一、中英文摘要

本研究為三年期計畫，其目的在設計一求解非線性規劃問題之全域最佳化運算軟體-GOPT。第一部分為評估目前常用的全域最佳化方法，再提出一逐段線性演算法（Piecewise Linearization Algorithm）與凸化策略（Convexification Strategies）求解一般非線性及整數規劃問題。此演算法可證明能收斂至全域最佳解。第二部分為依所提出的演算法發展最佳化運算軟體，此軟體係架構在LINGO上。此軟體的優點為：1、能求解一般非線性規劃問題得到全域最佳解。2、能求解包含非正數的非線性規劃問題。3、使用者可透過網路作遠端運算而不需下載軟體。4、具分散式運算功能。5、系統能整合啟發式演算法以增進系統運算效能。6、可以圖示運算過程與結果。

關鍵詞：逐段線性法、全域最佳化、非線性及整數規劃

Abstract

This study intends to design optimization software, GOPT, which can build the global optimum for general nonlinear and integer programming problems. The system performs the proposed piecewise linearization algorithm and convexification strategies to solve the problem and finally obtain a global optimum within the tolerance. The first part of this study reviews current global optimization methods then formulate a piecewise linearization algorithm proven to converge to a global optimum. The second part of this study is to design the optimization software based on the developed algorithms. The developed optimization software is built following the framework of LINGO. The advantages of the designed software are listed below:

1. Guarantee to obtain a global optimum of a nonlinear programming problem;
2. Allow a nonlinear programming problem to include non-positive variables;
3. Operate remote computation without downloading the software;
4. Support distributed computation;
5. Integrate with heuristic methods to enhance the computational efficiency;
6. Display the solution process and results graphically.

Keywords: piecewise linearization, global optimum, nonlinear and integer programming

二、前言

非線性規劃(NP)及整數規劃(IP)問題在工程與管理上的應用十分寬廣，目前求解這類問題常用的軟體有：LINGO，AMPL，GAM，MATLB，SAS，EXCEL。其中又以LINGO的使用最為普遍。這些軟體在求解 NP 及 IP 問題上常遇及的問題為：

1、對 NP 問題只能得到局部最佳解。

2、對 IP 問題只能處理線性模式問題而不易處理非線性整數規劃問題。

就第一個問題言，過去評量最佳化軟體的準則，多以解題速度為主。但隨著電腦解題速度的進步，解題品質已愈形重要。使用者已不能滿足於現有軟體的只能求得局部最佳解。就第二個問題言，由於非線性整數規劃問題在工業工程與機械的使用日益普遍，故現有軟體已不敷需求。最佳化軟體在業界及學界的市場甚大。單以LINGO 而言，其在全球學界的發行套數即超過十萬套，每年並有更新版推出，其每年產值超過一億美金。隨著管理與工程科學的日趨精緻，對最佳化軟體的需求將更擴大。

三、研究目的

本研究的目的即在發展一全域最佳化運算軟體。雖然許多管理上的最佳化問題，都屬於非線性規劃問題，目前尚無較好的最佳化運算軟體能求解此類問題，本研究提出以逐段線性法求解此問題以求得一全域最佳解。此類問題目標式和限制式

會包含帶有多次方的乘積項，決策變數可能是連續變數或整數，本法線性化技術是來自分段規劃法，先將多項式轉換成一連串絕對值項總和，再將這些絕對值項一一線性化，以將一個混和整數多項式規劃問題轉換成線性混和整數規劃問題。本研究所提出的方法可求得問題的近似全域最佳解。

這是三年期的研究計劃，目前理論準備工作多已完成，部份類型的檢測結果亦正確。本研究擬一方面對理論部份再予深究，另一方面則繼續進行軟體的設計。同時也將現有文獻上的各類 NP 及 IP 問題建為題庫，再予測試之以確保軟體之正確與穩定。

未來這一年的研究計劃，將著重於系統設計與系統效能改善方面，主要在於發展分散式演算法，並整合啟發式方法，並從例子測試與驗證本法與傳統方法的解題效果。

四、文獻探討

Currently, there are three commonly used approaches for solving signomial programming problems. The first approach is Multilevel Single Linkage Technique (MSLT) (Rinnooy and Timmer, 1987, Li and Chou, 1994); the second approach is Generalized Geometric Programming Technique (GGPT) (Maranas and Floudas, 1996); and the third approach is Reformulation Linearization Technique (RLT) (Sherali, 1998, Sherali and Tuncbilek, 1992). The features of these methods are briefly reviewed as follows.

(i) Multilevel Single Linkage Technique (MSLT) is a stochastic method for reaching a global optimum of a nonlinear program. Utilizing MSLT method, Li and Chou (1994) solved a design optimization problem by first generate enough starting points to search for most local optima within the feasible region. A global optimum is then found at a prespecified sufficiently high confidence level. The difficulty of MSLT approach is that it requires to solve a huge amount of nonlinear optimization problems based on various starting points. For instance, for solving the well known pressure vessel problem (Sandgren, 1990, Li and Chen, 1994) to obtain a global optimum at confidence level of 98%, it requires to solve 240 highly nonlinear programs.

(ii) Generalized Geometric Programming Technique (GGPT) could solve a signomial programming problem when all variables in are positive. By utilizing logarithmic/ exponential transformations employed in geometric programming (Beightler et al., 1979), the initial signomial program can be reduced to a problem where both the objective and constraints are decomposed into the difference of two convex functions. A global optimum can then be reduced through the successive refinement of a convex relaxation and the subsequent solution of a series of nonlinear convex optimization problems. A major difficulty of applying GGPT in solving a signomial programming problem is that the lower bounds on the variables are not permitted to be zero.

(iii) Reformulation Linearization Technique (RLT) developed by Sherali (1998), and Sherali and Tuncbilek (1992) derived a generic algorithm to solve a signomial program. Their algorithm generates nonlinear implied constraints by taking the products of bounding terms in the constraints set to a suitable order. The resulting problem is subsequently linearized by defining new variables, one for each polynomial term appearing in the problem. By incorporating appropriate bound factor products in their RLT scheme, and employing a suitable partitioning technique, a convergent branch and bound algorithm has been developed. Although RLT algorithm is very promising in solving a signomial programming problem, a major difficulty of RLT method is that it might generate a huge amount of new constraints.

五、研究方法

This project addresses a Signomial Programming (SP) problem which seeks a global optimum to a signomial objective function subject to a set of signomial constraint functions. Such optimization problems occur quite frequently in various engineering design, location-allocation, chemical process and management problems (Beightler et al. 1976, Sandgren 1990, and Fu et al., 1991). The mathematical formulation of this problem is given below.

SP:

Minimize $Z(X) = \sum_{p}^{T_o} c_p z_p$

subject to $\sum_{q}^{T_k} h_{kq} w_{kq} \le l_k$ (for each constraint k), $k = 1,2,\ldots,K$,

$z_p = x_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}}$,

$z_{kq} = x_1^{\beta_{kq1}} x_2^{\beta_{kq2}} \ldots x_n^{\beta_{kqn}}$,

where $\underline{x}_i \le x_i \le \bar{x}_i$ ($\underline{x}_i$ and $\bar{x}_i$ are respectively the lower and upper bounds of variables x_i) c_p, α_{pi}, h_{kq}, β_{kqi}, l_k are constants which are unrestricted in sign.

Suppose c_p, h_{kq}, l_k are non-negative constants, then SP is called a Posynomial Program (Bazara, 1993). If α_{pi}, β_{kqi} are positive integers and $c_p \ge 0$, $h_{kq} \ge 0$, $l_k \ge 0$, then SP is called a Polynomial Program (Sherali and Tuncbilek, 1992). Three strategies used in solving a signomial program are discussed below:

(1) Strategies of treating non-positive variables:

Consider the signomial terms $z_p = x_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}}$ in SP where $0 \le x_i \le \bar{x}_i$. If there exists $x_i = 0$ for $i \in \{1, 2, \ldots n\}$ then $z_p = 0$; if $x_i \ge \varepsilon$ for all $i \in \{1, 2, \ldots n\}$ where ε is a small noticeable positive value (as $\varepsilon = 10^{-6}$) then z_p can be represented as $1nz_p = \sum \alpha_{pi} x_i$. Here we use a set of inequalities to express z_p, described by the proposition below:

Proposition 1:

A signomial term $\mathrm{c}\mathrm{x}_1^{\alpha_1}\,\mathrm{x}_2^{\alpha_2} \ldots \mathrm{x}_n^{\alpha_n}$, where $c, \alpha_1, \alpha_2, \ldots, \alpha_n$ are real values and $0 \le x_i \le \bar{x}_i$, can be replaced by term cz, z, x_i and x_i should satisfy following inequalities:

(i) $0 \le z \le \bar{z}\lambda$ (for $c > 0$) or
$0 \ge -z \ge -\bar{z}\lambda$ (for $c < 0$)

(ii) $\bar{z}(\lambda - 1) + y \le z \le y$ (for $c > 0$) or
$\bar{z}(1 - \lambda) - y \ge -z \ge -y$ (for $c < 0$)

(iii) $\varepsilon\lambda_i \le \mathrm{x}_i \le \bar{\mathrm{x}}_i\, \lambda_i$ for $\mathrm{i} = 1, 2, \ldots,$ n

(iv) $0 \le \lambda \le \lambda_i$ for $\mathrm{i} = 1, 2, \ldots,$ n

(v) $\lambda \ge \sum_{i=1}^{n} \lambda_i - \mathrm{n} + 1$

(vi) $x_i^{`} + \bar{x}_i(\lambda_i - 1) \le x_i \le x_i^{`} + \bar{x}_i(1\text{-}\lambda_i)$

(vii) $1ny = \sum_{i=1}^{n} \alpha_{\mathrm{i}} 1nx_i$

(viii) $x_i^{`} \ge \varepsilon$

where $\bar{z}$ is a constant, $\bar{z} = \mathrm{Max}\ z$, ε is a positive noticeable small value, λ_i are 0-1 variables.

Proof:

If for any $i \in \{1, 2, \ldots n\}$, then from (iii) and (iv) to know $\lambda_i = \lambda = 0$, which results in $z = 0$ based on (i).

If $x_i \ge \varepsilon$ for all $i \in \{1, 2, \ldots n\}$ then from (iii) (iv) and (v) to know $\lambda_i = \lambda = 1$. We then have $x_i^{`} = x_i$ and $z = y = x_1^{\alpha_1}\, x_2^{\alpha_2} \ldots x_n^{\alpha_n}$ base on (ii), (vi) and (vii).

Both cases imply that $cx_1^{\alpha_1}\, x_2^{\alpha_2} \ldots x_n^{\alpha_n} = cz$ ■

(2) Strategies of piecewise linearization:

Hereby we discuss some propositions

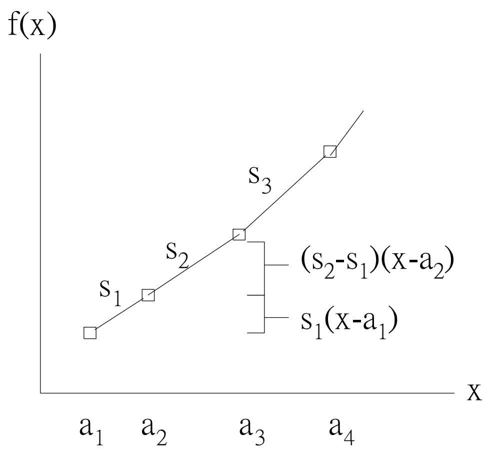
of formulating a non-zero signomial term by a piecewise linear function as follows:

Proposition 2:

Let $f(x)$ be the piecewise linear function of x, as depicted in Figure 1, where a_i, $i = 1,2,...,n$ are the break points of $f(x)$, and s_i, $i = 1,2,...,n$ are the slopes of line segments between a_i and a_{i+1}. $f(x)$ can be expressed as the sum of absolute terms:

$$f(x) = a_1 + s_1(x - a_1) + \sum_{i=2}^{n-1} \frac{s_i - s_{i-1}}{2}(|x - a_i| + x - a_i)$$

Figure 1



This proposition can be examined as follows:

(i) If $x = a_1$ then $f(x) = f(a_1)$.

(ii) If $x \le a_2$ then

$$f(x) = f(a_1) + \frac{f(a_2) - f(a_1)}{a_2 - a_1}(x - a_1)$$
$$= f(a_1) + s_1(x - a_1)$$

(iii) If $x \le a_3$ then

$$f(x) = f(a_1) + s_1(x - a_1) + s_2(x - a_2)$$

$$= f(a_1) + s_1(x - a_1) - s_1(x - a_2) + s_2(x - a_2)$$

$$= f(a_1) + s_1(x - a_1) + \frac{s_2 - s_1}{2}(|x - a_2| + x - a_2).$$

A posynomial term $z = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$ where x_i $(i = 1,2,...,n)$ are positive variables, $0 < a_{i1} \le x_i \le a_{im_i}$, can be approximately expressed as $\hat{z}$ below

$$\hat{z} = e^{b_1} + s_1(\sum_{i=1}^{n} \alpha_i \ln \hat{x}_i - b_1) +$$
$$\sum_{j=2}^{m} \frac{s_j - s_{j-1}}{2}(\left|\sum_{i=1}^{n} \alpha_i \ln \hat{x}_i - b_j\right| + \sum_{i=1}^{n} \alpha_i \ln \hat{x}_i - b_j)$$

where $b_1, b_2, ..., b_m$ are the break points of the function $\ln z$, s_j are the slopes of line segments between b_j and b_{j+1}; and $\ln \hat{x}_i$ is the linear approximation of $\ln x_i$, expressed as

$$\ln \hat{x}_i = \ln a_{i1} + t_{i1}(x_i - a_{i1}) +$$
$$\sum_{l=2}^{m_i} \frac{t_{il} - t_{i,l-1}}{2}(|x_i - a_{il}| + x_i - a_{il})$$

in which $a_{i1}, a_{i2}, ..., a_{im_i}$ are the break points of the function, and t_{il} are slopes of line segments between a_{il} and $a_{i,l+1}$.

(3) Strategies of convexification:

For signomial terms with specific features, there are some more computationally efficient convexification strategies. To simplify the expression, here we take a signomial term with three variables for instance to illustrate the convexification techniques. Consider the following propositions:

Proposition 3:

A twice-differentiable function $f(X) = cx_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$ is a convex function in one of the following conditions.

(i) $c \ge 0$, $\alpha_1, \alpha_2, \alpha_3 \le 0$, and α_i are even numbers for corresponding $x_i < 0$, i=1, 2, 3. (i.e, if $x_i < 0$, then α_i have to be an even number. Otherwise, α_i can be an odd or even number.)

(ii) $c < 0$, $0 \le \alpha_1, \alpha_2, \alpha_3 < 1$, $\sum_{i=1}^{3} \alpha_i \le 1$, and

$x_1, x_2, x_3 \geq 0$.

(iii) $c < 0$, $\alpha_1, \alpha_2, \alpha_3 \leq 0,$ and odd number of α_i are odd for corresponding $x_i < 0$, i=1, 2, 3. (Referring to Tsai et al., 2002)

Proof:

Denote $H(X)$ as the Hessian matrix of $f(X)$, and denote H_i as the ith principal minor of a Hessian matrix $H(X)$ of $f(X)$. The determinant of H_i can be expressed as

$$\det H_i = (-1)^i \left(\prod_{j=1}^{i} c\alpha_j x_j^{i\alpha_j - 2}\right)\left(1 - \sum_{j=1}^{i} \alpha_j\right) \quad \text{for}$$

$i = 1,2,3$.

(i) Since $c \geq 0$, $\alpha_1, \alpha_2, \alpha_3 \leq 0,$ and α_i are even numbers for corresponding $x_i < 0$ (i=1, 2, 3), we can get $\det H_1 \geq 0$, $\det H_2 \geq 0$, and $\det H_3 \geq 0$. Hence, $f(X)$ is convex.

(ii) Since $c < 0$, $0 \leq \alpha_1, \alpha_2, \alpha_3 < 1,$ $\sum_{i=1}^{3} \alpha_i \leq 1$, and $x_1, x_2, x_3 \geq 0$, we can have $\det H_1 \geq 0$, $\det H_2 \geq 0$, and $\det H_3 \geq 0$. Hence, $f(X)$ is convex.

(iii) If odd number of α_i are odd for corresponding $x_i < 0$ (i=1, 2, 3) , then $x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} < 0$. Since $c < 0$, $\alpha_1, \alpha_2, \alpha_3 \leq 0,$ and $x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} < 0$, we can have $\det H_1 \geq 0$, $\det H_2 \geq 0$, and $\det H_3 \geq 0$. Therefore, $f(X)$ is convex.

For a given signomial term z, if z can be converted into a set of convex terms satisfying Proposition 3, then the whole solution process is more computationally efficient. Under such a condition, z is unnecessary to do exponential-based decomposition.

Remark 1:

For a signomial term $z = cx_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$, $c \geq 0$, $\alpha_1, \alpha_2, \alpha_3 \leq 0,$ $x_1 < 0,$ $x_2, x_3 > 0$, if α_1 is even, thenz is convex. Otherwise, z can be expressed as

$z = cx_{11}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} + cx_{12}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$

($\underline{x_1} \leq x_{11} \leq 0,\ 0 < x_{12} \leq \overline{x_1}$) where $cx_{12}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$ is convex.

For instance, $z = x_1^{-1} x_2^{-2} x_3^{-1}$ with $-5 \leq x_1 \leq 5,\ 0 < x_2, x_3 \leq 5$ can be expressed as $z = x_{11}^{-1} x_2^{-2} x_3^{-1} + x_{12}^{-1} x_2^{-2} x_3^{-1}$ ($-5 \leq x_{11} \leq 0,\ 0 < x_{12} \leq 5$) where the term $x_{12}^{-1} x_2^{-2} x_3^{-1}$ is a convex term and need no transformation, and the term $x_{11}^{-1} x_2^{-2} x_3^{-1}$ can be transform into some proper forms solvable by Floudas' methods.

Remark 2:

For a signomial term $z = cx_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$, $c < 0$, $\alpha_1, \alpha_2, \alpha_3 \leq 0,$ $x_1 < 0,$ $x_2, x_3 > 0$, if α_1 is odd, thenz is convex. Otherwise,

$z = cx_{11}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} + cx_{12}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$

($\underline{x_1} \leq x_{11} \leq 0,\ 0 < x_{12} \leq \overline{x_1}$) where $cx_{11}^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$ is a convex term.

For instance, $z = -x_1^{-1} x_2^{-2} x_3^{-1}$ with $-5 \leq x_1 \leq 5,\ 0 < x_2, x_3 \leq 5$ can be expressed as $z = -x_{11}^{-1} x_2^{-2} x_3^{-1} - x_{12}^{-1} x_2^{-2} x_3^{-1}$ ($-5 \leq x_{11} \leq 0,\ 0 < x_{12} \leq 5$) where $-x_{12}^{-1} x_2^{-2} x_3^{-1}$ is a convex term and need no transformation, and $-x_{11}^{-1} x_2^{-2} x_3^{-1}$ can be transform into some proper forms solvable by Floudas' methods.

Following the above discussion, here we take a signomial term with three variables for instance to describe the strategy of convexification. The strategy can also be extended to convexify a signomial term containing *n* variables.

Consider a signomial term $cx_1^{\alpha} x_2^{\beta} x_3^{\gamma}$

composed of three variables, x_1, x_2, x_3, can be convexified by the following rules:

Rule 1. If $c > 0$, then let $cx_1^{\alpha} x_2^{\beta} x_3^{\gamma} = ce^{\alpha \ln x_1 + \beta \ln x_2 + \gamma \ln x_3}$.

Rule 2. If $c < 0$, $\alpha, \beta, \gamma \geq 0$, and $\alpha + \beta + \gamma \leq 1$, then $cx_1^{\alpha} x_2^{\beta} x_3^{\gamma}$ is already a convex term following Proposition 3. No convexification is required.

Rule 3. If $c < 0$, $0 \leq \alpha, \beta < 1$, $\gamma \geq 0$, $\alpha + \beta < 1$, and $\alpha + \beta + \gamma > 1$, then let $cx_1^{\alpha} x_2^{\beta} x_3^{\gamma} = cx_1^{\alpha} x_2^{\beta} y_3^{1-\alpha-\beta}$ and $y_3 = x_3^{\frac{\gamma}{1-\alpha-\beta}}$

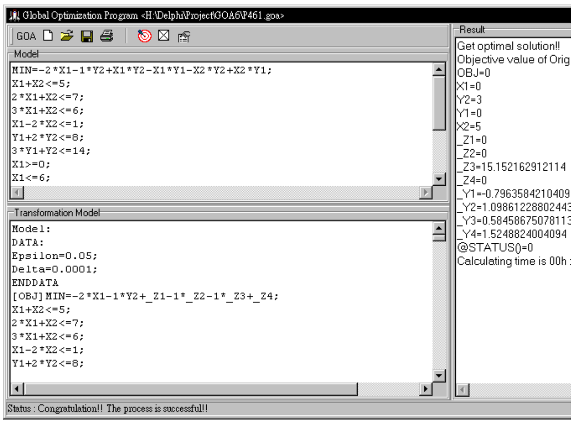


Figure 2 Transformation process

where $cx_1^{\alpha} x_2^{\beta} y_3^{1-\alpha-\beta}$ is regard as a convex term.

Rule 4. If $c < 0$, $\alpha, \beta, \gamma > 0$, and $\alpha + \beta + \gamma > 1$, then let $cx_1^{\alpha} x_2^{\beta} x_3^{\gamma} = cy_1^{\frac{\alpha}{\alpha+\beta+\gamma}} y_2^{\frac{\beta}{\alpha+\beta+\gamma}} y_3^{\frac{\gamma}{\alpha+\beta+\gamma}}$ where $y_1 = x_1^{\alpha+\beta+\gamma}$, $y_2 = x_2^{\alpha+\beta+\gamma}$, $y_3 = x_3^{\alpha+\beta+\gamma}$. $cy_1^{\frac{\alpha}{\alpha+\beta+\gamma}} y_2^{\frac{\beta}{\alpha+\beta+\gamma}} y_3^{\frac{\gamma}{\alpha+\beta+\gamma}}$ is a convex term.

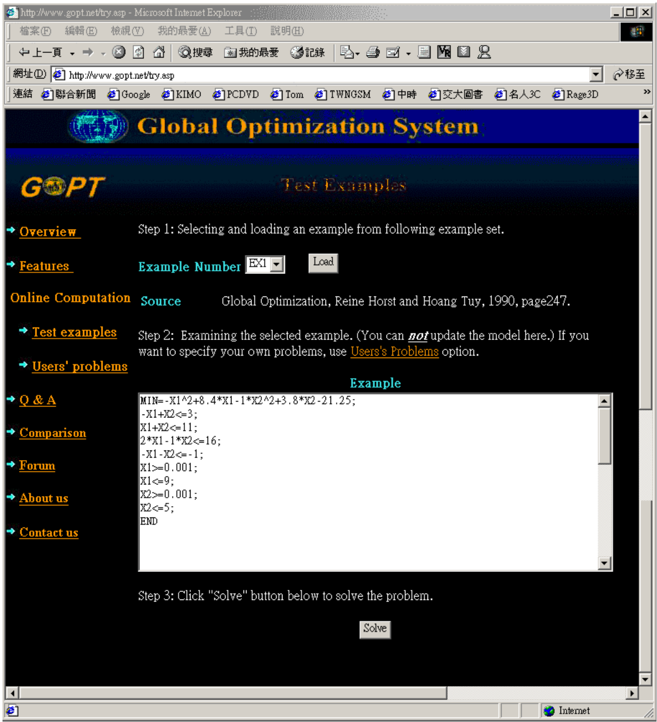


Figure 3 System interface

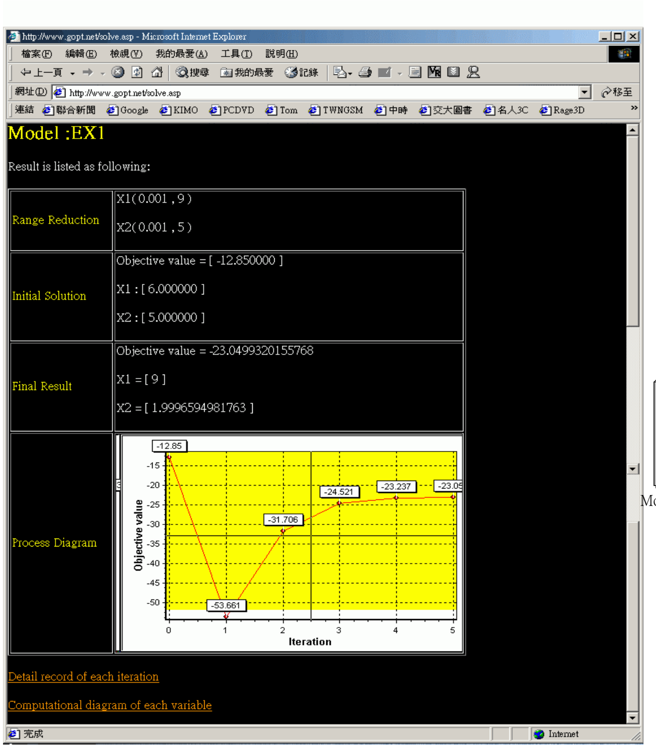


Figure 4 Computational results

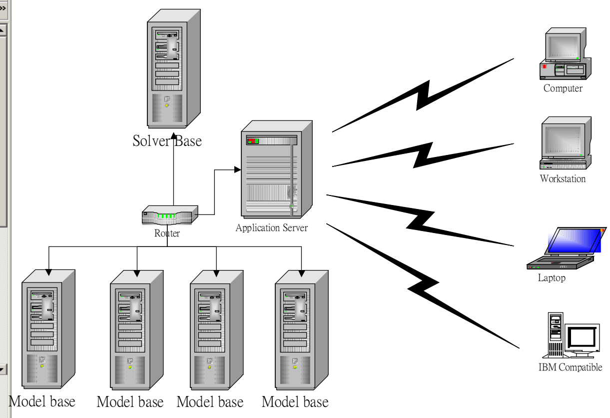


Figure 5 Distributed computation structure

六、結果與討論

This project is a three-year project and mainly to design a global optimization software and to develop the techniques for solving a signomial programming problem.

Current optimization methods of nonlinear programming problems have been reviewed and some optimization strategies such as strategies of treating non-positive variables, strategies of piecewise linearization, and strategies of convexification are developed. The proposed methods guarantee to obtain a global optimum within the tolerance. The system prototype is also done.

For solving a large scale optimization problem, we will emphasize on designing distributed computation algorithms and integrating heuristic methods in the next step. Furthermore, we will enhance the proposed algorithms for solving various optimization problems to find a global solution. The system has been announced and provided for testing and system stability and reliability are improved gradually.

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八、計畫成果自評

1、本研究已發展求解非線性規劃問題之全域最佳化方法，研究成果也已發表於國際國際知名期刊。

(i) Han-Lin Li, Jung-Fa Tsai, Nian-Ze Hu, A Distributed Global Optimization Method for Packing Problems, Journal of the Operational Research Society, 2002. (Forthcoming)

(ii) Han-Lin Li, Ching-Ter Chang and Jung-Fa Tsai, 2002, Approximately Global Optimization for Assortment Problems Using Piecewise Linearization Techniques, European Journal of Operational Research, 140, 584-589.

(iii) Jung-Fa Tsai, Han-Lin Li, and Nian-Ze Hu, 2002, Global Optimization for Signomial Discrete Programming Problems in Engineering Design, Engineering Optimization 34, 613-622.

(iv) Nian-Ze Hu, Jung-Fa Tsai and Han-Lin Li, 2002, A Global Optimization Method for Packing Problems, Journal of the Chinese Institute of Industrial Engineers.

(v) Han-Lin Li and Jung-Fa Tsai, 2001, A Fast Algorithm for Assortment Optimization Problems, Computers & Operations Research, 28, 1245-1252. (SCI)

2、協助與訓練博、碩士班研究生完成論文寫作。

(Jung-Fa Tsai, "Global Opimization of Generalized Geometric Programming Problems," PhD dissertation, National Chiao Tung University, 2003.)

3、將研究結果綜合目前廣為大家使用的數學軟體（LINGO），自行開發一套求解全域最佳化之軟體，並在網路上提供測試使用。(網址：http://www.GOPT.net）

4、目前所開發的系統能夠求解一般的多項式規劃問題，尚無法求解特殊三角函數問題。

5、為改善計算效能，以求解大規模的實際問題，系統需再整合啟發式演算法與分散式計算方法。