(II)

An investigation on the nonlinear buckling and postbuckling behavior of beam structures with open thin-walled cross section (II)

Abstract

A consistent co-rotational total Lagrangian finite element formulation and numerical procedure for the geometric nonlinear buckling and postbuckling analysis of generic thin-walled beams with open section is presented. Numerical examples are presented to demonstrate the accuracy and efficiency of the proposed method.

Keywords Co-rotational function, Thin-walled beam

1 Introduction

The buckling and postbuckling analysis of thin-walled beams with open section have been the subject of considerable research [1-5]. The object of this paper is to present a co-rotational total Lagrangian finite element formulation for the geometric nonlinear buckling and postbuckling analysis of generic thin-walled beams with open section.

The formulation of beam element proposed in [6~8] is modified and employed here. Here, the third order terms of twist rate of the beam axis is also considered.

An incremental-iterative method based on the Newton-Raphson method combined with constant arc length of incremental displacement vector is employed for the solution of nonlinear equilibrium equations. Numerical examples are presented to demonstrate the accuracy and efficiency of the proposed method

2 Finite element formulation
2.1 Basic assumptions

(1) The beam is prismatic and slender, and the Euler-Bernoulli hypothesis is valid.
(2) When the longitudinal normal strain at the centroidal axis relevant to the twist about the shear center axis is excluded, the unit extension of the centroid axis of the beam element corresponding to the rest of longitudinal normal strain is uniform.
(3) The cross section of the beam element does not deform in its own plane and strains within this cross section can be neglected.

2.2 Coordinate systems

In this paper, a co-rotational total Lagrangian formulation is adopted. In order to describe the system, we define four sets of right handed rectangular Cartesian coordinate systems:

(1) A fixed global set of coordinates, X_i^G ($i = 1, 2, 3$) (see Fig. 1).
(2) Element cross section coordinates, x_i^S ($i = 1, 2, 3$) (see Fig. 1).
(3) Element coordinates; x_i ($i = 1, 2, 3$) (see Fig. 1).
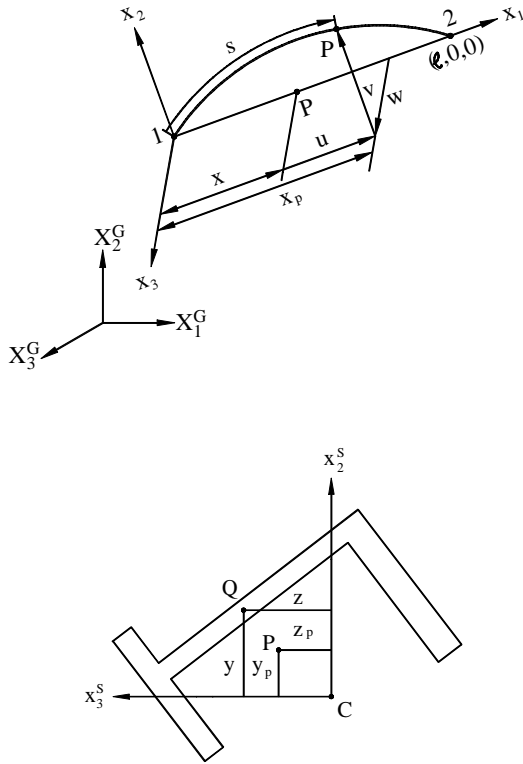(4) Load base coordinates, X_i^P ($i = 1, 2, 3$).

Fig. 1. Coordinate systems.

2.3 Kinematics of beam element

Let Q (Fig. 1) be an arbitrary point in the beam element, and P be the point corresponding to Q on the shear center axis. The position vector of point Q in the undeformed and deformed configurations may be expressed as

$$\mathbf{r}_0 = x\mathbf{e}_1 + (y - y_p)\mathbf{e}_2 + (z - z_p)\mathbf{e}_3 \qquad (1)$$

and

$$\mathbf{r} = x_p(x)\mathbf{e}_1 + v(x)\mathbf{e}_2 + w(x)\mathbf{e}_3$$
$$+ \theta_{1,x}\check{S}\mathbf{e}_1^S + (y - y_p)\mathbf{e}_2^S + (z - z_p)\mathbf{e}_3^S \qquad (2)$$

where y_p and z_p, and y and z are the x_2^S and x_3^S coordinates of point P and Q referred to the element cross section coordinates, respectively, $x_c(x)$, $v(x)$, and $w(x)$ are the x_1, x_2 and x_3 coordinates of point P, respectively, in the deformed configuration, $\check{S} = \check{S}(y, z)$ is the Saint Venant warping function for a prismatic beam of the same cross section, and $\mathbf{e}_i$ and $\mathbf{e}_i^S$ ($i = 1, 2, 3$) denote the unit vectors associated with the x_i and x_i^S axes, respectively. Note that $\mathbf{e}_i$ and $\mathbf{e}_i^S$ are coincident in the undeformed state.

The relation between the vectors $\mathbf{e}_i$ and $\mathbf{e}_i^S$ ($i = 1, 2, 3$) in the element coordinate system may be obtained as [6]

$$\mathbf{e}_i^S = \mathbf{R}\mathbf{e}_i \qquad (3)$$

Here, the lateral deflections of the shear center axis, $v(x)$ and $w(x)$, and the rotation about the shear center axis, $\theta_1(x)$, are assumed to be the Hermitian polynomials of x.

2.4 Element nodal force vector

The element nodal force corresponding to the implicit nodal parameters is obtained from the virtual work principle in the current element coordinates.

2.5 Element tangent stiffness matrices

The element tangent stiffness matrix corresponding to the explicit nodal parameters (referred to as explicit tangent stiffness matrix) may be obtained by differentiating the element nodal force vector with respect to explicit nodal parameters.

2.6 Load stiffness matrix

Here, the conservative moments generated by conservative force or forces (with fixed directions) are considered, and the ways for generating conservative moment and the corresponding load stiffness matrix proposed in [7] are employed and not repeated here.

2.7 Equilibrium equations

The nonlinear equilibrium equations may be expressed by

$$\mathbf{j} = \mathbf{F} - \lambda\mathbf{P} = 0 \tag{4}$$

where $\mathbf{j}$ is the unbalanced force between the internal nodal force $\mathbf{F}$ and the external nodal force $\lambda\mathbf{P}$, where λ is the loading parameter, and $\mathbf{P}$ is a reference loading.

2.8 Criterion of the buckling state

Here, the zero value of the tangent stiffness matrix determinant is used as the criterion of the buckling state. The tangent stiffness matrix of the structure is assembled from the element stiffness matrix and load stiffness matrix.

3 Numerical studies

An incremental-iterative method based on the Newton-Raphson method combined with constant arc length of incremental displacement vector [6] is employed for the solution of nonlinear equilibrium equations. The bisection method of the arc length is proposed in [8] is used here to find the buckling load. In order to initiate the secondary path, at the bifurcation point a perturbation displacement proportional to the first buckling mode is added [9].

The example considered here is a cantilever beam of monosymmetric angle section with a vertical force P applied at the apex of the centerline of the end cross section as shown in Fig. 2. The clamped end of the beam is warping free.
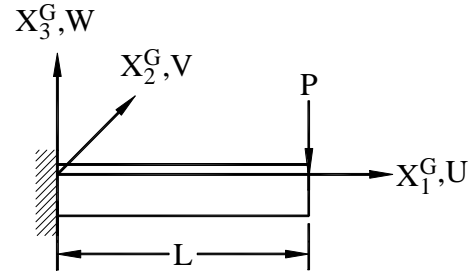
End cross section
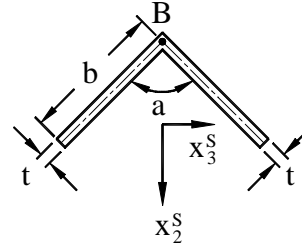
• Loading point

Fig. 2. Cantilever beam subjected to end force.

The geometry and material (Aluminum) properties are as follows: $L = 41.75\ in$, $b = 1\ in$, $t = 0.032\ in$, Young's modulus $E = 10^4\ ksi$, the shear modulus $G = 3.750 \times 10^3\ ksi$, the specific weight $\gamma = 169\ lb/ft^3$. The self-weight of the beam is 0.2613 lb. Three angle cross sections having included angles of $\gamma = 45.5'$, $71'$, and $91'$ are considered.

The present results are obtained using 40 elements. The load-deflection curves of of the present study together with the experimental results given in [1] are shown in Fig. 3. The initial deflections caused by the self-weight are excluded in Fig. 3. Discrepancies between the present solutions and the experimental results may be attributed to initial imperfections, and/or distortion of the root section, and/or yielding at the root of the beam due to excessive torsional twisting, which occur in experiments.
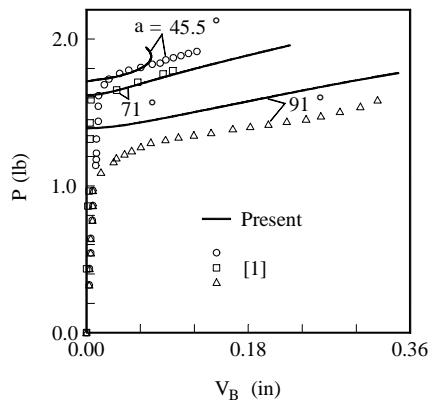
Fig. 3. Load-tip displacements for cantilever beam subjected to end force .

4 Conclusions

This paper has proposed a consistent co-rotational total Lagrangian finite element formulation and numerical procedure for the geometric nonlinear buckling and postbuckling analysis of generic thin-walled beams with open section. The third order term of twist rate in the element nodal forces is also considered. An incremental-iterative method based on the Newton-Raphson method combined with constant arc length of incremental displacement vector is employed for the solution of nonlinear equilibrium equations. The zero value of the tangent stiffness matrix determinant of the structure is used as the criterion of the buckling state. From the numerical examples studied, it is found that the agreement between the prebuckling displacements and buckling loads of the present study and those given in the literature is very good.

5 References

[1] Engel and J.N. Goodier, Measurements of torsional stiffness changes and instability due to tension, compression and bending, J. Appl. Mech. ASME (1953) 553-560

[2] S.P. Timoshenko and J.M. Gere, *Theory of Elastic Stability*, 2nd ed. McGraw-Hill, N.Y., (1963).

[3] D.O. Brush and B.O. Almroth, *Buckling of Bars, Plates and shells*, McGraw-Hill, N.Y., (1975).

[4] H. Ziegler, *Principles of Structural Stability*, Birkhauser Verlag Basel., (1977).

[5] B.A. Izzuddin and D.L. Smith, *Large-displacement analysis of elastoplastic thin-walled frames, part I: formulation and implementation and part II: verification and application*, J. Struct. Engrg. ASCE 122 (1996) 905-925.

[6] K.M. Hsiao, *Corotational total Lagrangian formulation for three-dimensional beam element*, AIAA Journal 30 (1992) 797-804.

[7] K.M. Hsiao, R.T. Yang and W.Y. Lin, *A consistent finite element formulation for linear buckling analysis of spatial beams*, Comput. Methods Appl. Mech. Engrg. 156 (1998) 259-276.

[8] K.M. Hsiao and W.Y. Lin, *A co-rotational finite element formulation for buckling and postbuckling analysis of spatial beams*, Comput. Methods Appl. Mech. Engrg. (accepted) (1999).

[9] T. Matsui and O. Matsuoka, *A new finite element scheme for instability analysis of thin shells*, Internat. J. Numer. Meths. Engrg. 10 (1976) 145-170.