can provide another useful tool for the distinction purpose. Many previous researches have devoted to develop methods for testing or searching for chaos in time series of various areas (for instance, [18, 19, 22, 27, 4, 11, 8, 6, 13]). However, it is still argued that testing for chaos is more artistic than scientific and that no recipe will guarantee success for every case and even going through a battery of tests, the conclusions are seldom definitive [1]. Nonetheless, it is always challenging to gain better insight and understanding of the underlying dynamics and it would help if one could develop simple methods to test for chaos in time series. The main objective of this study is to develop a parsimony procedure to test for chaos by using as few effective indexes as possible.

2. Developmet of parsimony testing procedure

The above illustrations by Eqs. (1) through (4) have suggested that it is easy to incorrectly think a random system as chaos or a chaotic system as random by only judging their one-dimensional (trace) plots because they are very much alike. Therefore, we must attempt other indexes that could make more distinction. We already know that the simplest determinism of chaotic time series has each value dependent solely on its immediate predecessor; thus, through the reconstruction of the state space, some of its geometric plots would reveal very unique patterns, which can be served for the distinction purposes. The two- and three-dimensional state space plots in Figs. 2 and 3 are good examples of such geometric plots. Other known geometric plots in chaos and time series literatures include return maps (plots of each local maximum versus the previous maximum), phase-space plots (slopes of the trajectories), Poincare maps (or Poincare movies), iterated function system (IFS) clumpiness maps, autocorrelation function plots, probability distributions, and power spectra. The known statistics include the largest Lyapunov exponent, Kolmogorov entropy, Hurst exponent, relative complexity, capacity dimension, embedding dimension, correlation dimension, and delay time. By investigating the characteristics of the above-mentioned geometric plots and statistics, we attempt to establish a "parsimony" testing procedure (Fig. 6) that includes as few geometric plots or statistics as possible. Details of the parsimony testing procedure are narrated as follows:

Step 1. Examine the largest Lyapunov exponent (LE) for the original data

If the LE is negative, the time series will converge toward a stable sink (or equilibrium fixed points, see for instance [15]). If the LE is zero, the time series is periodic in the sense that the trajectories will converge to a period-k sink (k is greater or equal to 2). If the LE is positive, the time series can be quasi-periodic, chaotic or stochastic, then we go to step 2.

Step 2. Examine the power spectrum of the original data

If the power spectrum is narrow and has only few (two or three) dominant sharp peaks, it must be quasi-periodic. In case that it is a broadband spectrum, it can be chaotic or stochastic, then we go further to step 3.

Step 3. Compare the IFS clumpiness between the original and surrogate data

If the IFS clumpiness map of the original data visually differs from the surrogates, then we have evidences (but not proof) that the time series is not stochastic [22]. In this case, we can probably say that the original time series exhibits chaotic structures.

In sum, the first step is to rule out the fixed points or periodic time series. The second step further rules out the possibility of quasi-periodic data. The final step is to make distinction between chaoticity and stochasticity once we already know that they are not fixed points, periodic, or quasi-periodic.

3. Validations

To validate our proposed parsimony testing procedure, we make use of nine well-known time series data generators: FIX.DAT is fixed-point time series generated by Eq. (7). Both FEIGEN.DAT and SINE.DAT are periodic time series generated by Eqs. (8) and (9). Both TWOSINE.DAT and THREESIN.DAT are quasi-periodic time series generated by Eqs. (10) and (11). Both NOISE.DAT and RANDOM.DAT are stochastic time series generated by Eqs. (3) and (12). Both HENON.DAT and LORENZXZ.DAT are chaotic time series generated by Eqs. (1) and (2). We produce 2,000 iterates for each of the nine time series generators by using the chaos data analyzer [28] and then examine the largest Lyapunov exponents in the first step, look for the power spectra in the second step and compare the IFS clumpiness between original and surrogate data in the final step.

$$X_n=\text{constant} \tag{7}$$

$$X_{n+1}=3.5699456X_n(1-X_n) \tag{8}$$

$$Sin\ (t/10) \tag{9}$$

where t is an integer value from 0 to 1999.

$$Sin\ (t/2)+cos(gt/2) \tag{10}$$

where t is an integer value from 0 to 1999,g= $(\sqrt{5}-1)/2$

$$Sin\ (t/2)+cos(gt/2)+sin(ht/2+\pi/4) \tag{11}$$

where t is an integer value from 0 to 1999, g= $(\sqrt{5}-1)/2$

$$X = \sqrt{-2\ln r_1}\,\sin 2\pi r_2 \qquad (12)$$

Table 1 reports the largest Lyapunov exponents of these nine generators. As what we anticipated, the first step has successfully identified the FIX.DAT (LE<0) as an equilibrium fixed-point time series and SINE.DAT (LE=0) and FEIGEN.DAT (LE=0) as periodic time series. Therefore, we go to step two by further examining the power spectra for the remaining time series data as presented in Fig. 7. We find that only TWOSINE.DAT (Fig. 7 (a)) and THREESIN.DAT (Fig. 7 (b)) have spectra with few dominant peaks, which are strong evidences for quasi-periodic time series, consistent with what we anticipated. Finally, we go to step three to further distinguish chaoticity from stochasticity for the remaining time series by comparing the IFS clumpiness maps between the original and surrogate data. We find that the original data of NOISE.DAT and RANDOM.DAT (Fig. 8(a) and (c)) do not differ their IFS clumpiness from the surrogates (Fig. 18 (b) and (d)), indicating that NOISE.DAT and RANDOM.DAT are stochastic data. By contrast, the original data (Fig. 8(e) and (g)) obviously differ their IFS clumpiness from the surrogates (Fig. 8(f) and (h)), supporting that HENON.DAT and LORENZXZ.DAT are chaotic data. These IFS clumpiness tests also agree to the known properties of the remaining time series. In other words, all the nine time series data generators have successfully validate our proposed parsimony testing procedure.

4. Applications and Discussions

After having the proposed parsimony procedure validated by the known time series data generators, we further apply it to test some selected eight stations of the I-35 Freeway whether or not chaotic structures also exhibit in their one-minute flow dynamics. Table 2 reports the positive LE values for all stations, which rule out the traffic dynamics being fixed points or periodic in the first-step test. The power spectra for the eight stations are all broadband, which rule out the flow time series being quasi-periodic in the second-step test. Fig. 9 illustrates the power spectrum for station 32. The IFS clumpiness maps for the eight stations all reveal apparent differences between the original and surrogate data (Fig. 10), which further rule out the flow time series data being stochastic in the third-step test. As a consequence, we can conclude that all the one-minute flow dynamics for the remaining eight stations on the I-35 Freeway's morning rush hours also exhibit chaotic structures.

The word "chaos" was introduced by Li and Yorke (1975) to designate the nonlinear systems that have aperiodic behavior more complicated than equilibrium (fixed points) and periodic or quasiperiodic motions. In fact, a related concept of chaos -- the "strange attractor" was introduced by Ruelle and Takens (1971) who emphasized more the complicated geometry of the attractor in phase space than the complicated nature of the motion itself. The theoretical works by these mathematicians supplied many of the ideas and approaches that were late applied in physics, celestial mechanics, chemistry, biology, and other fields (Robinson, 2004). In traffic flow study, Disbro and Frame (1989) utilized chaos theory to describe the traffic flow phenomena. Dendrinos (1994), Iokibe, *et al* (1995), Frison and Abarbanel (1997), Zhang and Jarrett (1998) and Lan, *et al* (2003) also found that the short-term traffic flows have nonlinear chaotic phenomena. Addison and Low (1996, 1998) found that some specific parameter values of the GM car-following models exist in chaotic dynamics. The application of our parsimony procedure to test the chaoticity for I-35 Freeway one-minute traffic flow time series supports the previous chaotic traffic flow studies.

Our proposed parsimony testing procedure only employs three indexes, largest Lyapunov exponent, power spectra and IFS clumpiness maps. Some known time series data generators are used to validate the proposed procedure, which is then further applied to testing for chaotic traffic phenomena. The results reveal that chaotic structures exist in the nature of one-minute flow time series. Note that such conclusions are based on the empirical analysis of the morning rush-hour one-minute flows, drawn from some 8 stations of the United States I-35 Freeway. In order to have more general and robust conclusions, traffic data from different roadways (including surface roads) require further exploration. Moreover, traffic data from other periods to cover very low (e.g., midnight) to moderate (e.g., off-peak hours) flow conditions also deserve further investigation. Finally, the one-minute flow data can be summed up and converted to longer-term flow dynamics (e.g., five-minute, ten-minute, etc.). It is interesting to test if the chaotic structures still exist in such longer-term flow dynamics.

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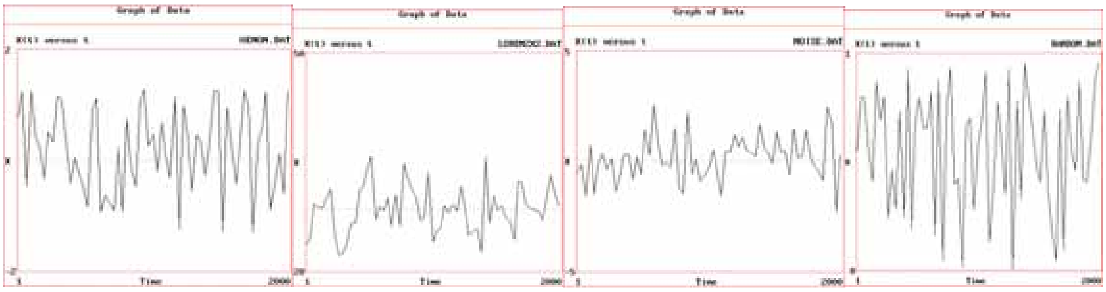
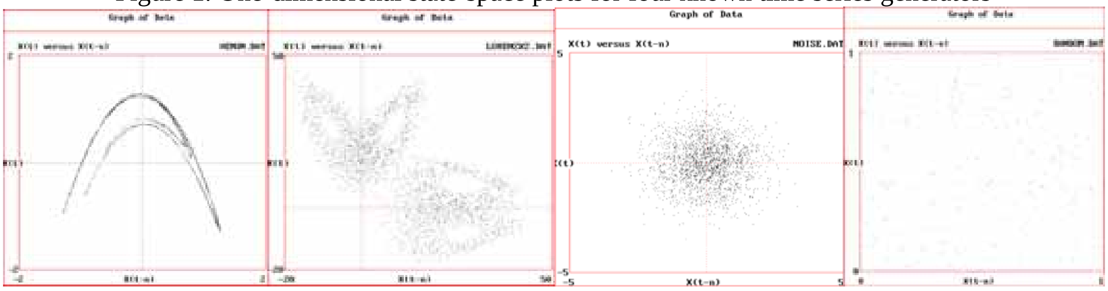


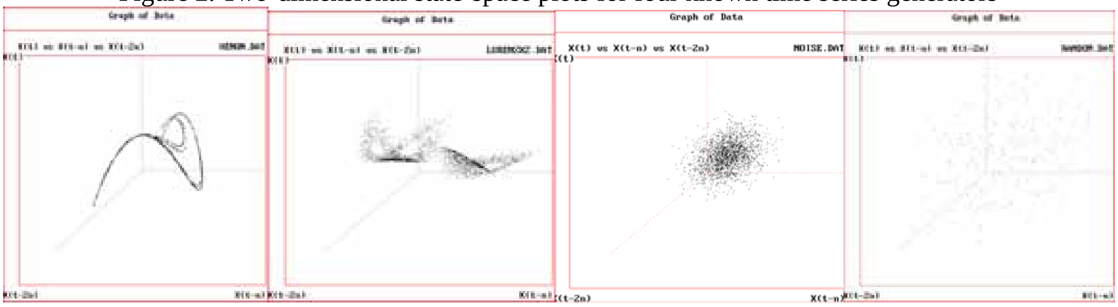
(a) Henon (b) Lorenz (c) White noise (d) Random

Figure 1. One-dimensional state-space plots for four known time series generators



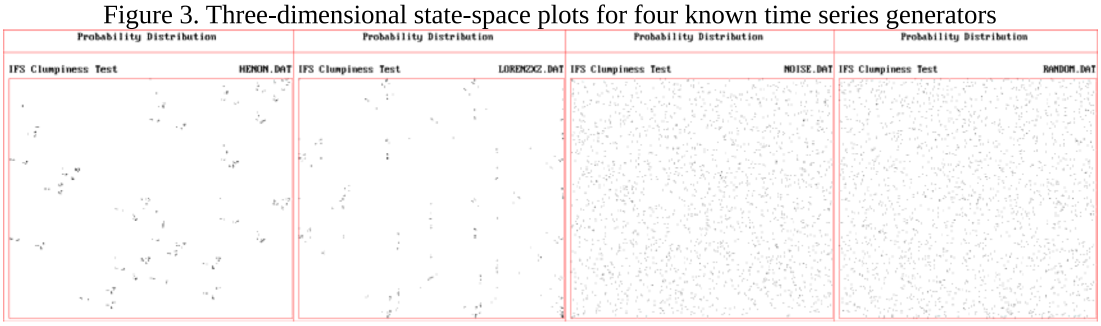
(a) Henon (b) Lorenz (c) White noise (d) Random

Figure 2. Two-dimensional state-space plots for four known time series generators



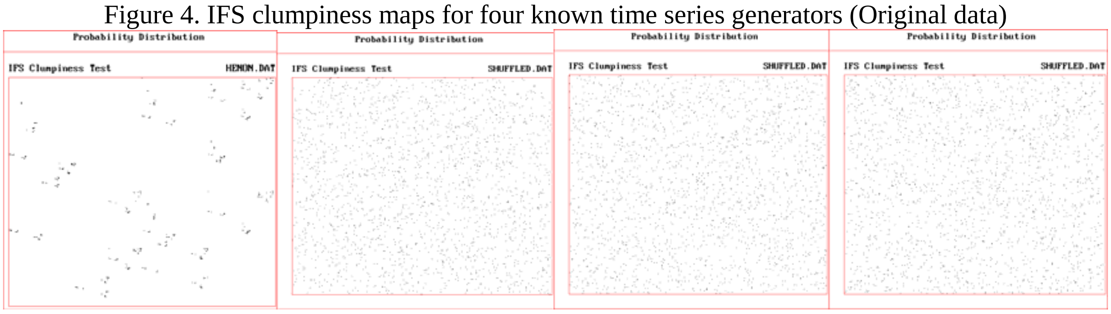
(a) Henon (b) Lorenz (c) White noise (d) Random

Figure 3. Three-dimensional state-space plots for four known time series generators



(a) Henon (b) Lorenz (c) White noise (d) Random

Figure 4. IFS clumpiness maps for four known time series generators (Original data)



(a) Henon (b) Lorenz (c) White noise (d) Random

Figure 5. IFS clumpiness maps for known time series generators (Surrogate data)

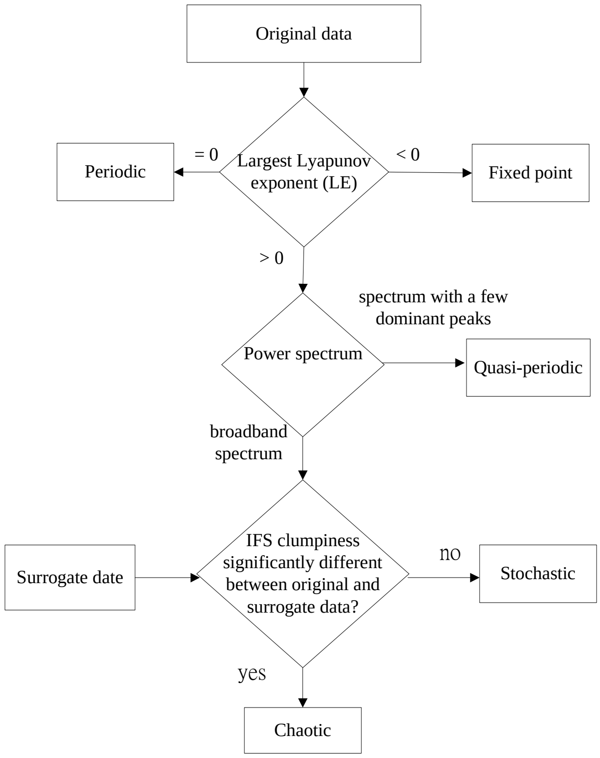
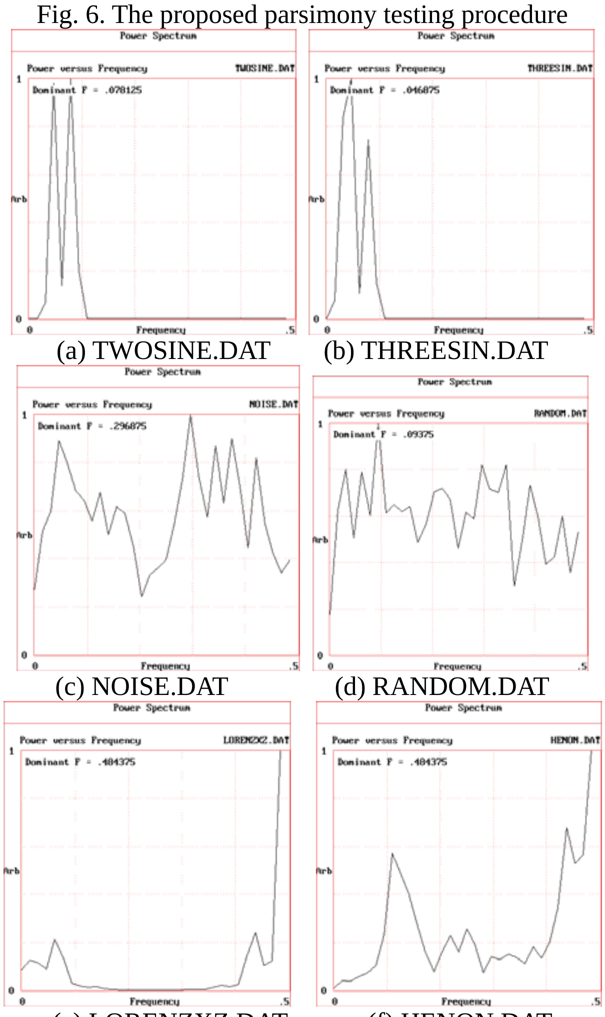


Fig. 6. The proposed parsimony testing procedure

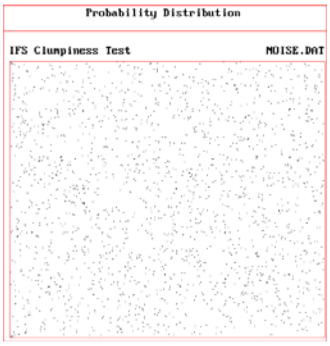


(a) TWOSINE.DAT (b) THREESIN.DAT

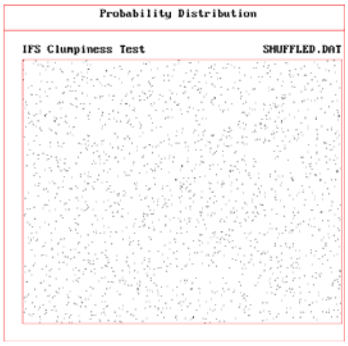
(c) NOISE.DAT (d) RANDOM.DAT

(e) LORENZXZ.DAT (f) HENON.DAT

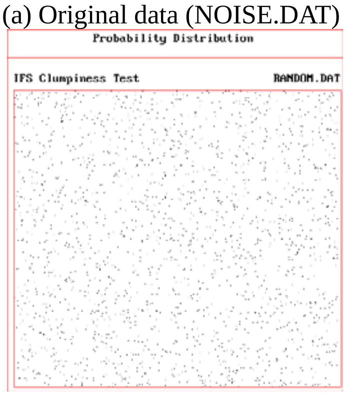
Fig. 7. Power spectra for known time series generators



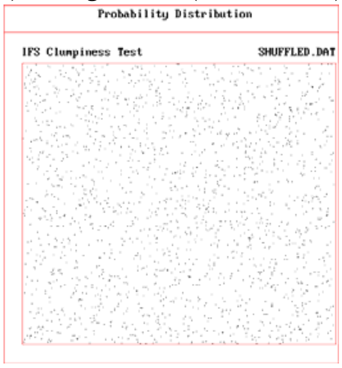
(a) Original data (NOISE.DAT)



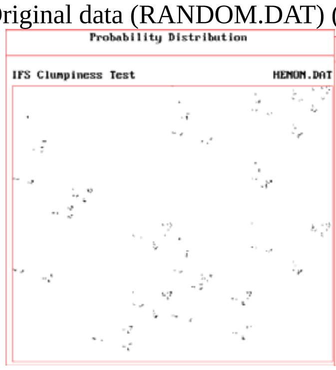
(b) Surrogate data (NOISE.DAT)



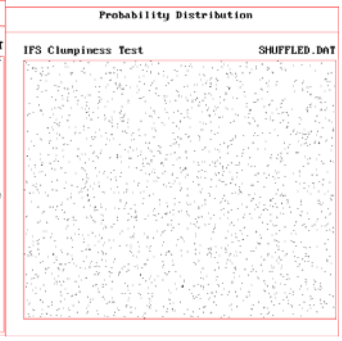
(c) Original data (RANDOM.DAT)



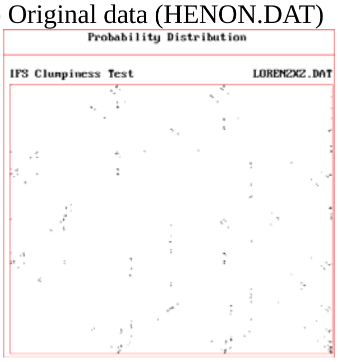
(d) Surrogate data (RANDOM.DAT)



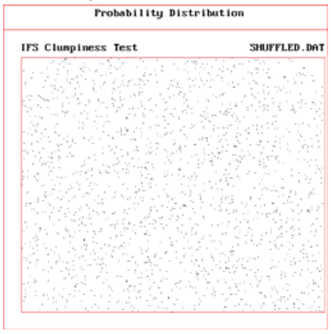
(e) Original data (HENON.DAT)



(f) Surrogate data (HENON.DAT)



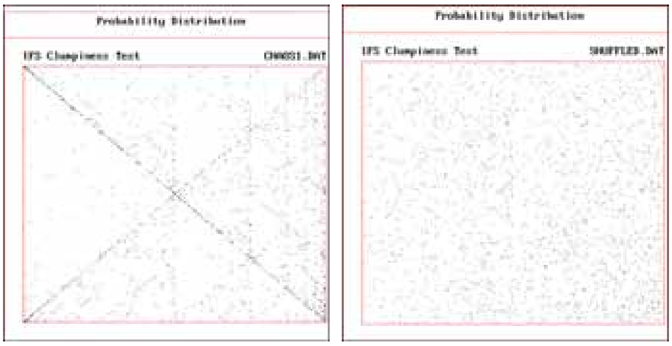
(g) Original data (LORENZXZ.DAT)



(h) Surrogate data (LORENZXZ.DAT)

Fig. 8. IFS clumpiness maps for known time series generators

Fig. 9. Power spectrum for one-minute flow time series at station 32



(a) Original data (b) Surrogate data

Fig. 10. IFS clumpiness maps for one-minute flow time series at station 32

Table 1. Largest Lyapunov exponents for known time series generators

Time series generator	Known property	Largest Lyapunov exponent (LE)
FIX.DAT	Fixed point	-0.1
SINE.DAT	Periodic	0.0
FEIGEN.DAT	Periodic	0.0
TWOSINE.DAT	Quasi-periodic	0.5
THREESIN.DAT	Quasi-periodic	0.5
NOISE.DAT	Stochastic	0.5
RANDOM.DAT	Stochastic	0.7
LORENZXZ.DAT	Chaotic	0.1
HENON.DAT	Chaotic	0.6

Table 2. Largest Lyapunov exponents for one-minute flow time series in US I-35 Freeway

Station no.	Average lane-flow (veh./min)	Largest Lyapunov exponent (LE)
32	16.8	0.6
45	23.7	0.7
44	24.8	0.7
43	28.9	0.7
41	29.9	0.7
52	31.9	0.3
53	32.5	0.4
56	33.2	0.4