

行政院國家科學委員會專題研究計畫 成果報告

具有 AR(1) 誤差線性迴歸模型的廣義及 feasible 廣義截斷  
平均數

計畫類別：個別型計畫

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執行期間：92 年 08 月 01 日至 93 年 07 月 31 日

執行單位：國立交通大學統計學研究所

計畫主持人：陳鄰安

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# 行政院國家科學委員會補助專題研究計畫成果報告

具有AR(1)誤差線性迴歸模型的廣義及feasible廣義截斷平均數

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共同主持人：

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中 華 民 國 9 0 年 1 0 月 3 1 日

# 行政院國家科學委員會專題研究計畫成果報告

## 具有AR(1)誤差線性迴歸模型的廣義及feasible廣義截斷平均數

計畫編號: NSC92-2118-M-009-006

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主持人:陳鄰安

研究人員:

執行機構及單位:國立交通大學統計研究所

### 一、中文摘要

我們提出廣義及插入法廣義截斷平均數來處理具有自我相關的線性迴歸問題，我們導出大樣本理論結果，此一結果提供了穩健性估計量其具有廣義估計性質的估計量。

Abstract

We propose generalized and pseudo generalized trimmed means for the linear regression with AR(1) errors model. These will play the role of robust type generalized and pseudo generalized estimators for this regression model. Their asymptotic distributions are developed.

### 二、Results and Discussions

For some regression models such as linear regression with AR(1) errors or the seemingly unrelated regression model, the generalized least squares estimator (GLSE) and the pseudo generalized least squares estimator (PGLSE) have the advantage that their variances (or asymptotic variances) are smaller than that of the least squares estimator (LSE). However, the GLSE and the PGLSE are sensitive to departures from normality and to the presence of outliers. Hence, extending these concepts to robust estimation is an interesting topic in regression analysis. The concept of developing robust type generalized estimators in regression analysis is not new. Koenker and Portnoy (1990) introduced this interesting idea and developed the generalized M-estimators for the estimation of regression parameters of the multivariate regression model. Although considering only generalized estimation, their approach initiated interest in robust type generalized and pseudo generalized estimators for estimation of regression parameters.

Rather than multivariate regression, we consider the linear regression with AR(1) errors model

$y_i = x_i' \beta + \epsilon_i$  and  $\epsilon_i = \rho \epsilon_{i-1} + e_i$  where  $|\rho| < 1$ ,  $e_i$  are i.i.d. variables with mean zero and

variance  $\sigma^2$  and  $x_i$  is a known design p-vector with value 1 in its first element.

From the regression theory on the estimation of  $\beta$ , it is known that, when  $\rho$  is known, the GLSE and, when  $\rho$  is unknown, the PGLSE have (or asymptotically have) the same covariance matrix, which is smaller than that of the LSE.

To see the sensitivity of the GLSE and the PGLSE, let  $X' = (x_1', \dots, x_n')$  and  $\Omega = \text{Cov}(\epsilon)$  with  $\epsilon = (\epsilon_1, \dots, \epsilon_n)'$ . The GLSE and the PGLSE both have a (asymptotic) covariance matrix of the form  $\sigma^2 \text{inv}(X' \text{inv}(\Omega) X)$ . The sensitivity is clear from the fact that  $\sigma^2$  could be arbitrary large when  $e_i$  has a heavy tailed distribution.

The fact that  $\sigma^2$  is sensitive to the error distribution motivates us to consider robust estimators that have a (asymptotic) covariance matrix of the form  $\gamma \text{inv}(X' \text{inv}(\Omega) X)$ , where robustness means that  $\gamma$  is insensitive to heavy tailed distributions.

Based on the regression quantiles of Koenker and Bassett (1978), we will introduce the generalized trimmed mean (GTM) and the pseudo generalized trimmed mean (PGTM) to play the role of robust type generalized and pseudo generalized estimators for the linear regression with AR(1) errors model.

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Self Evaluation: Extending the concepts of BLUE and generalized estimation for linear regression model, we propose a concept of robust type generalized estimator. This paper has been submitted for publication and is accepted by *Statistics and Probability Letters of Statistics*, 31, 942 (2003).