行政院國家科學委員會專題研究計畫　成果報告

大夫涅耳數值的微晶片雷射之時空動力研究(2/2)

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大夫涅耳數值的微晶片雷射之時空動力研究（2/2）

Spatiotemporal dynamics in a large-Fresnel-number microchip laser

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一、中文摘要

本計畫研究大夫涅耳數值的微晶片雷射中橫模形態動力學。其中的主題包括(1) 激發分布的影響(doughnut shape versus top hat)，(2) 橫模形態與共振腔腔內 Q 因子(cavity Q-value)的關係，(3) 橫模的頻率間距(transverse mode spacing)對形態的時空動力之影響(spatio-temporal dynamics)。

關鍵詞：雷射、橫模形態、形態發生學、時空動力學、光學旋渦

Abstract

We experimentally observed the formation of high-order transverse patterns in a microchip laser with a high degree of frequency degeneracy. With a doughnut pump profile, the spontaneous transverse patterns are well localized on the *Lissajous* trajectories. The observed transverse patterns are reconstructed very well by using the coherent state of quantum theory. The good reconstruction suggests that the laser resonators may be designed to obtain a more thorough understanding of the quantum-classical connection.

Keywords: laser, transverse pattern, pattern formation, classical-quantum correspondence

二、緣由與目的

It is well-known that the paraxial wave equation for the spherical laser resonators has the identical form with the Schrödinger equation for the two-dimensional (2D) harmonic oscillator [1]. The eigenfucntion of the 2D quantum harmonic oscillator can be analytically expressed as Hermite-Gaussian function with Cartesian symmetry (x,y) or Laguerre-Gaussain function with cylindrical symmetry (r,ϕ) [2]. Since the functional forms of the 2D quantum oscillator and the spherical resonators are similar, the higher transverse modes of the spherical resonators can be in terms of Hermite-Gaussian (HG) modes or Laguerre-Gaussian (LG) modes.

The wave functions of HG mode native to a spherical resonator are given by

$$HG_{m,n}(x,y;\varpi_o)=\frac{1}{\sqrt{2^{m+n-1}\pi\, m!\, n!}}\frac{1}{\varpi_o}H_m\left(\frac{\sqrt{2}\,x}{\varpi_o}\right)H_n\left(\frac{\sqrt{2}\,y}{\varpi_o}\right)\exp\left[-\frac{(x^2+y^2)}{\varpi_o^2}\right] \quad (1)$$

with the resonance frequencies

$$\nu_{l,m,n}=l(\Delta\nu_L)+(m+n+1)(\Delta\nu_t)\ , \quad (2)$$

where $H_n(\cdot)$ is a Hermite polynomial of order n, ϖ_o is the laser beam waist, l is the longitudinal mode index, m and n are the transverse mode indices, $\Delta\nu_L$ is the longitudinal mode spacing, and $\Delta\nu_T$ is the transverse mode spacing. For a plano-concave resonator, as shown in Fig. 1, the transverse mode spacing is given by

$$\Delta\nu_T=\Delta\nu_L\left[\frac{1}{\pi}\cos^{-1}\left(\sqrt{1-\frac{d}{R}}\right)\right] \quad . \quad (3)$$

where d is the cavity length and R is the radius of curvature of the output coupler. Recently, we use a doughnut-shaped pumped profile to generate the LG modes in an a-cut Nd:YVO_4 laser [3] and to generate the elliptical modes in a c-cut Nd:YVO_4 laser [4]. Both *LG* and elliptical modes can be considered as the superposition of the degenerate HG eigenmodes $HG_{K,N-K}(x,y;\varpi_o)$, where $K=0,1,2....N$. For example, the $LG_{0,\pm N}$ is given by

$$LG_{0,\pm N}(x,y;\varpi_o)=2^{-N/2}\sum_{K=0}^{N}\binom{N}{K}^{1/2}(\pm i)^K\, HG_{K,N-K}(x,y;\varpi_o) \quad (4)$$

Since the LG modes are formed by the superposition of the degenerate HG eigenmodes, we previously setup the resonator length to be as short as possible for reaching single-longitudinal mode operation, i.e. $\Delta\nu_L >> \Delta\nu_T$.

In this work, we study the spontaneous 2D transverse modes in a frequency degenerate cavity. As indicated in Eq. (3), adjusting the cavity length *d* may result in the ratio $\Delta\nu_L/\Delta\nu_T$ to be an integer *S*. These cavity configurations constitute a high degree of frequency degeneracy, as lowering (raising) the longitudinal mode index *l* by *K,* while simultaneously raising (lowering) the sum of the transverse mode indices *n*+*m* by $S\times K$, will leave the frequency unaltered. Although configurations with a high degree of frequency degeneracy have been shown to allow closed geometric trajectories [5], so far the 2D transverse modes studied in such cavities are never performed. The experimental results reveal that the spontaneous 2D transverse modes in a frequency degenerate cavity are associated with the *Lissajous* wave patterns. The observed transverse patterns can be reconstructed very well.

三、結果與討論

The system schematic and the pump profile in the laser system are shown in Fig. 1. The experimental laser cavity that consists of one planar Nd:YVO4 surface, high-reflection coated at 1064 nm and high-transmission coated at 809 nm for the pump light to enter the laser crystal, and a spherical output mirror is analogous to the one described in Ref. [3]. The gain medium in the experiment is a-cut 2.0 at.% 1 mm length Nd:YVO4 microchip crystal. The absorption coefficient of the Nd:YVO4 crystal is about 40 mm^{-1} at 809 nm. The radius of curvature of the output coupler is 10 mm with the reflectivity of 98%. The pump source is a 1-W fiber-coupled laser diode (Coherent, F-81-800C-100) with a 0.1-mm of core diameter. The pump spot size on the crystal is controlled to be around 0.15~0.2 mm. Note that the output intensity profile from an ordinary fiber-coupled laser diode is a top-hat distribution. The top-hat pump profile, as usual, leads to a complicated multi-transverse HG mode without locking. With the special coupling condition, a fiber-coupled laser diode can have a doughnut output profile. Previously, we used the doughnut pump profile to successfully generate the pure LG mode.

For a stable cavity, the minimum integer *S* is in general equal to 3. With *R*=10 mm, the condition of $\Delta\nu_L/\Delta\nu_T=3$ can be obtained by setting the cavity length to be $d\approx 7.5$ mm. Under the condition of $\Delta\nu_L/\Delta\nu_T=3$, the transverse patterns are found to change drastically with the fine tune of the output coupler. Slightly adjusting the pump position and finely tuning the output coupler, several typical transverse patterns on the concave mirror are obtained and shown in Fig.2 (a)-(f). Although the different sharp pattern is obtained at a different cavity length, each pattern is related with a specific value of the cavity length and can be reproduced as long as the pump profile is a doughnut distribution. The difference between the cavity lengths of two sharp patterns is approximately 30-μm. The difference of the cavity lengths for different sharp patterns mainly arises from the fact that the effective cavity length depends on the order of the transverse mode, even though the dependence is very weak. The range of the cavity length for each sharp pattern to be structure stable is around 10-μm. When the cavity length does not meet the range of the locking modes, the transverse pattern is usually irregular and vague. It can be seen that the observed patterns are completely unlike a HG or LG mode. Interestingly, these patterns are well localized on the *Lissajous* figures that are classical periodic orbits for a 2D anisotropic harmonic oscillator with commensurate frequencies. The measurement of the optical spectrum evidences that all observed patterns are single frequency emissions. In other words, the transverse pattern formation can be interpreted as a spontaneous process of cooperative frequency locking. However, it is worthwhile to note that the frequency locking in the present experiment occurs among different transverse order with the help of different longitudinal order, while the generation of $LG_{0,\pm N}$ modes is the frequency locking within the same family of transverse modes operating in a single longitudinal mode.

Since the present cavity is set at the length of $\Delta\nu_L/\Delta\nu_T=3$, the family of the transverse modes $HG_{pK,q(N-K)}(x,y;\varpi_o)$ for a given index *N* and $p-q=\pm 3$ can be frequency locked by different longitudinal index $l=L\mp K$ for a given index *L*, where *K=0,1,2….N*. Substituting $m=pK$, $n=q(N-K)$, $l=L\mp K$, $p-q=\pm 3$, and $\Delta\nu_L/\Delta\nu_T=3$ into Eq. (2), the laser frequency of the family $HG_{pK,q(N-K)}(x,y;\varpi_o)$ can be found to be locked at $\nu_{L,N}=L(\Delta\nu_L)+(qN+1)(\Delta\nu_T)$. From the numerical analysis, the transverse patterns shown in Fig. 2 are found to be associated with the partially coherent states:

$$\Psi_{N,M}^{p,q}(x,y;\varpi_o)=\left[\sum_{K=J}^{N-J}\binom{N}{K}\right]^{-1/2}\sum_{K=J}^{N-J}\binom{N}{K}^{1/2} HG_{pK,q(N-K)}(x,y;\varpi_o) \quad (5)$$

where the index $M = N - 2J + 1$ represents the number of eigenstates used in the state $\Psi_{N,M}^{p,q}(x,y;\varpi_o)$. Note that the condition of $p - q = \pm 3$ must be satisfied to correspond to the observed patterns. Fig. 3 shows the numerically reconstructed patterns for the results shown in Fig. 2. It is clear that only 3~5 eigenstates are already sufficient to localize the wave patterns on the classical trajectories, even for high-order periodic orbits. In fact, Eq. (5) is the special case in the representation of the SU(2) coherent state [4,6,7] that is used to make the connection between wave functions and classical periodic trajectories in 2D confined systems. The words "partially coherent state" mean a partial sum of the SU(2) coherent state. The good agreement between the experimental and reconstructed patterns confirms that the interrelation between wave optics and geometrical optics is somewhat similar to that between quantum and classical mechanics. Such an analogy enables us to employ quantum theory in analyzing the formation of high-order laser transverse modes [4]. Reversely, the laser resonators can be deliberately designed to simulate the quantum phenomenon in mesoscopic physics [8-10]. Recently, Doya *et al* [10] have introduced the paraxial approximation to establish an analogy between light propagation along a multimode fiber and quantum confined systems. We believe that these analogies will continue to be exploited for understanding the physics of mesoscopic systems.

四、 結論

In conclusion, the formation of high-order transverse patterns in a laser resonator with a high degree of frequency degeneracy has been investigated. With a doughnut pump profile, the spontaneous transverse patterns are found to be associated with the *Lissajous* trajectories. With the partially coherent states, the observed transverse patterns can be explained very well. The nice explanation suggests that the laser resonators with identical functional form can be used to attain a more thorough understanding of the quantum-classical connection.

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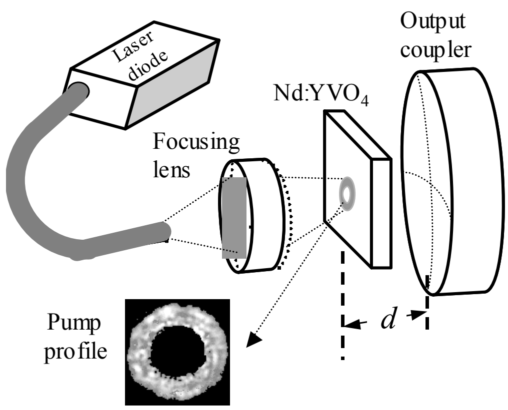

Fig.1

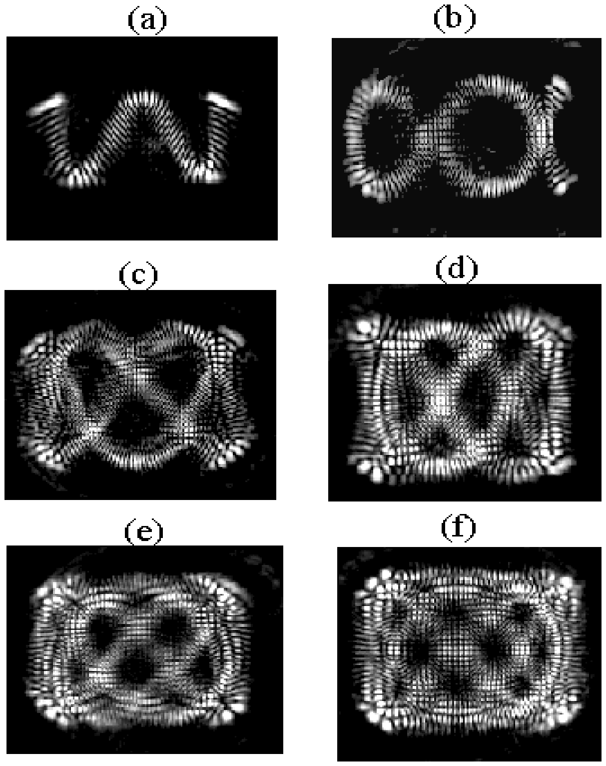

Fig.2

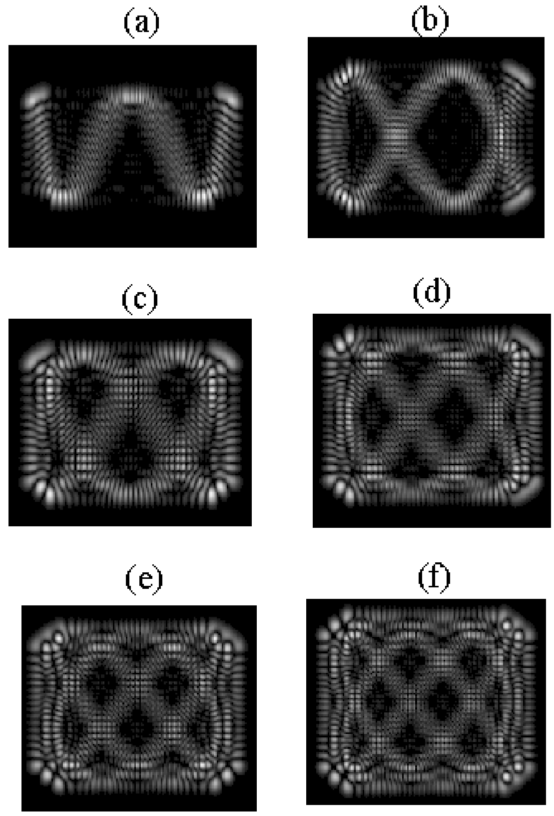

Fig.3