行政院國家科學委員會專題研究計畫　成果報告

自行車傳動系統齒數比最佳化設計(2/2)

計畫類別：個別型計畫
計畫編號：NSC92-2212-E-009-006-
執行期間：92 年 08 月 01 日至 93 年 07 月 31 日
執行單位：國立交通大學機械工程研究所

計畫主持人：曾錦煥

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摘要

本研究計畫將提供一系統化的過程，尋找自行車傳動系統的最佳齒數比，使其具有最佳之人因及機構設計的特性。此設計流程將騎乘者人因特性及傳動系統本身的機械特性納入設計需求，設計者依據此設計流程，即可設計出一組符合其需求之最佳齒數比。在設計的過程中，個別騎乘者的舒適度為目標函數的基礎；而齒盤間輔助上鏈結構的數目則被視為限制條件。

ABSTRACT

This project is going to provide the design procedure of the gear ratios of a freewheel-type bicycle transmission. This design procedure treats the ergonomic behaviors of cyclists and the mechanical behaviors of the transmission as the design requirements. Using this design procedure, a designer is able to obtain the optimum gear ratios. In this design process, the comfort of the cyclist is regarded as the bases of cost function. On the other hand, the number of shifting path between two sprockets is treated as a constraint.

Keywords: Bicycle; transmission; gear ratio; design procedure; optimum.

INTRODUCTION

The quality of a freewheel-type transmission depends on its ergonomic and mechanical behaviors. Lots of previous studies mentioned the ergonomic behaviors of cyclists and the mechanical behaviors of freewheel-type transmission [Seabury et al., 1977; Hagberg et al., 1981; Whitt and Wilson, 1982; Coast and Welch, 1985; Hull et al., 1988; Kyle, 1988; Marr, 1989; Feng et al., 1995; Wang et al., 1996; Neptune et al., 1997; Cheng, 1998; Cheng and Tseng, 1998; Neptune and Hull, 1999]. According to these studies, several ergonomic and mechanical costs functions as well as constraints are defined in this study. This study also proposes an integrated design procedure consisting of the ergonomic analysis, mechanical analysis and the optimization. Following this procedure, a designer can design a freewheel-type bicycle transmission with optimally ergonomic and mechanical behaviors systematically.

ERGONOMIC ANALYSIS

A freewheel-type bicycle transmission provides several shifting strategies, and the ergonomic behavior of a freewheel-type bicycle transmission depends on the shifting strategy chosen by a cyclist. The gear ratios in the optimum shifting strategy perform the best ergonomic behaviors of a freewheel-type bicycle transmission. The cost function of each shifting strategy can be defined as,

$$C = W_{\mu}\mu_W^i + W_{\sigma}\left(\sigma_W^i\right)^2.$$

The mean and standard deviation of cadence spreads in the i^{th} shifting strategy can be expressed as,

$$\mu_W^i = \frac{\sum_{j_{TR}=1}^{N_i} W_{cr_{i,j_{TR}}}}{N_i},$$

$$\left(\sigma_W^i\right)^2 = \sum_{j_{TR}=1}^{N_i}\left(W_{cr_{i,j_{TR}}} - \frac{\sum_{j_{TR}=1}^{N_i} W_{cr_{i,j_{TR}}}}{N_i}\right)^2.$$

The shifting strategy with the minimum cost is the optimum shifting strategy, whose cost is regarded as the cost of a freewheel-type transmission.

MECHANICAL ANALYSIS

In order to improve the percentage of successful gear shifting, the chainrings (cogs) in the main stream on the market are designed with the shifting paths. The more shifting paths are on the chainring (cog), the faster the gear shifting is. The shifting path can be divided into two types: straight and non-straight. The method for estimating the number of shifting path is described as follows:

Straight shifting path

While up-shifting, as shown in Fig. 1(a), the transit chain, which leaves the smaller chainring (cog) at T_S, may engage the tooth between points A and B on the larger chainring (cog). While down-shifting, the transit chain, which engages the smaller chainring (cog) at T_S, may leave the tooth between points C and D on the larger chainring (cog). $\angle T_S OA$ and $\angle COT_S$ are defined as the minimum phase angles; $\angle T_S OB$ and $\angle DOT_S$ are defined as the maximum phase angles.

For a given relative angle, every teeth on the smaller chainring (cog) T_S and every teeth T_L, between minimum phase angle and maximum phase angle, on the larger chainring (cog) is checked if there is a straight up-shifting or down-shifting path between T_S and T_L. By discretely scanning the relative angles, the numbers of non-straight up-shifting or down-shifting paths at each relative angle are obtained.

Non-straight shifting path

Cheng and Tseng proposed a method finding the maximum numbers of non-straight shifting paths that can be designed on a chainring (cog) [Cheng, 1998; Cheng and Tseng, 1998]. This method is modified and described briefly as follows:

While up-shifting, as shown in Fig. 1(b), the transit chain, which leaves the smaller chainring (cog) at T_S, may engage the tooth between points A and B on the larger chainring (cog). While down-shifting, the transit chain, which engages the smaller chainring (cog) at T_S, may leave the tooth between points C and D on the larger chainring (cog). $\angle T_S OA$ and $\angle COT_S$ are defined as the minimum phase angles; $\angle T_S OB$ and $\angle DOT_S$ are defined as the maximum phase angles.

For a given relative angle, every teeth on the smaller chainring (cog) T_S and every teeth T_L, between minimum phase angle and maximum phase angle, on the larger chainring (cog) build a non-straight up-shifting or down-shifting path between T_S and T_L. By discretely scanning the relative angles, the numbers of non-straight up-shifting or down-shifting paths at each relative angle are obtained.

DESIGN PROCEDURE

Based on the ergonomic and mechanical characteristics described above, a design procedure of freewheel-type transmission with optimally ergonomic and mechanical behavior is developed as follows:

Step 1. Form a design model

First of all, define the number of chainrings and cogs. Second, decide the weights of mean and standard deviation for calculating the ergonomic behavior. Third, choose the optimum cadence w_i and the duplicating ratio r for designing. Then, determine the maximum and the minimum gear ratios should be provided and estimate the numbers of teeth on the largest and the smallest chainring and cog. The design variables are the differences of the numbers of teeth between every two chainrings and cogs. This arrangement guarantees the numerical order of the numbers of teeth on chainrings (cogs).

Step 2. Form an optimization model

The optimization problem is stated as minimizing the cost function of ergonomic behavior and subjecting to constraints of mechanical behavior:

Minimize:

cost of optimum shifting strategy.

Subject to:

numerical order of the numbers of teeth on the smallest one and its neighbor,

numbers of up-shifting and down-shifting paths, straight or non-straight, at one relative angle at least, of every two neighbor gears are greater than zero,

standard deviation of cadence spreads is smaller than 2.5.

Considering the integral design variables in the optimization problem, a zero-order method has been chosen, that is a genetic algorithm (GA) [Holland, 1975]. A binary-string gene is used to represent the design variables. The number of bits occupied by each design variable in a binary string needs to be defined before optimization. While evaluating the genes, the binary string is decomposed to the design variables according to the occupied numbers of bit by each design variable. Then, the feasibility and cost of the design variables can be obtained. Larger defined number of bits creates more extensive design domain, results in a smaller proportion of the feasible domain to the design domain and spends more computing time.

Step 3. Optimize the design model

First of all, GA initializes a population of genes. Then, GA enters an iterative scheme, which is called a generation, as follows: Each gene is evaluated from the cost function and the constraints. Infeasible genes are valued at a penalty of 10000. According to the costs of genes and the probability of crossover and mutation, GA selects the genes for crossover and mutation from the population, generates new offspring, replaces the worse genes in the population by offspring, and then creates a new population. The process is continued until the criterion of the number of generations is satisfied. Finally, the gene with the minimum cost in the population of the last generation represents the optimum numbers of teeth on chainrings and cogs.

EXAMPLES

In this section, a commercial freewheel-type transmission is chosen for testing the design procedure. The numbers of teeth on the largest and smallest chainrings and cogs in the commercial transmission are specified for the new design. Then the design procedure estimates the numbers of teeth on the other chainrings and cogs in the new design.

Table 1 lists the comparisons between the new designs and the commercial transmission. In an ergonomic aspect, the means of the cadence spreads of the new designs and the commercial transmission are almost the same; nevertheless, both new designs reduce the standard deviation of the cadence spreads of the commercial transmission. In a mechanical aspect, only a few of numbers of shifting paths in the new designs are different with those in the commercial transmission. Consequently, this design procedure produces better designs than the commercial transmission.

CONCLUSIONS

This work presents a systematic procedure for designing a freewheel-type bicycle transmission. This procedure takes the mean and standard deviation of cadence spreads of the optimum shifting strategy as the cost function, and meantime treats the numbers of shifting paths as the constraints. The design examples are put to proof that this procedure generates the freewheel-type transmission with better ergonomic and mechanical behaviors than the commercial product.

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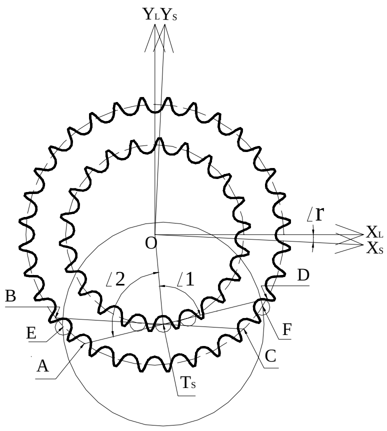
Number of	Design A																
Teeth on chainrings	44		32		22												
Up-shifting path		4		4													
Down-shifting path		4		2													
Teeth on cogs	32		28		24		20		18		16		14		12		11
Up-shifting path		4		4		4		2		2		2		2		1	
Down-shifting path		4		4		4		2		2		2		2		1	
Mean	10.05																
Standard deviation	2																

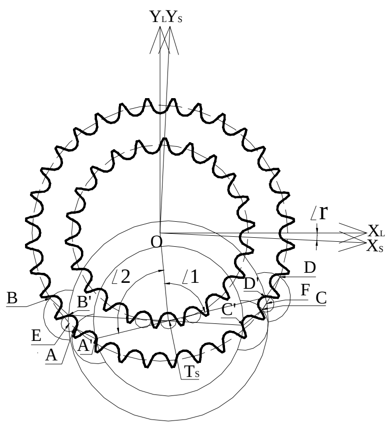
Number of	Design B																
Teeth on chainrings	44		32		22												
Up-shifting path		4		4													
Down-shifting path		4		2													
Teeth on cogs	32		28		24		22		19		17		15		13		11
Up-shifting path		4		4		2		2		2		2		2		2	
Down-shifting path		4		4		2		2		2		2		1		1	
Mean	10.05																
Standard deviation	1.99																

Number of	Commercial product																
Teeth on chainrings	44		32		22												
Up-shifting path		4		4													
Down-shifting path		4		2													
Teeth on cogs	32		28		24		21		18		16		14		12		11
Up-shifting path		4		4		3		3		2		2		2		1	
Down-shifting path		4		4		3		3		2		2		2		1	
Mean	10.05																
Standard deviation	2.07																

Table 1 Optimum designs of a freewheel-type bicycle transmission.



(a) Straight shifting path.



(b) Non-straight shifting path.

Fig. 1 Shifting path.